# Projective Ring Planes and the Fundamental Prespace

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**Abstract:** This contribution aims at giving an algebraic geometrical description of the concept of the fundamental prespace, first introduced by one of us [1] on a field-theoretical level. A key element of our approach is the concept of projective planes defined over rings [2–5]. The most remarkable and fascinating feature of such planes is the fact that two distinct points/lines need not have a unique connecting line/meeting point, which enables us to introduce two distinct kinds of non-incidence. Namely, two distinct points/lines of a (finite) ring plane are called neighbour if they are joined by/meet in at least two different lines/points; otherwise, they are called distant (or, by some authors, also remote). The character of the neighbour relation is embodied in the properties of the ring, being the identity relation, equivalence relation and/or non-transitive relation according as the ring is a field (i.e., a ring where each non-zero element is invertible), a local ring and/or a ring featuring more than one maximal ideal. Accordingly, there exist three distinct sectors of the fundamental prespace, viz. the classical, semi-clasical and quantum, respectively. It is surmised that the neighbour relation will enable us to geometrize such concepts as quantum non-locality and quantum entanglement. Projective planes defined over quadratic extensions of the Galois fields [6,7] will serve as an elementary illustration of the theory.

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