THE (AMAZING) ARCHITECTURE OF FINITE PROJECTIVE RING LINES[1]

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ABSTRACT: I will present an interesting diagrammatic exposition of some intriguing properties of projective lines defined over finite rings, which emerged as a by-product of our recent applications of these remarkable geometries in quantum physics [1–9]. The corner-stone concept will be a free cyclic submodule (fcs) over a finite ring and a "tree" comprising all the fcs's characterizing a given line. I will first show how one can fine-tune the neighbour relation by taking into account the cardinality of shared pairs of the fcs's representing given points. This will be illustrated by examples of projective lines defined over local rings; here, in the case of rings of order 8 we find two different kinds of a projective line, and as many as four kinds for order 16 (with 8 zero-divisors). The difference between the individual kinds of lines will be shown to be intimately related with the number of pairs not lying on any fcs generated by an admissible pair — so-called "outliers". I will then proceed to demonstrate that there exist finite rings (some of) whose outliers even generate fcs's. The smallest case when this occurs is the non-commutative ring of order 8 (the ring of ternions), but also some other examples — several non-commutative rings of order 16 — will be given to illustrate the phenomenon. After demonstrating the important role of modular ring lines in understanding the commutation algebra of some generalized Pauli groups, the talk will be rounded off with a brief outline what novelties we (can) encounter when passing to geometries over nearrings.

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3. Planat, M., Baboin, A.-C., and Saniga, M.: 2008, Multi-Line Geometry of Qubit-Qutrit and Higher-Order Pauli Operators, International Journal of Theoretical Physics, in press (arXiv.org:0705.2538].

4. Planat, M., and Saniga, M.: 2008, On the Pauli Graph of N-Qudits, Quantum Information and Computation, Vol. 8, No. 1-2, 127-146 (arXiv:quant-ph/0701211).

5. Saniga, M., Planat, M., and Pracna, P: 2008, Projective Ring Line Encompassing Two-Qubits, Theoretical and Mathematical Physics, in press (arXiv:quant-ph/0611063).

6. Saniga, M., and Planat, M.: 2007, Projective Line over the Finite Quotient Ring GF(2)[x]/(x^3 – x) and Quantum Entanglement: Theoretical Background, Theoretical and Mathematical Physics, Vol. 151, No. 1, 474-481 (arXiv:quant-ph/0603051).

7. Saniga, M., Planat, M., and Minarovjech, M.: 2007, Projective Line over the Finite Quotient Ring GF(2)[x]/(x^3 – x) and Quantum Entanglement: The Mermin "Magic" Square/Pentagram, Theoretical and Mathematical Physics, Vol. 151, No. 2, 625-631 (arXiv:quant-ph/0603206).

8. Saniga, M., and Planat, M.: 2007, Multiple Qubits as Symplectic Polar Spaces of Order Two, Advanced Studies in Theoretical Physics, Vol. 1, No. 1, 1-4 (arXiv:quant-ph/0612179).

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[1] A seminar talk to be given at the Institute of Discrete Mathematics and Geometry, Vienna University of Technology, Vienna (Austria), on March 13 (or 14), 2008.