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The second rise of general relativity in astrophysics

Abolition of a postulate and solutions for the relativistic compact objects without maximum mass and an energy content larger than that implied by their gravity

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The field equations, which are the mathematical basis of the theory of general relativity, provide us with a much larger variety of solutions to model the neutron stars and other compact objects than are used in the current astrophysics. We point out some important consequences of the new kind of solutions of the field equations, which can be obtained if the astrophysical usage of general relativity is not constrained, and outline an impact of these solutions on the models of internal structure of compact objects. If general relativity is not constrained, it enables to construct the stable object, with the outer surface above the event horizon, of whatever large mass. A new concept of relativistic compact object is a consequence of newly discovered property of gravity, yielded by the field equations in a spherically symmetric configuration of matter: in comparison with the Newtonian case, a particle is more effectively attracted by a nearer than a more distant matter.

Keywords: General relativity; Oppenheimer–Volkoff limit; neutron stars.

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1. General Relativity in Astrophysics

Einstein's general theory of relativity[3,4] has been a necessary tool to describe the internal structure of relativistic compact objects (RCOs, hereinafter) like neutron stars. The mathematical basis of this theory, the Einstein field equations (EFEs, hereinafter), provide us with a variety of solutions which can be used in a modeling of the hypothetical RCOs. However, only a tiny part of these solutions has been used in describing of origin or structure of real RCOs. In this paper, we deal with the solutions which have not been used in astrophysics. We point out their existence, analyze probable cause of their ignorance, and outline the properties of the RCOs modeled using these solutions.

Within the current astrophysics, the models of neutron stars or supermassive compact objects (the meaning of this term will be clarified later) that are spherically or axially symmetric are constructed. For the sake of simplicity and larger clarity of our explanation, we will consider, in this paper, only the spherically symmetric objects.

Except of other demands, the solution regarded today as usable in the relativistic astrophysics must obey the demand that it is regularized, i.e. it implies the distribution of matter down to the object's center, where usually the energy density and pressure reach their maximum. Since there are many more solutions, as we demonstrate below, the demand can be regarded as the postulate choosing just one specific possibility.

The postulate has several consequences, which can be regarded as its different forms. All this is mutually related and we can either accept the postulate and all its consequences or the postulate and all its consequences are abolished as a whole.

Perhaps the most frequently reported consequence is the claim that the gravitational acceleration of a test particle (TP) in the gravitational field of an RCO is *linearly* proportional to the RCO mass. (According to the exact formula giving the acceleration in the general relativity, the claim is valid approximately in distances much larger than the radius of RCO's event horizon.) When we speak about the mass, we mean the quantity which is calculated, according to the well-known Einstein formula, as the ratio of energy inside the object and quadrate of the speed of light in vacuum, whereby the energy is the integral of energy density, figuring in the EFEs, through the entire volume of the object.

The postulate appears to be also closely related to the demand of the Minkowski metrics inside the spherically symmetric material shell. Further, the postulate implies that the values of g_{rr} component of metric tensor in the interval from 0 to -1, for $---+$ signature of the tensor, are excluded (or the values ranging from 0 to $+1$ are excluded for $+++-$ signature).

In the case of spherical symmetry, the first kind of solutions to describe the neutron stars was found by Oppenheimer and Volkoff[10] who utilized the work by Tolman[15] which just occurred that time. Later, more sophisticated forms of the equation of state entering the EFEs have been suggested and improved. The description of the metrics published by Tolman and used by Oppenheimer and Volkoff has, however, remained the same and we refer to this metrics as the TOV (Tolman–Oppenheimer–Volkoff) sub-class of solutions of the EFEs.

In 2011, Chinese researcher Jun Ni[8] published a kind of solutions of the EFEs which appeared to be the super-class of solutions with respect to the TOV sub-class. The extent of the TOV and Ni's classes of solutions is schematically illustrated in Fig. 1. (The figure is described more in Sec. 2.) The blue area continues beyond the right and upper shown borders to infinity. In Ni's solutions, the orientation of the gravity is normal, i.e. attractive everywhere, and energy density and pressure in each part of the object are positive, but the postulate requiring the distribution of matter down to the RCO's center is not obeyed.

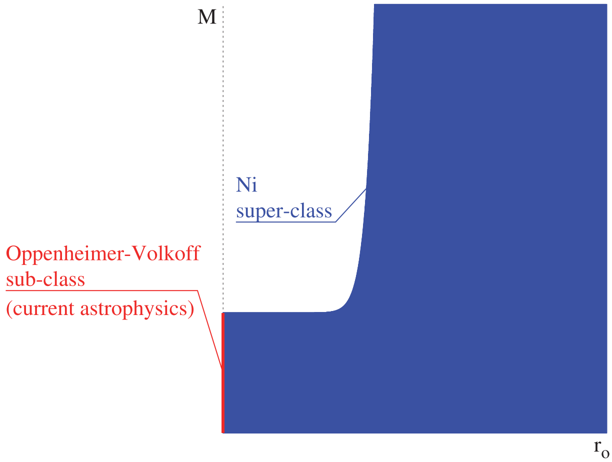

Fig. 1. (Color online) A scheme illustrating the extent of currently acceptable and used (red vertical abscissa) and further possible (blue area) solutions of Einstein field equations to construct the models of stable, spherically symmetric, relativistic compact objects. The whole class of solutions with the attractive orientation of gravity and positive energy density and pressure was found by Ni[8] (Ni super-class of solutions). A sub-class, bordering the super-class at left, was found by Oppenheimer and Volkoff.[10]

What is, however, a reason for the postulate? Does its ignorance result in a violation of one or more well-established laws in physics, like energy-conservation or momentum-conservation laws? In every case, there should be given a clear reason for the rejection of a huge majority — Ni's super-class — of the EFEs' solutions. In this paper, we attempt to develop a discussion about a possible applicability of Ni's super-class in relativistic astrophysics.

We do not find any problem, any violation of a physical law, when the postulate is not obeyed. We give several arguments that there is no reason, in fact, to establish this postulate. We describe some consequences of its abolition. Using Ni's solutions, we can construct the models of the stable RCOs.[5–7] It can be expected that we are able to explain several mysterious phenomena in the relativistic astrophysics with the help of these models. It seems there are many more applications of general relativity (GR) in astrophysics than people have believed.

2. Ni's Class of the Solutions

In 2011, Ni[8] solved the same problem as Oppenheimer and Volkoff[10] before the Second World War. The single difference in his approach to numerically solve the EFEs was the starting point of the numerical integration of the TOV equations. While Oppenheimer and Volkoff started the integration at the center of the object, Ni started this integration in a finite star-centric distance and performed the integration in two parts: stepping inward and outward. (So, we should, maybe, rather speak about Ni's strategy than about Ni's solution.) Interestingly, not only outward,

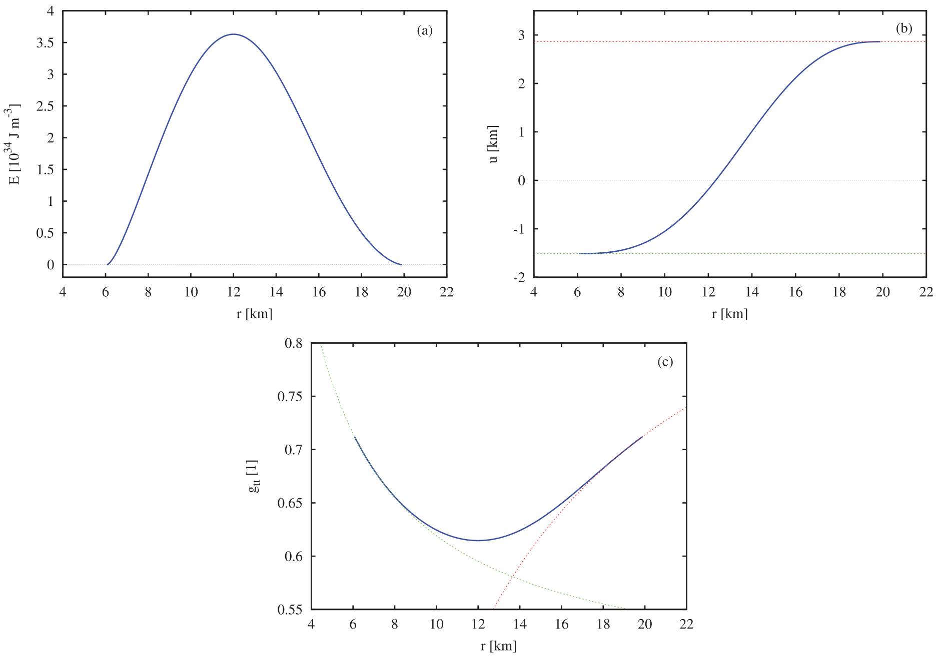

Fig. 2. (Color online) The behavior of energy density (plot (a)), auxiliary metric function u (b), and component g_{tt} of metric tensor (c) in an example of RCO. Inside the RCO body, the behavior is obtained via numerical integration of the field equations (thick, solid, blue curves). In plots (b) and (c), u and g_{tt} functions yielded by the outer Schwarzschild solution, which are tailored to the RCO-interior functions, are also shown with a dotted straight line or curve. The function drawn with the red (green) line or curve is tailored to the interior function at the outer (inner) physical border of the RCO. In the intervals of distances $r \leq R_{\rm in}$ and $r \geq R_{\rm out}$, function u is constant, whereby $u = u_{\rm in}$ in the region $r \leq R_{\rm in}$ and $u = u_{\rm out}$ in $r \geq R_{\rm out}$.

but also the inward processed integration ended with the zero energy density and pressure in a finite star-centric distance. This result implies an existence of not only an outer, but also an inner physical border of the RCO.

Constructing several hundreds of models of neutron stars considering Ni's solutions as well as the Oppenheimer–Volkoff neutron stars,[5] the following details were found. If one does not devote any special attention to the initial conditions, the numerical integration of the EFEs from a finite object-centric distance downward almost always ends with the zero energy density and pressure in a finite star-centric distance. There is only a single combination of initial values entering the integration, for the RCO of given mass, which ends with the finite (maximum) energy density and pressure in the object's center. We can force this solution to occur if we start the integration at the center.

Otherwise, both energy density and pressure are zero at the inner physical border of the object, with radius $R_{\rm in}$, and increase with the increasing radial distance up to certain distance, $r_{\rm o}$, where they reach the maximum values. The behavior of the energy density in an example of RCO is shown in Fig. 2(a). In this example,

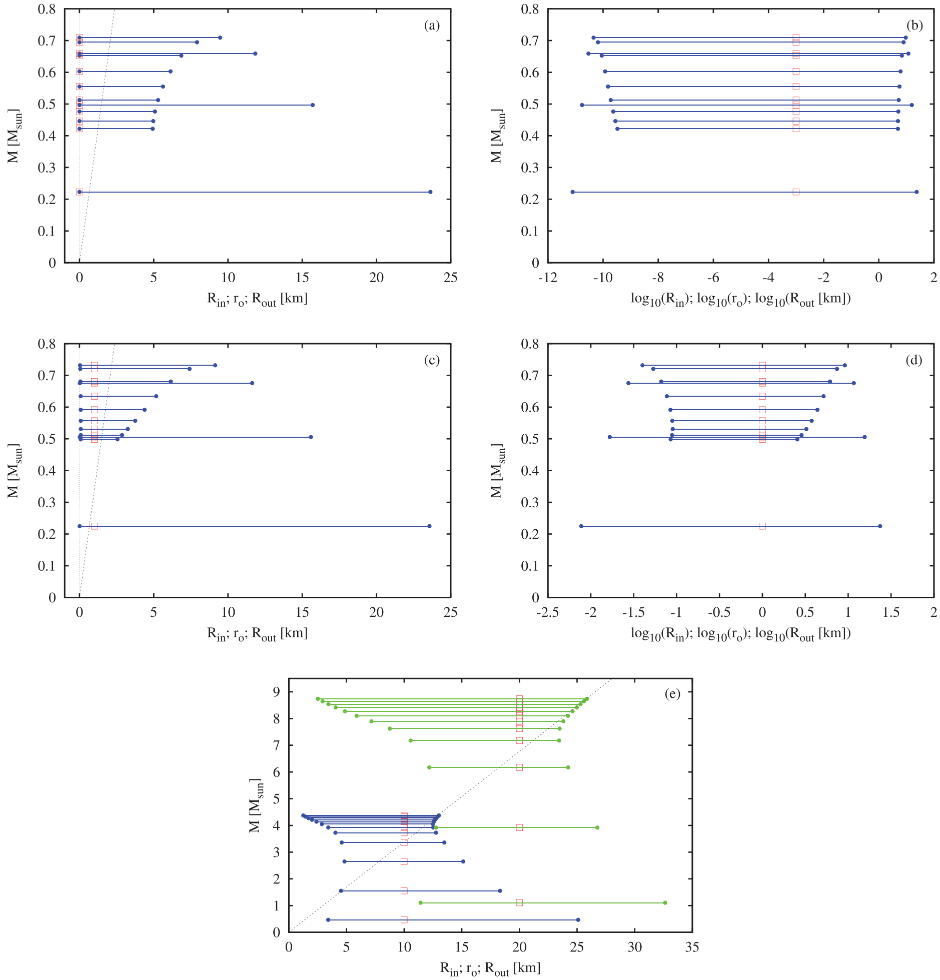

Fig. 3. (Color online) The size, from inner to outer physical border, of RCOs consisting of cold, degenerated, neutron gas of various masses. The physical borders are shown with the full blue (or green in plot (e)) circles connected with the horizontal abscissa of the same color for a given sequence of models. The distance of zero gravity and maximum energy density and pressure, $r_{\rm o}$, is indicated with an empty red square on the corresponding abscissa. Specifically, we show the extent for four values of the distance $r_{\rm o}$ and a sequence of RCO masses: $r_{\rm o} = 1$ m (plot (a) in linear and plot (b) in decadic-logarithm scale), $r_{\rm o} = 1$ km (plot (c) in linear and plot (d) in decadic-logarithm scale), $r_{\rm o} = 10$ km (plot (e), only in the linear scale; blue abscissas), and $r_{\rm o} = 20$ km (plot (e), only in the linear scale; green abscissas). The dotted black straight line shows the dependence of the Schwarzschild gravitational radius on the mass.

$r_{\rm o} = 12$ km. In radial distance $r > r_{\rm o}$, energy density and pressure decrease and vanish at the outer physical border of RCO, with radius $R_{\rm out}$.

In Fig. 3, there are four sequences of RCO models, for $r_{\rm o} = 1$ m, 1 km, 10 km, and 20 km. Each sequence contains the models for a set of the object's masses. The similar sequences of RCO models constructed for $r_{\rm o} = 1$ cm, 1 mm, or, generally for $r_o \to 0$ are indistinguishable from the corresponding models for $r_o = 1$ m in the linear scale (Fig. 3(a)).

If distance r_o is small, like in Figs. 3(a)–(d), there are two solutions for the same mass. Interestingly, we can construct the models with mass only up to a certain maximum limit. (For this limiting mass and given r_o, there is only single solution.) This maximum mass limit is consistent with the conclusion drawn by Oppenheimer and Volkoff[10] and the proof of the limit by Rhoades and Ruffini[12] and several other authors later for $r_o \to 0$.[a]

However, if r_o exceeds a certain value, there is only a single solution for given mass (Fig. 3(e)). In the case of relatively low-mass objects of this kind that have the same r_o, their volume decreases, i.e. inner radius increases and outer decreases, with increasing mass. When the mass of these equal-r_o objects exceeds a certain critical value, then the object volume increases, i.e. the inner radius decreases and outer increases, with increasing mass.

At this increase, it seems that the outer radius asymptotically approaches the Schwarzschild gravitational radius for given object, R_g, whereby $R_{\mathrm{out}} \to R_g$ in the limit $M \to \infty$ (the right full circles corresponding to the outer RCO radius in Fig. 3 approach the black dotted straight line showing the Schwarzschild gravitational radius of the RCO of given mass). At the moment, this asymptotic approach is not proved. It is only an indication deduced from the observations of the relationship between the RCO's mass and RCO's outer radius in a large set of the numerical models of stable RCOs. If it was once proved, it would mean that a collapse of the object to shrink within its event horizon is energetically forbidden (only infinitely massive objects could collapse onto the horizon) and black holes would be only some fictitious objects yielded by the old theory. The skepticism of Einstein and Eddington concerning the collapse of an object below its event horizon may still occur reasonable.

The collapse of a massive stellar object below its event horizon was suggested by Oppenheimer and Volkoff[10] when they did not find any solution of the EFEs for the object without an internal source of energy having the mass above a certain limit, which is today known as the Oppenheimer–Volkoff limit. These authors did not, however, try to construct a model of neutron star starting the numerical integration of relevant equations in a nonzero distance. This is an analogy of for example, the following situation. Imagine that one wants to construct a model describing the structure of a common, non-relativistic star with mass 1 $M_\odot$. If he or she starts the integration of the well-known equations of stellar structure in, say, star-centric distance of the Venus' orbit in the Solar System, he or she obviously obtains a model of red-giant star, but no model of main-sequence star of 1 $M_\odot$. If one insisted on the universality of the starting distance at the Venus' orbit, he or she would conclude that there do not exist the main-sequence stars of 1 $M_\odot$ in the universe.

[a]The TOV equations contain the fractions that are divergent for $r = 0$, therefore we speak only about approaching the center.

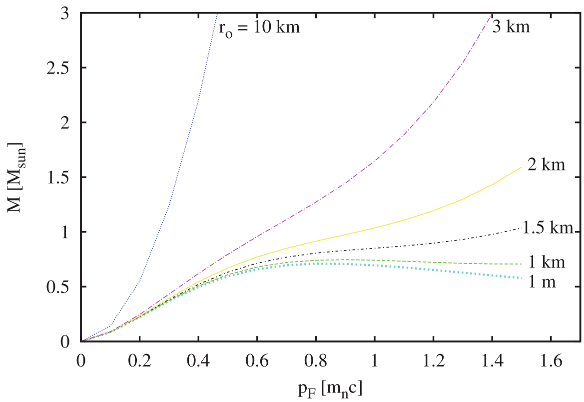

Fig. 4. (Color online) The dependence of the mass of RCO on the input value of Fermi impulse, p_F, in the starting point of numerical integration in distance r_o. The dependence is shown for several sets of models, each set with the same distance r_o, in which the energy density and pressure reach their maximum. The specific values of distance r_o label the corresponding curves.

It has often been claimed that the collapse of a dead object with the mass above the Oppenheimer–Volkoff limit below its event horizon was proved in the work by Oppenheimer and Volkoff[10] and/or the work by Oppenheimer and Snyder.[9] A careful study of these works reveals that this is not true. Oppenheimer and Volkoff did not find the solution for the objects with mass above a certain mass limit (and, remember, for $r_\mathrm{o} \to 0$). However, the argument that somebody did not find a solution does not mean that the solution can never be found.

Oppenheimer and Snyder did not either prove any collapse of a massive star without the internal energy source. Reading their paper one can find that they accepted the claim made in the previous paper by Oppenheimer and Volkoff that, after the exhaustion of thermonuclear source of energy, a massive enough star will contract indefinitely. Using this claim as an assumption entering their work, Oppenheimer and Snyder *described* the collapse. One can see that the description cannot be regarded as any proof because there is not, in their work, considered any gradient of pressure which could resist the gravitational attraction. In a proof, one must demonstrate that there is an unbalance between the gravity and gradient of pressure, whereby the former is stronger, and this is impossible if no gradient of pressure is evaluated.

The proof of maximum mass of neutron star by Rhoades and Ruffini[12] is correct, of course, but these authors implicitly assumed the object as the full sphere, therefore the proof is valid only for the objects with $r_\mathrm{o} \to 0$.

The problem of the maximum mass can be clarified considering a large set of RCO models constructed using Ni's solution of the EFEs. For each model, we started the numerical integration in distance r_o. In more detail, we fixed the value of r_o and constructed a set of models varying the initial Fermi impulse, p_F, in r_o. The dependence of mass, M, on the Fermi impulse, p_F, is shown in Fig. 4. Individual curves in this figure correspond to the specific values of r_o. We can see that the dependence $M = M(p_F)$ has actually a maximum for r_o up to ~1 km. In the resolution of Fig. 4, the curves for $r_o < 1$ m would be practically indistinguishable from that for $r_o = 1$ m (dotted cyan curve; this curve corresponds to that in Fig. 1 in the paper by Oppenheimer and Volkoff).[10] Our set of RCO models also supports the conclusion that there is the maximum mass, but only for those RCOs with r_o approximately smaller than 1 km (and, therefore, also those with $r_o \to 0$). This is, however, only a part of all solutions offered by the GR.

In Fig. 4, we can see that the RCO of given mass can acquire, theoretically, a variety of configurations, with various values of r_o. However, when an object constituted by a given, fixed numbers of particles (given rest mass, in fact) is formed, what a configuration does it acquire in reality? It is reasonable to apply the principle of minimum energy and answer this question: the object with a given rest mass will acquire the configuration with the minimum total energy.

The total energy of neutron star or whatever RCO, W, is the integral of energy density, E, through the whole volume of the RCO body, i.e.

$$W = 4\pi \int_{R_{\rm in}}^{R_{\rm out}} E r^2 \, dr. \tag{1}$$

To find the minimum total energy, we need to study the relationship between W and r_o for a variety of RCO models with the same number of particles constituting the object. Such RCOs have identical rest mass (sum of the rest masses of the constituting particles) and, therefore, the rest energy, W_o, whereby

$$W_o = 4\pi m_n c^2 \int_{R_{\rm in}}^{R_{\rm out}} n r^2 \sqrt{-g_{rr}} \, dr. \tag{2}$$

In relation (2), m_n is the rest mass of neutron, c is the speed of light in vacuum, n is the number density of neutrons, and $4\pi r^2 \sqrt{-g_{rr}}\, dr$ is the proper volume of the thin spherical layer with radius r. The number density can be calculated as[2]

$$n = \frac{8\pi}{3h^3} (m_n c)^3 \sinh^3 \frac{\tau}{2}. \tag{3}$$

In the last relation, h is the Planck constant and τ is an auxiliary function related to the Fermi impulse, p_F, as

$$\tau = 4 \ln \left[\frac{p_F}{m_n c} + \sqrt{1 + \left(\frac{p_F}{m_n c} \right)^2} \right]. \tag{4}$$

Energy density, E, depends on variable τ as

$$E = \frac{\pi m_n^4 c^5}{4h^3} (\sinh \tau - \tau). \tag{5}$$

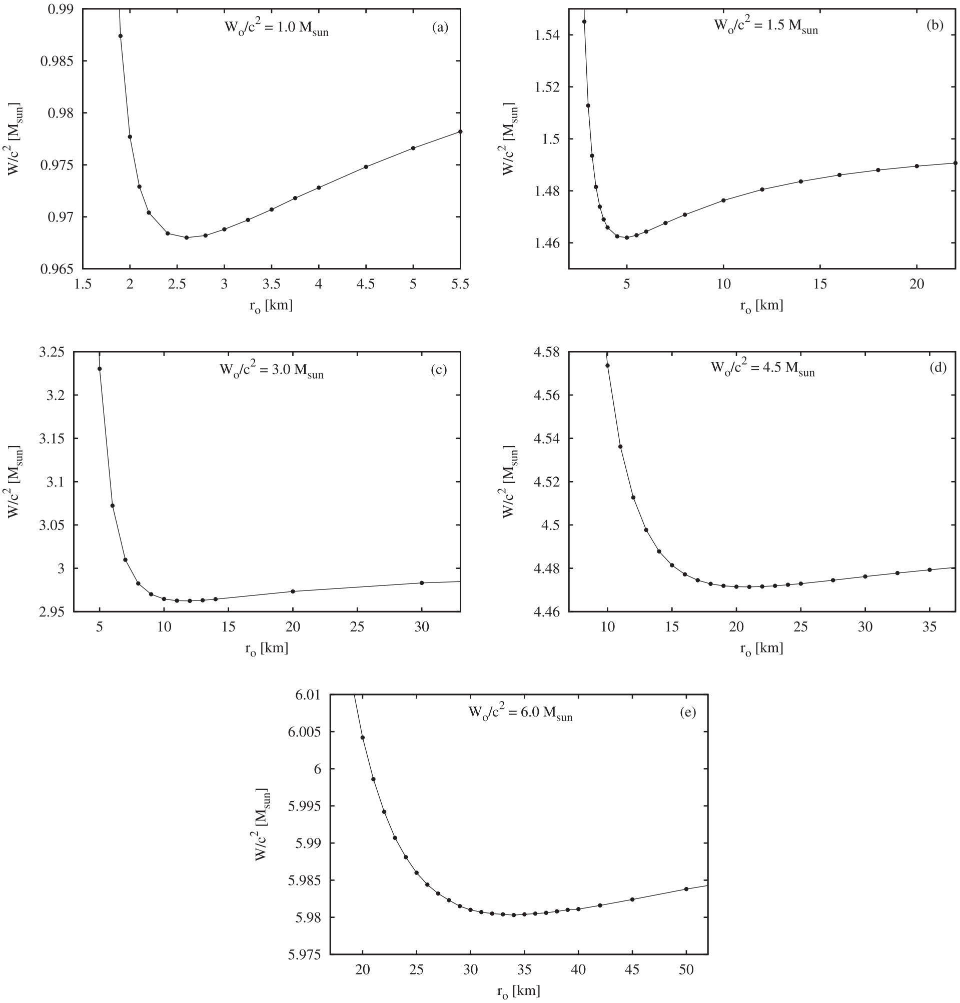

Fig. 5. The dependence of total energy, W, of the RCOs constituted by the same number neutrons and, therefore, the same rest energy on the distance r_{o} of zero gravity. Specifically, the dependence is shown for the value of rest energy corresponding to the rest mass equal to 1.0 (plot (a)), 1.5 (b), 3.0 (c), 4.5 (d), and 6.0 $\mathrm{M}_{\odot}$ (e).

For several values of the RCO's rest energy, the relation between W and r_{o} is shown in Fig. 5. Actually, the dependence of W on r_{o} has a single minimum. We can expect that the RCO with the given number of constituting particles (neutrons in the case of the considered equation of state) will acquire just the configuration with the value of r_{o} corresponding to the minimum total energy, W. (Since the rest energy, W_{o}, is the same in all models of given sequence, the minimum free energy, $W - W_{\mathrm{o}}$, occurs at the same value of r_{o} as the minimum of the total energy, W.) The extent of the minimum-energy RCOs is shown in Fig. 6.

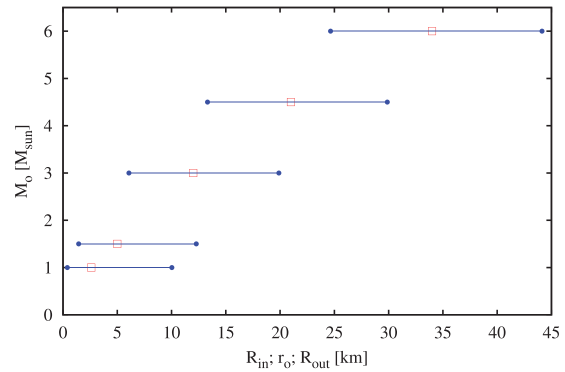

Fig. 6. (Color online) The size, from inner to outer physical border, of RCOs consisting of cold, degenerated, neutron gas of various rest masses. Only the models with the minimum total energy for a given rest mass are shown. The physical borders are shown with the full blue circles connected with the horizontal abscissa of the same color. The distance of zero gravity and maximum energy density and pressure, $r_{\rm o}$, is indicated with an empty red square on the corresponding abscissa.

3. Gravity Inside the Spherically Symmetric Material Shell

To clarify the unusual behavior of state quantities in an RCO constructed on the basis of Ni's solution of EFEs, let us analyze the curvature and, hence, a strength of gravitational action in a thin, spherically symmetric, material shell (SSMS) as yielded by the EFEs. Of course, one can find the answer in many textbooks on GR that the metrics inside the SSMS is the Minkowski, i.e. flat metrics, implying zero gravitational attraction on a TP. However, if we do not postulate the Minkowski metrics, the EFEs alone, via Ni's solution, provide us with a much larger variety of metrics describing the curvature in the internal void of the SSMS. Below, we analyze just these further possibilities.

We note that the postulate of the Minkowski metrics inside an SSMS cannot mean any disappearance of gravitational action of individual parts of the shell's material. Gravity of nonzero mass entities cannot disappear. The postulate obviously means that we require such a configuration of spacetime inside the SSMS which results in zero net gravity. A partial gravity of the matter in every volume of shell is required to be exactly eliminated by the gravity of other shell's volume as in a non-relativistic shell with the gravity described by Newtonian physics (see a more detailed explanation below).

Within the GR, there is not known any principle that could lead, in general, to the special configuration of spacetime curvature resulting in the mutual elimination of partial gravitational actions and zero total net gravity in the shell. On the contrary, the gravity inside the shell is described by Ni's solution within this theory. Hence, the postulate of the Minkowski metrics inside the SSMS enters astrophysics from outside of the GR.

When investigating the actual net gravitational action inside the internal void of SSMS, let us deal, at first, with the SSMS in the Euclidean space of Newtonian

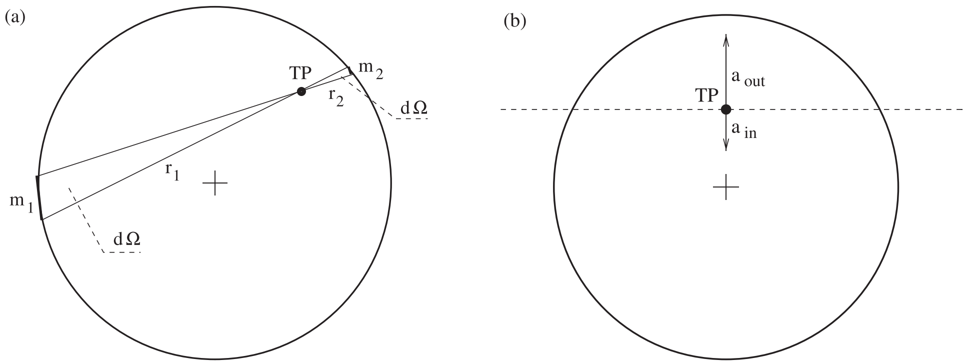

Fig. 7. The schemes to explain some aspects of the gravitational attraction of a test particle by an infinitesimally thin, spherically symmetric, material shell (shown with a circle in each scheme). The scheme in plot (a) helps us to analyze the gravitational acceleration of the test particle due to the action of two small volumes, having masses m_1 and m_2 and being in distances r_1 and r_2, respectively, from the particle. From the position of the particle, both volumes are seen within the same infinitesimal space angle $d\Omega$. In plot (b), the shell is divided by a plane (indicated with the dashed line) crossing the test particle and perpendicular to its radius vector to two parts. The arrows oriented outward and inward indicate the net gravity of the first or second part of the shell acting on the particle. In a curved space, the size of the outward oriented acceleration, $|a_{\rm out}|$, is larger than the size of inward oriented acceleration, $|a_{\rm in}|$, according to the Ni's solution of the field equations.

physics. In this case, the net gravitational acceleration of a TP, situated wherever inside the shell, can easily be calculated and equals zero. Specifically, we can consider two infinitesimal volumes of the shell seen from the position of the TP within the infinitesimally small space angle $d\Omega$, whereby the second volume is exactly in the opposite direction than the first volume with respect to the TP (Fig. 7(a)). Mass m_1 of the first volume is proportional to the size of the space angle $d\Omega$. Since in the spherical coordinate system $O(r\vartheta\varphi)$, $d\Omega = r_1^2 \sin\vartheta\, d\vartheta\, d\varphi$, mass $m_1 = \rho r_1^2 \sin\vartheta\, d\vartheta\, d\varphi\, dr$, where ρ is the mass density of the shell and r_1 is the distance of the first volume from the TP. The size of the acceleration due to mass m_1, according to the Newton gravitational law, is linearly proportional to the mass and reciprocally proportional to the quadrate of distance r_1, i.e. to r_1^{-2}. Therefore, distance r_1 vanishes in the calculation of acceleration. The same is valid for the size of acceleration due to mass m_2 of the second volume. Thus, the acceleration does not depend on the position of the TP inside the shell. Both accelerations are of equal magnitude, but oppositely oriented, therefore their sum is zero. For whatever small volume of the shell, there can be found the corresponding volume in the opposite direction which eliminates the gravity of the first volume. As a consequence, the net gravitational acceleration is zero.

Let us further consider a TP in the case of significantly curved space described by the GR. Specifically, we consider the TP being in the rest which is attracted by the infinitesimally small volume with mass m_1. This volume can be regarded as point-like mass particles with this mass. Except for the TP and volume, no other object is present. To calculate the acceleration of the TP, we use the equation of geodesic

$$\frac{d^2x^\alpha}{ds^2} = -\Gamma^\alpha_{\beta\gamma}\frac{dx^\beta}{ds}\frac{dx^\gamma}{ds}, \tag{6}$$

which can be re-written using relation $dx^\alpha/ds = (dt/ds)(dx^\alpha/dt)$ (see, e.g. Straumann, 2013, p. 59)[14] to

$$\frac{d^2x^\alpha}{dt^2} = \left(\Gamma^4_{\beta\gamma}\frac{dx^\alpha}{dt} - \Gamma^\alpha_{\beta\gamma}\right)\frac{dx^\beta}{dt}\frac{dx^\gamma}{dt}. \tag{7}$$

Because of the considered spherical symmetry, the TP accelerates in radial direction. The radial component of its acceleration is

$$\begin{aligned}\frac{d^2r}{dt^2} &= (-\Gamma^1_{11} + \Gamma^4_{14} + \Gamma^4_{41})\left(\frac{dr}{dt}\right)^2 - \Gamma^1_{22}\left(\frac{d\vartheta}{dt}\right)^2 - \Gamma^1_{33}\left(\frac{d\varphi}{dt}\right)^2 - \Gamma^1_{44}c^2\\ &= \left(-\frac{1}{2}\frac{d\lambda}{dr} + \frac{d\nu}{dr}\right)\left(\frac{dr}{dt}\right)^2 + re^{-\lambda}\left(\frac{d\vartheta}{dt}\right)^2 + r\sin^2\vartheta\, e^{-\lambda}\left(\frac{d\varphi}{dt}\right)^2 - \frac{c^2}{2}e^{\nu-\lambda}\frac{d\nu}{dr},\end{aligned} \tag{8}$$

where we used usual denotation $g_{11} = g_{rr} = -e^\lambda$ and $g_{44} = g_{tt} = e^\nu$. In the considered static case, $\lambda = \lambda(r)$ and $\nu = \nu(r)$. For the TP being in rest, i.e. $dr/dt = d\vartheta/dt = d\varphi/dt = 0$, the formula reduces to

$$\frac{d^2r}{dt^2} = -\Gamma^1_{44}c^2 = -\frac{c^2}{2}e^{\nu-\lambda}\frac{d\nu}{dr}. \tag{9}$$

Oppenheimer and Volkoff[10] replaced auxiliary metric function λ with other auxiliary metric function u related to the latter as

$$u = \frac{1}{2}r(1 - e^{-\lambda}). \tag{10}$$

In the vacuum outside the shell's body, this function is constant. Its behavior inside the RCO body in an example of RCO is shown in Fig. 2(b). In the limit of weak field, i.e. when $r \gg |u|$, the relations of the GR must converge to the corresponding relations in the Newtonian physics and this convergence requires the relation

$$u = \frac{c^2}{G}m \tag{11}$$

between u and the mass of gravitating object, m. G is the gravitational constant.

Obviously, the metrics in a vicinity of point-like volume is the outer Schwarzschild metrics (OSM)[13] with $e^{-\lambda} = e^\nu = 1 - 2u/r$, therefore relation (9) can be given in the more explicit form

$$\begin{aligned}\frac{d^2r}{dt^2} &= -\left(1 - \frac{2u}{r}\right)\frac{c^2u}{r^2} = -\left(1 - \frac{2Gm}{c^2r}\right)\frac{Gm}{r^2}\\ &= -G\rho\sin\vartheta\, d\vartheta\, d\varphi\, dr + \frac{2G^2\rho^2}{c^2}r(\sin\vartheta\, d\vartheta\, d\varphi\, dr)^2.\end{aligned} \tag{12}$$

Using the last relation, the formally calculated difference of the sizes of accelerations due to both mass m_2 and mass m_1 is

$$\left|\frac{d^2r_2}{dt^2}\right| - \left|\frac{d^2r_1}{dt^2}\right| = \frac{2G^2\rho^2}{c^2}(r_1 - r_2)(\sin\vartheta\, d\vartheta\, d\varphi\, dr)^2. \tag{13}$$

This difference is positive for $r_1 > r_2$, therefore the size of the acceleration in the direction of the second volume is larger than that in the direction of the first volume. The second volume is closer to the TP, therefore the acceleration is more effective in the direction of nearer matter.

The GR is a nonlinear theory, therefore the above calculation of the size of acceleration is valid only in two separate cases, (1) when the isolated first volume and TP constitute the system of interacting entities and (2) when the isolated second volume and the TP constitute the system. If we considered the system consisting of both masses, m_1 and m_2, and the TP and we could not calculate the resultant net acceleration, $|d^2r_2/dt^2| - |d^2r_1/dt^2|$, as the simple algebraic sum of the partial results. Because of the nonlinearity of GR, the result for the isolated first volume and TP is no longer valid when the second volume is present. Nevertheless, we introduced relation (13) giving the formal difference of the partial accelerations, since it indicates, more transparently than the rigorous calculation, that (1) the acceleration in the void inside the relativistic SSMS can be expected to be nonzero (in contrast to the Newtonian physics), and (2) it is likely oriented in the direction of matter which is closer to the TP. The actual acceleration due to the whole shell can rigorously be obtained solving the EFEs for the appropriate distribution (in shell) of the matter, which was first done by Oppenheimer and Volkoff and later, with an other strategy, by Ni.

Ni's solution of the EFEs implies the outward oriented net gravitational attraction inside the relativistic SSMS and this fact can be seen in the behavior of the metrics in the shell's internal void tailored to the metrics gained via the numerical integration of the EFEs for the shell's body. The former metrics appears to be (as one expects taking into account the Birkhoff theorem[1]) the OSM, $g_{rr} = -(1 - 2u_{\mathrm{in}}/r)^{-1}$ and $g_{tt} = K_{\mathrm{in}}(1 - 2u_{\mathrm{in}}/r)$, but with $u_{\mathrm{in}} < 0$. K_{in} is an integration constant. If $u_{\mathrm{in}} < 0$, acceleration (12) can be rewritten as

$$\frac{d^2r}{dt^2} = -\left(1 - \frac{2u_{\mathrm{in}}}{r}\right)\frac{c^2u_{\mathrm{in}}}{r^2} = \left(1 + \frac{2|u_{\mathrm{in}}|}{r}\right)\frac{c^2|u_{\mathrm{in}}|}{r^2}. \tag{14}$$

Its orientation implied by $d^2r/dt^2 > 0$ is opposite that of the TP in the external vacuum with $u = u_{\mathrm{out}} > 0$, which is (relation (12)) $d^2r/dt^2 < 0$.

If it is postulated that $r_{\mathrm{o}} = 0$, then function $u \geq 0$ everywhere. It means that the values of g_{rr} component of metric tensor in the interval from -1 to 0 are excluded as one can find analyzing relation (10).

The outward oriented acceleration of the TP inside the relativistic SSMS (except of the exact center) can also be deduced in the following way. Let us divide the SSMS with a plane passing through the TP and perpendicular to its radius vector

(Fig. 7(b)). The plane separates the upper and lower parts of the SSMS. It is obvious that the net gravitational action of upper (lower) part must be oriented outward (inward) from (to) the shell's center. The gravity of each small volume of the upper part is larger than the corresponding volume of the lower part in the opposite direction, because the distance between the upper volume and TP is always shorter than that between the lower volume and TP. Hence, the net gravity by the upper part must be larger than the net gravity of lower part (Fig. 7(b)). It implies that the total net acceleration is oriented upward.

The outward oriented gravity of relativistic SSMS means that the spherical layers of an RCO with the radii larger than the RCO-centric distance of TP, i.e. the upper layers, attract the TP outward from the RCO center (in contrast to a Newtonian spherically symmetric object, where the net gravity of upper layers is zero). The layers with the radius shorter than the size of the TP's radius vector, i.e. the lower layers, attract the TP toward the RCO center, of course (as in the Newtonian object). Consequently, if the TP is situated inside the body of the RCO (Ni's hollow sphere) and near its outer border, then the net mass of lower layers is much larger than the net mass of the upper layers and gravity of the former overwhelms the gravity of the latter. The TP is attracted toward the sphere's center (Fig. 8; the TP in the uppermost position). On the contrary, if the TP is situated inside the RCO body, but near its inner border (the bottom position of the TP in Fig. 8), the gravity of relatively large net mass of upper layers dominates over that of small net mass of lower layers and the TP is attracted outward from the center. If the TP is situated in distance r_o from the center of the RCO, then the net gravity of lower layers is just balanced by that of upper layers and the total net gravity is zero.

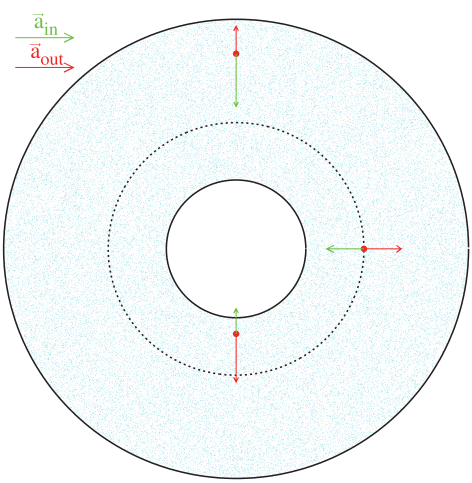

Fig. 8. (Color online) The scheme indicating the acceleration of test particle (red dot) due to the net gravity of inner ($\mathbf{a}_{in}$; green arrow) and outer ($\mathbf{a}_{out}$; red arrow) layers of a RCO. The acceleration is analyzed in three positions of the test particle inside the RCO's body. The dotted circle indicates the distance r_o of zero gravity, where $|\mathbf{a}_{in}| = |\mathbf{a}_{out}|$.

In distance $r < r_o$, every volume of RCO matter is attracted outward from the center and RCO is kept stable by the gradient of pressure which is opposite relatively to that in a distance $r > r_o$ (or in the Newtonian-physics objects).

Eventually, we note that the demand, that the metrics in the internal cavity of SSMS is the Minkowski metrics, faces the problem of discontinuity of metrics in the inner physical border of the shell. In the Minkowski metrics, the radial and time components of metric tensor equal $-1/g_{rr} = g_{tt} = 1$. In an infinitesimally thin SSMS, this metrics is demanded to be smoothly tailored to the OSM, which is generally given as $-1/g_{rr} = g_{tt} = 1 + C/r$, where C is a nonzero constant if the mass of the SSMS is nonzero. Such a tailoring is impossible. In the case of a thick SSMS, the behavior of metric tensor is given by Ni's solution of the EFEs, whereby also converges to the OSM at the inner border of the shell with a nonzero constant C. The smooth tailoring is impossible also in this case.

4. The Central Singularity

As mentioned in Sec. 3, the metrics in the void of relativistic SSMS and, hence, in the internal cavity of a RCO, is the OSM characterized with $-1/g_{rr} = g_{tt}/K_{\text{in}} = 1 - 2u_{\text{in}}/r$. This metrics has a singular point in $r = 0$, since $g_{tt} \to \infty$ for $r \to 0$. In the Ni's solution of the EFEs, the stable RCO has no event horizon, therefore this singularity is a naked singularity. Except of the Big-Bang singularity, the naked singularity is forbidden by the cosmic censorship.[11]

When analyzing the singularity, we have to realize that the metrics in the internal cavity of RCO is shaped by the energy contained in the matter constituting the RCO and this energy is finite and fixed for a given RCO. Intuitively, the singularity likely occurs due to an action of a finite entity on a zero volume (of the sphere having radius r, whereby $r \to 0$).

Since a TP is accelerated outward from the center in the RCO internal cavity, the central singularity is the Big-Bang type singularity, which is not in any conflict with the cosmic censorship. We emphasize that the TP is not repelled by anything in the point $r = 0$. The latter is a point of empty space, which cannot act on any particle in its vicinity. The active agent accelerating the particle is the circumambient matter of the RCO.

The singularity due to a finite action cannot be true singularity. Let us do a thinking experiment: we shoot a particle, situated in the cavity, toward the center. Since the GR is not linear theory, it is difficult to describe the behavior of the particle with an energy from the whole energy interval, from zero to infinity. However, the behavior can be easily deduced in two limiting cases. If the kinetic energy of the particle is so small that its influence on the metrics in the cavity is negligible in comparison to that of the matter constituting the RCO, the shot particle approaches the center at a finite distance and is returned back by the gravity of the RCO matter. On the contrary, if the energy is so high that the influence of the particle highly dominates over that of the RCO matter, then the metrics inside the cavity is reshaped by the particle; it becomes the OSM with respect to the particle and the central singularity simply disappears. Then the particle can pass through the point in $r = 0$, but this point is no longer any singular point. The experiment implies that the central singularity is only an abstract singularity, which figures in our mathematical description of cavity metrics, but it can never be visited by any material object and, hence, detected by any observer.

5. Representation of Function u

In course of gauging the metric quantities figuring in the GR to reach a convergence of this theory to the Newtonian gravitation in the limit of a weak gravitational field, function u established by Oppenheimer and Volkoff (relation (10)) must be related to mass m according to relation (11). This gauging resembles, for example, that of relativistic mass, m, with the corresponding rest mass, $m_{\rm o}$, which are related according to relation $m = m_{\rm o}/\sqrt{1 - v^2/c^2}$. If we know the rest mass (Newtonian physics), mass m of an object can be found as $m = m_{\rm o}$ in the situation of a motion of object with a non-relativistic speed, v, i.e. a speed negligible in comparison to the speed of light, c.

For a relativistic speed, identity $m = m_{\rm o}$ is, however, no longer valid and, in this context, we should ask, if relation (11) remains valid in a significantly curved spacetime of GR. Or, using Einstein's formula $W = mc^2$ relating the energy W with mass m, if there is valid $c^2u/G = W/c^2$ in a strong field.

The identification of c^2u/G to W/c^2 might be, in addition, evoked by the following circumstance. It is well known that the EFEs can be rewritten to the form in which one of the EFEs is

$$\frac{du}{dr} = \frac{1}{2}\kappa E r^2, \tag{15}$$

where $\kappa = 8\pi G/c^4$ is the Einstein gravitational constant. Hence,

$$u = \frac{1}{2}\kappa \int_{r_1}^{r_2} E r^2 \, dr \tag{16}$$

and this integral seems to be identical to (1) if we identify u to $\kappa W/(8\pi) = GW/c^4 = Gm/c^2$, distance r_1 to $R_{\rm in}$, and distance r_2 to $R_{\rm out}$. However, the GR itself provides us only with the differential form (15). The extent of integration, from r_1 to r_2, to obtain the integral counterpart (16) of differential equation (15) is not given by the theory and any physical reason to identify r_1 and r_2 to $R_{\rm in}$ and $R_{\rm out}$, respectively, is absent. In definition relation (10), we see that function u depends, directly, only on radial distance r and g_{rr}-component of metric tensor. It clearly implies that it is a metric, not energetic quantity.

According to Eq. (15), an increment of function u is proportional to $Er^2\, dr$. Coincidentally, a mass increment corresponding to the mass content in the thin shell of radius r is also proportional to $Er^2\, dr$. When one chooses the appropriate constant of proportionality, this coincidence enables an identification of u to mass.

However, such identification, if done, is only a postulate, whereby this postulate leads to the following contradiction.

In the region above the outer physical border of RCO, function u is positive. In the internal cavity of RCO, the value of u is negative as we demonstrated in Sec. 3. Therefore, the function decreases with a decreasing radial distance inside the RCO body, whereby there is a distance, r_u, where $u = 0$.

In the region of distances $r < r_u$, it is valid that $u < 0$ and this inequality would imply a negative mass if we represented this quantity in accord with relation (11). We remind that quantity m in this relation was represented as the mass within radius r and u was the value of this function in distance r. At the same time, it would imply a negative energy in the part of object delimited by $R_{\rm in} < r < r_u$ since the energy is related to the mass according to Einstein's formula $W = mc^2$. However, energy W concentrated within the sphere of radius r is positive. It is the integral of energy density through the volume delimited by the spheres of radii $R_{\rm in}$ and r in this case. Since the energy density is positive everywhere inside the RCO body, the integral must necessarily be positive.

The above-described discrepancy clearly implies that quantity u cannot be interpreted in terms of mass or energy because the GR would then be an intrinsically inconsistent theory. In any theory, the given quantity cannot be negative according to an argument and, at the same time, positive according to other argumentation. This is valid regardless the solution of the EFEs is or is not applicable in reality. (The given quantity cannot be represented in a certain way in a kind of solution and in a different way in other kind of solution of the same equations.)

We note, there exist some solutions of the EFEs with a negative energy density, mass, and function u in the GR, when constructing a model of compact objects. These solutions are not, of course, applicable to the real objects, but are correct from the mathematical point of view and, thus, there is no problem with the consistency of GR. The above-described discrepancy concerning energy W is however a different kind of problem.

6. Nonlinear Dependence of Gravitational Acceleration on Mass and the Metric Mass-Equivalent

In the GR, the gravitational acceleration of a particle being in rest (or moving with a speed negligible in comparison to the speed of light), is given by relation (12), in the case of gravitating object with $r_{\rm o} = 0$, i.e. the full-sphere compact object. In this case, it appears that the gravitational acceleration is linearly proportional to the mass $m(r)$ inside the object-centric sphere of radius r for whatever distance r.

However, as soon as we consider $r_{\rm o} > 0$, the gravitational acceleration is no longer linearly proportional to the mass inside the sphere of radius r, which is calculated by Einstein's formula $m = W/c^2$ and energy W is the integral of the energy density through the corresponding volume. Of course, there must be a relation between the acceleration and mass (or energy), since both quantities are calculated with the help of quantities the behaviors of which are the result of the same solution of the EFEs. However, this relation can be expected to be complicated.

Although the relation between the acceleration of a TP in the vicinity of RCO and the RCO's mass or energy is complicated, the dynamical characteristics of the TP has often been and, we can expect, further will be reported to with the help of the auxiliary metric quantity u and, therefore, with the help of mass since $u = m$ in the system of units with $c = 1$ and $G = 1$, which are frequently used in the GR.

Because of the above-mentioned circumstance and in purpose of a more fluent communication, we define "metric mass-equivalent", M_μ. By the definition, this mass-equivalent is linearly proportional to the auxiliary quantity u_{out} in the region where the metrics can be approximated by the OSM. Specifically,

$$M_\mu = \frac{c^2}{G} u_{\mathrm{out}}. \tag{17}$$

The metric mass-equivalent is identical to the "mass", m, related to the energy according to the Einstein formula $W = mc^2$, in the Newtonian physics. The metric mass-equivalent is defined mainly in the context of gravitational acceleration of a particle situated in vacuum outside of a compact object having a discrete outer border or outside the central condensation of relativistic radiation sphere (see Sec. 7). It is assumed that the term "metric mass-equivalent" will be most frequently used just in this context.

Nevertheless, we also define the generalized metric mass-equivalent, in an arbitrary distance r, by relation

$$m_\mu(r) = \frac{c^2}{G} K_\nu u(r). \tag{18}$$

In more detail, we distinguish among the "object-interior metric mass-equivalent", when $R_{\mathrm{in}} \le r \le R_{\mathrm{out}}$, "internal-cavity metric mass-equivalent", when $r < R_{\mathrm{in}}$, and "metric mass-equivalent" or "outer-space metric mass-equivalent", when $r > R_{\mathrm{out}}$ (therefore, $m_\mu(r > R_{\mathrm{out}}) \equiv M_\mu$). The object-interior metric mass-equivalent can be positive as well as negative and internal-cavity metric mass-equivalent is always negative. In region $r \ge R_{\mathrm{out}}$, $K_\nu = 1$. In $r \le R_{\mathrm{in}}$, constant K_ν must be determined with the help of the model of compact object demanding the continuity of metrics in R_{in}.[b] Relation (18) with constant K_ν could be formally generalized also for the object's interior. Here, the numerical integration of field equations yields the values g_{tt} and u in a given distance r. Since $g_{tt} = K_\nu(1 - 2u/r)$, we can calculate K_ν and, subsequently, m_μ according to (18).

The acceleration due to an action of an RCO on a TP situated above its outer physical border can be calculated by using the metric tensor yielded by the outer Schwarzschild solution. If the TP is in rest, the acceleration is given by the first or

[b]It appears that the strength of gravity in the region $r \le R_{\mathrm{in}}$ is different from that in the region $r \ge R_{\mathrm{out}}$. Constant G/K_ν can be regarded as the gravitational constant relevant to the region $r \le R_{\mathrm{in}}$.

second form of relation (12), in which we replace $m \to M_\mu$. In a large RCO-centric distance, where $2u/r \ll 1$ and term $2u/r$ is negligible, the acceleration can be well approximated as

$$\frac{d^2r}{dt^2} = -\frac{c^2u}{r^2} = -\frac{GM_\mu}{r^2}. \tag{19}$$

This relation clearly demonstrates the linear dependence of d^2r/dt^2 on M_μ. However, all numerically constructed models of RCOs with $r_\mathrm{o} > 0$ imply that $M_\mu < M$. So far, mass M can be several orders of magnitude larger than the corresponding metric mass-equivalent M_μ (see Sec. 7).

7. Relativistic Radiation Sphere, Its Central Condensation and Huge Corona

The simplest equation of state is, perhaps, that for a radiation fluid, which says that the energy density, E, is the triplicate of pressure, P, i.e. $E = 3P$. (We note that this equation is valid for a fluid or gas consisting of whatever kind of particles, even of Fermions, if their kinetic energy largely exceeds their rest energy, therefore the latter is negligible.) It appears that there are also stable solutions for the relativistic objects consisting solely of radiation. If the objects are exactly spherically symmetric and stable, we refer to these objects as the relativistic radiation spheres (RRSs, hereinafter). If we allow $r_\mathrm{o} > 0$, the RRSs can contain whatever large amount of energy, W, formally corresponding to mass $M = W/c^2$.

The behaviors of some quantities characterizing an example of RRS are shown in Fig. 9. We are aware that this is only a toy model. In reality, the temperature in some layers is appropriate for an occurrence of reaction in which two photons are converted to a particle-antiparticle pair. The opacity is then expected to be different of that in a pure radiation fluid. If there occurs an asymmetry in annihilation of such pairs, the layers can contain a common hydrogen gas and, subsequently, some nuclear reactions can alter the composition. These processes are ignored in the example model as well as in considerations presented below.

Our modeling implies that an RRS has a central condensation (CC) with the energy density and pressure that can be much larger than those in a surrounding environment. At the border of CC, the state quantities may decrease about several orders of magnitude. The energy content inside the CC of RRS can be so high that the temperature of the border — photosphere of the CC — can be extraordinarily high (many trillions of Kelvin). The luminosity of the CC can also be tremendous. According to our investigation, the CC mass can exceed up to $1.89774 \cdot 10^6$ times the metric mass-equivalent corresponding to u_out above the outer border of CC.[c]

[c]The EFEs contain the fractions with form $1-2u/r$ in the denominator. Hence, there must always be valid that $2u/r < 1$ in the numerical integration. Otherwise, the integration would have to cross the singular point corresponding to $2u/r = 1$. An attempt to create a model of RRS with the mass of CC exceeding the metric mass-equivalent more than $1.89774 \cdot 10^6$ times always ended in a failure of the numerical integration because it acquired the direction to cross the singular point.

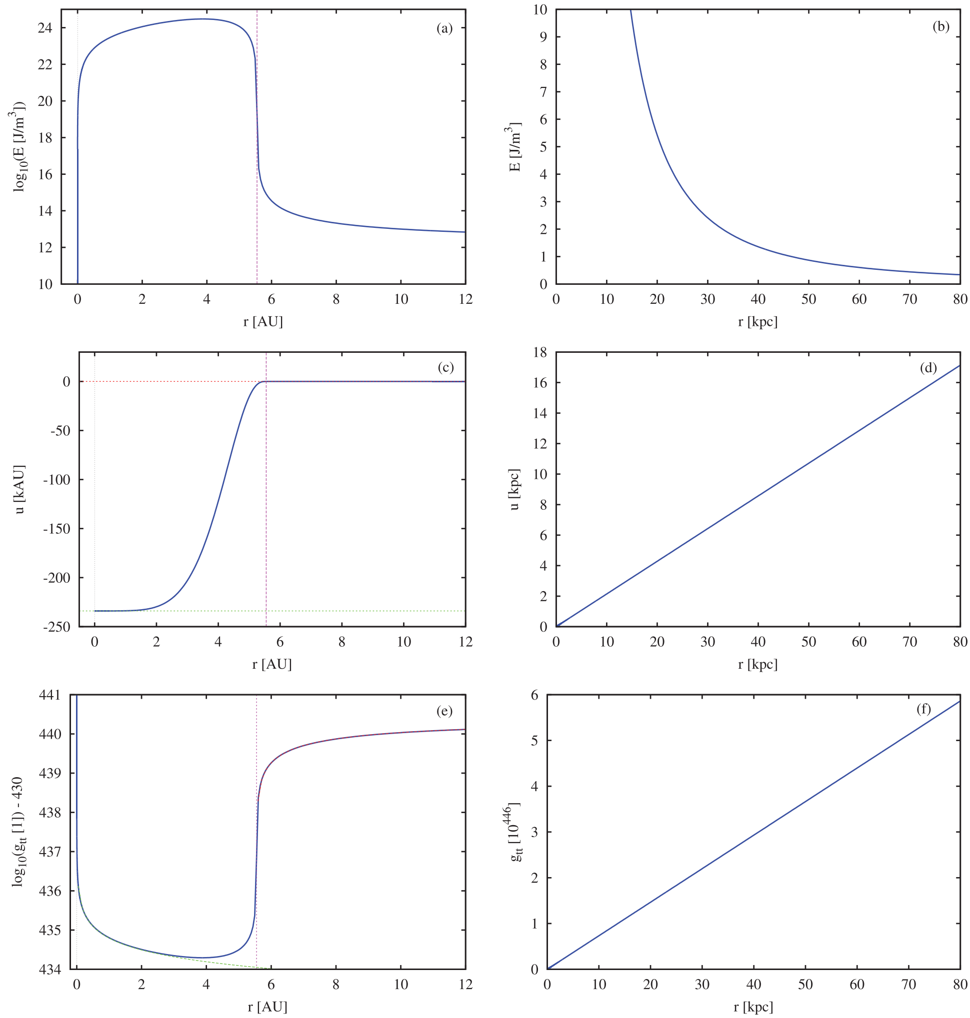

Fig. 9. (Color online) The behavior of energy density (plots (a) and (b)), auxiliary quantity u ((c) and (d)), and g_{tt}-component of metric tensor ((e) and (f)) in an example of RRS. The left plots, (a), (c), and (e), show the behavior inside the CC (thick blue curve) and in its vicinity. In plot (c), the constant values u_{out} and u_{in} tailored to the quantity u at the outer and inner border are shown with a dotted red and green curve, respectively. In plot (e), component g_{tt} of metric tensor in the outer Schwarzschild solution in forms $g_{tt} = 1 - 2u_{\mathrm{out}}/r$ and $K_{\mathrm{in}}(1 + 2|u_{\mathrm{in}}|/r)$ are drawn with the dotted red and green curves, respectively. The dashed, violet, vertical abscissa in plots (a), (c), and (e) indicates the outer border of the CC, in distance $r = R_{\mathrm{cc}}$. The right plots, (b), (d), and (f), show the behavior of E, u, and g_{tt} of the same object, obtained via numerical integration, in the distance scale of several tens of kilo-parsecs.

Let us denote the radius of the border of CC by R_{cc}. In distances $r > R_{\mathrm{cc}}$, the energy density and pressure are negligible in comparison with those in the CC. The metrics in this region (but not that in $r \gg R_{\mathrm{cc}}$, see the text below) can be well approximated with the OSM. We can also construct the RRS models with an inner border of the CC. Within this border, there is also a region with a negligible energy density and pressure where the spacetime can be well approximated with the OSM.

Energy density is, however, nonzero in region above the border of the CC. The radiation fluid constitutes, here, a thin but extremely extending "corona". (The static solution of the EFEs implies the extent of the corona up to an infinity. In a very large distance scale, the static solution could no longer approximate the reality well, if we wanted to use it in a modeling of a real object of the large size. The cosmological expansion of the universe would no longer be negligible.) The net energy content inside a sphere of radius R, whereby $R \gg R_{\mathrm{cc}}$, typically exceeds, in several orders of magnitude, the energy content in the CC.

In the region of the corona, the energy emitted by the CC is dispersed to a large volume and we can expect that the radiation spreads in the same way as in a free space, therefore the energy density will decrease as $E \propto r^{-2}$. Actually, we demonstrated[7] that quantities E, P, u, and g_{tt} can be described with the help of the asymptotic solution of the EFEs for $r \gg R_{\mathrm{cc}}$: $E = 3/(7\kappa r^2)$ (this behavior is identical to that in Fig. 9(b) which was obtained via numerical integration of the EFEs), $P = P_{\mathrm{o}}/r^2$, $u = 3r/14$ (identical to that in Fig. 9(d)), and $g_{tt} = r/R_\nu$ (identical to that in Fig. 9(f)). P_o and R_ν are constants.

The gravitational field generated by an RRS is qualitatively different than a field of a Newtonian, point-like mass object. To demonstrate this fact, let us imagine a disk consisting of stars embedded inside the corona of RRS. The stars orbit the center of the RRS in circular orbits. (Such a disk resembles that in spiral galaxies.) To calculate the speed of a star in the disk as the function of its distance from the RRS center, we use again the equation of geodesic (6). For a sake of simplicity, we assume that the mass of the disk is negligible in comparison to the mass proportional to the energy of the part of corona situated within the sphere of radius identical to the RRS-centric distance of the star.

Specifically, the radial acceleration of the star in distance r from the RRS center can be calculated according to relation (8). If the considered star moves in the x–y plane in a circle, then the radial acceleration, d^2r/dt^2, as well as velocity components dr/dt and $d\vartheta/dt$ are zero, and $d\varphi/dt$ is a constant independent on time. We denote $d\varphi/dt = \omega$. Subsequently, relation (8) reduces to

$$0 = e^{-\lambda}\left(r\omega^2 - \frac{c^2}{2}\frac{de^\nu}{dr}\right) \tag{20}$$

from which

$$\omega^2 = \frac{c^2}{2}\frac{1}{r}\frac{de^\nu}{dr}. \tag{21}$$

Taking into account the above-derived metrics of the RCO's corona, i.e. $e^\nu = r/R_\nu$ and $de^\nu/dr = 1/R_\nu$, the quadrate of the angular speed of the star, ω^2, equals

$$\omega^2 = \frac{c^2}{2R_\nu}\frac{1}{r} \tag{22}$$

and subsequently, the orbital speed of the star, $v_\star = \omega r$, equals

$$v_\star = c\sqrt{\frac{r}{2R_\nu}}. \tag{23}$$

The speed increases with increasing distance from the disk center.

8. Summary

We demonstrated that the collapse of very massive stars without any internal energy source below their event horizon, i.e. a formation of black holes, is not predicted on the basis of the GR, but, on the contrary, the assumption of the collapse occurred due to a large constraint of the application of this theory in the relativistic astrophysics.

The constraint has implicitly appeared in the astrophysics of RCO in several ways. One of these is the general identification of the mass of object, m_r, within the sphere of radius r to Oppenheimer–Volkoff auxiliary function u when units $c = 1$ and $G = 1$ are used. However, while mass m_r is related to the energy, function u is related directly to the metrics of spacetime. The EFEs imply that the mass as another form of energy, which is the integral of energy density through an appropriate volume, is always positive. On the contrary, quantity u is also negative in a region inside the compact object.

The consequence of the identification of function u to mass m_r is that the values from -1 to 0 of g_{rr}-component of metric tensor are forbidden. There is, however, no reason why the values from this interval should be excluded in relativistic astrophysics.

In the real world, an RCO obviously acquires the configuration with the minimum total energy. According to the GR, if it is not constrained, this configuration appears to be not a full sphere with the maximum energy density and pressure in the RCO center, but its matter is distributed within a hollow sphere having not only an outer, but an inner physical border, too. The inner border and a cavity inside occur due to the change of orientation of the gravitational attraction to be directed outward in the most inner part of the object. We explained the effect of the orientation of the gravitational attraction outward: the gravitational action of a nearer matter is more effective than that of a more distant matter in the significantly curved space. Consequently, the upper material layers in the RCO attract a TP outward from the object's center, in contrast to the Newtonian physics where the net gravity of every of these layers is zero.

The center of internal void of the RCO is a singular point from the mathematical point of view. We demonstrated that this is only an abstract singularity figuring in our description of the metrics inside the void. It can never be entered by a particle and, hence, detected.

If the complete GR is allowed to be used in constructing of the models of RCOs, then there is no upper-mass limit. Not only the neutron stars having a mass exceeding two, three, or even many more solar masses can exist, but the super-massive compact objects — RRSs — with a mass of many million solar masses, like those observed in the centers of quasars and galaxies, are possible. Their outer surface is situated above the event horizon, therefore they can have an extra bright and luminous photosphere. We also found that the RRS possesses a huge "atmosphere" of radiation — corona — with the gravity exceeding, in a large distance scale, the gravity of the CC RRS about many orders of magnitude.

The gravitational acceleration and, hence, dynamical characteristics of the objects in a vicinity of RCO or CC of RRS are derived from the size of metric quantity c^2u/G given in the unit of mass. According to the models of RCO, the actual mass, related to the energy by the well-known Einstein's formula between the energy and mass, is larger than c^2u/G. The mass in the CC of RRS can exceed this quantity up to six orders of magnitude. If we imagine a CC with c^2u/G equal to, e.g. 10^9 $M_\odot$ that has lost its energy in the same rate as a bright quasar during the whole age of the universe, it could spent only a small fraction of its total initial energy.

If the currently valid constraint of the usage of GR in relativistic astrophysics is abolished, we noticed no violation of any well proved physical law, like the laws of energy or momentum conservation. Thus, we mean that the constraint of the GR should be abolished and the solutions of the EFEs found by Ni,[8] which yield the model of RCO in the form of hollow sphere and without the upper mass limit, can also be used in the relativistic astrophysics.

The further observational evidence, in the future, will force us to decide whether we will go on with the replacing of the GR by various other concepts or this theory will be used to describe all astrophysical phenomena, which can be described within it.

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