HOMALOIDAL WEBS, SPACE CREMONA TRANSFORMATIONS AND THE DIMENSIONALITY AND SIGNATURE OF MACRO-SPACETIME

An Outline of the Theory

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1. Introduction

No phenomenon of natural sciences seems to be better grounded in our everyday experience than the fact that the world of macroscopic physical reality has three dimensions we call spatial and one dimension of a different character we call time. Although a tremendous amount of effort has been put so far towards achieving a plausible quantitative elucidation of and deep qualitative insight into the origin of these two puzzling numbers, the subject still remains one of the toughest and most challenging problems faced by contemporary physics (and by other related fields of human inquiry as well). Perhaps the most thought-provoking approach in this respect is the one based on the concept of a transfinite, hierarchical fractal set usually referred to as the Cantorian space, $\mathcal{E}^{(\infty)}$. In its essence, $\mathcal{E}^{(\infty)}$ is an *in*finite dimensional quasi-random geometrical object consisting of an *in*finite number of elementary (kernel) fractal sets; yet, the *expectation* values of its both topological and Hausdorff dimensions are *finite*. The latter fact motivated El Naschie [1,2] to speculate not only about the total dimensionality of spacetime, but also about its enigmatic signature. His reasoning goes, loosely speaking, as follows. It is assumed that the effective topological dimension of $\mathcal{E}^{(\infty)}$, $\langle n \rangle$, grasps only spatial degrees of freedom, whereas its averaged Hausdorff dimension, $\langle d \rangle$, incorporates also the temporal part of the structure. These two dimensions are interconnected, as both depend on the Hausdorff dimension of the kernel set, $d_c^{(0)}$. And there exists a unique value of the latter, viz. $d_c^{(0)} = 1/2$, for which $\langle n \rangle = 3$ (space) and $\langle d \rangle = 4$ (spacetime)!

2. Cremonian Pencil-Spacetimes

In our recent papers [3,4], we approached this issue from a qualitatively different, but conceptually similar to the latter, algebraic geometrical point of view. This approach is based on our theory of pencil-spacetimes [5–13]. The theory identifies spatial coordinates with pencils of lines and the time dimension with a specific pencil of conics. Already its primitive form, where all the pencils lie in one and the same projective plane, suggests a profound connection between the observed number of spatial coordinates and the internal structure of time dimension [5–7,9,11–13]. A qualitatively new insight into the matter was acquired by relaxing the constraint of coplanarity and identifying the pencils in question with those of *fundamental* elements of a Cremona transformation in a three-dimensional projective space [3,4]. The correct dimensionality of space (3) and time (1) was found to be uniquely tied to the so-called *quadro-cubic* Cremona transformations – the *simplest* non-trivial, non-symmetrical Cremona transformations in a projective space of three dimensions. Moreover, these transformations were also found to fix the type of a pencil of fundamental conics, i.e. the global structure of the time dimension.

A space Cremona transformation is a rational, one-to-one correspondence between two projective spaces [14]. It is determined in all essentials by a *homaloidal* web of rational surfaces, i.e. by a linear, triply-infinite family of surfaces of which any three members have only one free (variable) intersection. The character of a homaloidal web is completely specified by the structure of its *base* manifold, that is, by the configuration of elements which are common to every member of the web. A quadro-cubic Cremona transformation is the one associated with a homaloidal web of quadrics whose base manifold consists of a (real) line and three isolated points. In a generic case, discussed in detail in [3], these three base points (B_i, i=1,2,3) are all real, distinct and none of them is incident with the base line ($\mathcal{L}^{\mathrm{B}}$). In the subsequent paper [4], we considered a special 'degenerate' case when one of B_i lies on $\mathcal{L}^{\mathrm{B}}$. It was demonstrated that the corresponding fundamental manifold still comprises, like that of a generic case, three distinct pencils of lines and a single pencil of conics; in the present case, however, one of the pencils of lines incorporates $\mathcal{L}^{\mathrm{B}}$, and is thus of a different nature than the remaining two that do not. As a consequence, the associated pencil-space features a kind of intriguing anisotropy, with one of its three macro-dimensions standing on a slightly different footing that the other two. Being examined and handled in terms of the transfinite Cantorian space approach, this macro-spatial anisotropy was offered a fascinating possibility of being related with the properties of spacetime at the microscopic Planck scale [4].

If this spatial anisotropy is a real characteristic of the Universe, then its possible manifestations, whatever bizarre and tantalizing they might eventually turn out to be, must obviously be of a very subtle nature as they have so far successfully evaded any experimental/observational evidence. Yet, conceptually, they deserve serious attention, especially in the light of recent progress in (super)string and related theories [15]. For alongside invoking (compactified) extra spatial dimensions to provide a sufficiently-extended setting for a possible unification of all the known interactions, we should also have a fresh look at and revise our understanding of the three classical macro-dimensions we have been familiar with since the time of Ptolemy.

3. Conclusion

The concept of Cremonian spacetimes represents a very interesting and fruitful generalization of the pencil concept of spacetime by simply raising the dimensionality of its projective setting from two to three. When compared with its two-dimensional sibling, this extended, three-dimensional framework brings much fresh air into old pressing issues concerning the structure of spacetime, and allows us to look at the latter in novel, in some cases completely unexpected ways. Firstly, and of greatest importance, this framework offers a natural qualitative elucidation of the observed dimensionality and signature of macro-spacetime, based on the sound algebro-geometrical principles. Secondly, it sheds substantial light at and provides us with a promising conceptual basis for the eventual reconciliation between the two extreme views of spacetime, namely physical and perceptual. Thirdly, it gives a significant boost to the idea already indicated by the planar model that the multiplicity of spatial dimensions and the generic structure of time are intimately linked to each other. Finally, being found to be formally on a similar philosophical track as the fractal Cantorian approach, it grants the latter further credibility.

Acknowledgement–This work was partially supported by the NATO Collaborative Linkage Grant PST.CLG.976850.

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