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A solution to unconstrained Einstein's equations for a relativistic radiation sphere

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Abstract. We describe an application of a solution to unconstrained Einstein's field equations (Ni, 2011) as a qualitative model for a relativistic radiation sphere, an object that may resemble a quasar with an extended galactic-scale corona that resembles behaviors explained by dark matter.

Keywords. Relativity, (galaxies:) quasars: general, (cosmology:) dark matter

In 2011, Ni published the solution of the Einstein (1915; 1916) field equations (EFEs), solved in course to model a stable neutron core with no upper mass limit. Ni could achieve this result because he did not establish some additional, post-Einstein postulates, which constrained ('newtonized') the general relativity (GR) in its application to model the relativistic compact objects. With the help of the Ni's solution, we can show that the upper mass limit of neutron stars is not any consequence of the GR, as generally believed, but this limit and, consequently, the black holes are the consequence of the constraint of GR. The upper mass limit does not occur in the continuum of the Ni's solution, which is the super-class to the Oppenheimer and Volkoff solutions. (For a further reasoning, see Neslušan 2015; sabotin.ung.si/∼agomboc/IAUS324_presentations/Posters/Poster28_Neslusan.pdf.)

Here, we outline the concept of stable super-massive object consisting of radiation - relativistic radiation sphere (RRS). Using the EFEs for the spherical symmetry (the same as Oppenheimer and Volkoff used) with the equation of state for the radiation, $E = 3P$ (E − energy density, P − pressure), we construct an example of such object. The behaviors of its E, u, and g_{tt} are shown in Fig. 1. Quantity u is other form of g_{rr}, whereby $u = r(1 + 1/g_{rr})/2$. g_{rr} and g_{tt} are the components of metric tensor. Note: Equation $E = 3P$ is also valid for a gas consisting of fermions, which have the rest energy negligible in comparison to their total energy.

In the E-behavior (Fig. 1a), we see a central condensation (CC) having the radius (5.55 AU) slighly larger than the radius of the Jupiter's orbit. The total energy inside the CC equals $W = 5.55 \times 10^{59}$ J ($W/c^2 = 3.10 \times 10^{12}\,\mathrm{M}_\odot$). It is such a large energy that the

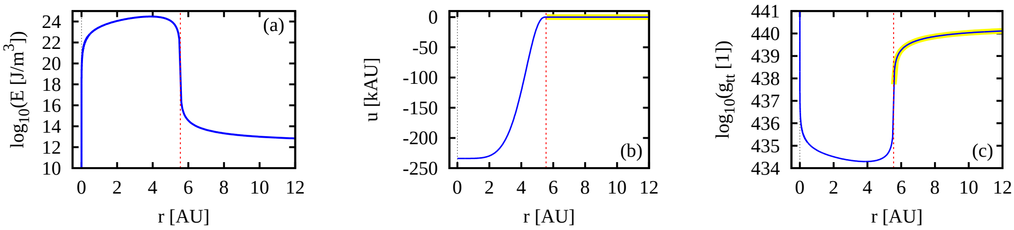

Figure 1. The behaviors of energy density, E (plot a), auxiliary function u related to g_{rr}-component of metric tensor (b), and g_{tt}-component of metric tensor (c) in an example of the RRS. The behaviors are plotted by the solid blue curves. The fit of the behavior by the corresponding outer Schwarzschild metrics in plots b and c is shown with the thick yellow curve. The dashed red vertical line indicates the border of the CC of object.

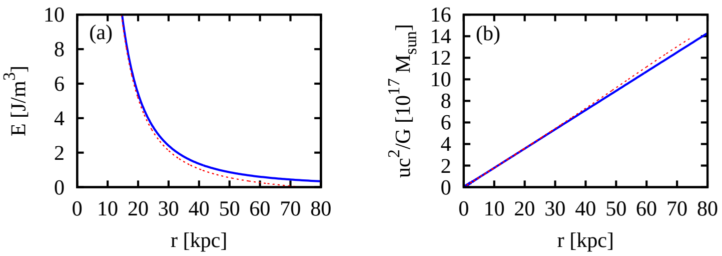

Figure 2. The behaviors of energy density, E, and quantity uc^2/G in the scale of tens of kiloparsecs. The behavior for the zero value of cosmological constant Λ is plotted with the solid blue line and that for this constant equal to $8 \times 10^{-44}\,\mathrm{m}^{-2}$ with the dashed red curve.

CC photosphere emitting the radiaton with the luminosity of bright quasar (e.g. 3C 273 having luminosity $\sim 3.0 \times 10^{40}\,\mathrm{J\,s}^{-1}$) during the age of the universe (13.799 Gyr) would spend only 2.3% of this energy.

At the border of the CC, E suddenly decreases about 6 orders of magnitude. Beyond the border, it is non-zero, but negligible, in a certain interval of distance, in comparison to that inside the CC, therefore the metrics outside can be well approximated with the outer Schwarzschild metrics (yellow curves in Fig. 1b and 1c). The trajectory of an object in a vicinity of the CC would practically be a Keplerian cone section. In our example, the orbit would be like that in the Newtonian potential generated by the central body of mass $2.8 \times 10^8\,\mathrm{M}_\odot$ (mass of 3C 273).

The energy of the RRS is spread to a large distance. In the distance scale of several kiloparsecs, the net energy of the RRS in form of a 'corona' exceeds the energy concentrated in the CC and increases linearly with the radial distance (Fig. 2a). The quantities uc^2/G (c is the speed of light and G is the gravitational constant) and g_{tt} also lineraly increase with this distance in the RRS corona when the cosmological constant Λ is zero (the behavior of uc^2/G is shown in Fig. 2b). The trajectories of some objects, e.g. stars, moving around the RRS's center in the kiloparsec-scale distance are considerably different from the Keplerian orbits. The gravitational effect by the corona resembles that we assign to the dark matter in a galaxy.

The extremely concentrated energy in the CC implies an extremely high luminosity of the CC, like that observed at quasars. In the concept of quasar as the CC of RRS, the assumption of physically problematic super-Eddington accretion, which is accepted for some quasars, would not be necessary.

Here, we presented the RRS as an hypothetical object. One can nevertheless ask if some real counterparts could form in the universe. We can speculate that the RRSs could form in the cosmological era of radiation, when the universe was filled in with a fluid similar to that inside the CC of RRS. Our modeling demonstrates that the most massive RRSs could contain enough energy, in their CCs, to survive until the present and the CCs could permanently emit the radiation with a quasar luminosity.

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