Two-Qutrits from Miniquaternions

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It is well known that the maximum cardinality of mutually unbiased bases in Hilbert spaces of finite dimensions d cannot be greater than $d+1$ and that this limit is reached if d is a power of a prime, $d=p^m$. For p odd, the proof of this property lies with an additive Fourier transform over Galois *fields* $GF(d=p^m)$ (Wootters and Fields, 1989). For p=2, lifts of the base ring Z_4, the set of integers modulo 4, to Galois *rings* $GR(4^m)$ is used instead (Klappenecker and Roetteler, 2003). Here we study lifts of the base ring Z_3 to *near-fields* $NF(3^m)$, with a view of unveiling a new class of quantum states for the corresponding dimensions. In the simplest case of miniquaternions (m=2), the relevant automorphism (permutation) group is employed to get the trace map from $NF(3^2)$ to Z_3 and find the corresponding additive characters. In the qutrit bases so obtained, tensorial products of the vectors are found to be related to unextendible product bases and bound entanglement (e.g. Bennett et al, 1999). As nearfields (and nearrings) are of great importance for coordinatizing (non-Desarguesian) projective planes (Saniga and Planat, 2004), and the latter play a crucial role in the *classical* coding theory, we surmise that these structures will also play a similar role in *quantum* coding and computing.