GEOMETRY OF PSYCHO(PATHO)LOGICAL SPACE-TIMES: A CLUE TO RESOLVING THE ENIGMA OF TIME?

METOD SANIGA
Astronomical Institute of the Slovak Academy of Sciences, SK-059 60 Tatranská Lomnica, The Slovak Republic

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Abstract – The theory of pencil-generated space-times is demonstrated to represent a cogent, mathematically-based framework for understanding the so-called altered states of consciousness in their temporo-spatial aspects. It is shown, in particular, that the three most extreme groups of 'pathological' perceptions of time, that is the feeling of timelessness, time standing still as well as the experience of the pure past, can well be modelled by singular (space)time configurations generated by the pencil of conics featuring two degenerate conics and two real base points. Being speculative, the paper is also offered as a stimulus to our quest for other possible links between mathematics and physics on one side and psychology, psychiatry and philosophy on the other.

Keywords: projective geometry, pencils of conics, psychopathology of time and space

1. INTRODUCTION

One of the most striking and persistent symptoms of the so-called 'altered' states of consciousness are undoubtedly distortions in the perceptions of time and space (see, for example, Jaspers, 1968; Minkowski, 1970; Melges, 1982; Sims, 1955; Tellenbach, 1956a,b; Binswanger, 1933; Scheller, 1957; Fischer, 1929; Kloos, 1954; Huber, 1955; and Ciompi, 1961). Time is frequently reported as flowing faster or slower, expanded or contracted, and may even be experienced as being severely discontinuous ('fragmented'). In extreme cases, it can stop completely (events occurring immediately after each other seem to be separated by an 'eternity') or expand unlimitedly (i.e. items from the past, present and future are equally accessible without the intervals and seriality that separate them in the 'ordinary' state(s)). The sense of space is likewise powerfully affected. Space can appear amplified or compressed, condensed or rarefied, or even changing its dimensionality; it can, for example, become just two-dimensional ('flat'), acquire another dimensions, or be reduced to a dimensionless point in consciousness.

As yet, there exists no conceptual framework that would provide us with satisfactory explanations of these phenomena. Physics itself, although being the most sophisticated scientific discipline in describing the 'objective' world, is not even able to account for the ordinary arrow of time, let alone its other, more pronounced 'pathologies' mentioned above. Nor does it offer a plausible and convincing interpretation why space has just three dimensions – giving more conceptual challenges than satisfactory answers (see, e.g., Duff, 1997). This failure of current paradigms to accommodate the vast reservoir of the phenomena described above was a principal motivation in our quest for a rigorous and selfconsistent scheme, which ultimately led into what we call the theory of pencil-generated space-times (Saniga, 1996a,b; 1998a-e).

Interestingly, it is precisely a profound and intimate link between the internal structure of the temporal coordinate and the number of spatial dimensions that is one of the most crucial and distinguished features exhibited by this theory (Saniga, 1996b; 1998a). In the particular case of the pencil of conics having two singular conics and a couple of real base points (Saniga, 1998a) we have found that the (very) existence of the arrow of time is conditioned by the *three*-dimensionality of space and vice versa, i.e. any extinction of the temporal arrow is accompanied by a change in the number of observable spatial coordinates (Saniga, 1998a, Sec. 4; see also Saniga, 1996b). As already briefly listed in Saniga (1998a), there are a few different modes in which the transitions to arrow-free space-time configurations can be performed, the resulting configurations differing from each other in the character of the 'internal distortion' of the temporal coordinate and/or in the number of observable spatial dimensions. The paper aims at taking a detailed look at the qualitative properties of the structure of these 'distorted' space-times and a subsequent demonstration of their striking resemblance with the spatio-temporal characteristics exhibited by some well-pronounced groups of 'altered' states of consciousness.

2. A VERY BRIEF OVERVIEW OF THE THEORY

In this section we will highlight only the most crucial points of the theory of pencil-generated space-times; those readers who wish to have a more detailed look at the corresponding mathematical formalism are referred to our papers Saniga (1996a,b; 1998a), especially to the last one.

We start with the definitions of the temporal and spatial dimensions which, at the most formal level, are completely identical: [1]

$$\Theta^{\vartheta} \equiv \vartheta_1 \Theta^{(1)} + \vartheta_2 \Theta^{(2)} = 0, \tag{1}$$

where $\Theta^{(1)} = 0$ and $\Theta^{(2)} = 0$ represent two distinct geometrical objects of a projective plane. The difference between time and space appears as soon as the character of these objects is concerned, for

$$\Theta^{(1,2)} = \begin{array}{l} \nearrow \quad L_{\breve{x}}^{(1,2)} \equiv \sum_i \zeta_i^{(1,2)} \breve{x}_i \qquad \text{for spatial dimensions,} \\ \\ \searrow \quad Q_{\breve{x},\breve{x}}^{(1,2)} \equiv \sum_{i \leq j} c_{ij}^{(1,2)} \breve{x}_i \breve{x}_j \qquad \text{for temporal dimensions;} \end{array} \tag{2}$$

here $\zeta_i^{(k)}$ and $c_{ij}^{(k)}$ ($k = 1, 2$) are regarded as fixed quantities, while the $\breve{x}_i$ are viewed as variables ($i, j = 1, 2$ and 3). In other words, a spatial coordinate is generated by a range (pencil) of *linear* elements (as $L_{\breve{x}} = 0$ defines a curve of first degree, i.e. a line), whereas a temporal dimension is induced by a pencil of *non-linear* objects ($Q_{\breve{x},\breve{x}} = 0$ standing for a second degree object, i.e. a conic). Space, in our approach, is thus

[1] The symbols and notation used throughout this paper are the same as in Saniga (1998a).

a simpler concept than time for there is only one type of pencil of projective lines, whereas, in general, there exist several distinct kinds of pencils of projective conics, differing from each other in the number and character of singular conics; the latter figure is very sensitive to the projective plane's ground field and for the field of real numbers it amounts to nine (Saniga, 1998a, Sec. 6).

In general, any pencil of lines can serve as a spatial dimension and, similarly, any pencil of conics can stand for a temporal coordinate. This original symmetry is broken after one singles out a particular pencil of conics and postulates that only those pencils of lines can generate spatial coordinates whose carriers lie on the singular conics of the conics' pencil selected (such pencils will be referred to as s-pencils). There is, however, only one kind of non-singular projective conic with a non-empty image, which means that our temporal coordinate is still (internally) structureless. The final task, that is endowing the temporal with its arrow-like structure, is accomplished simply by affinizing the projective plane; stipulating, at the same time, that out of all potential spatial dimensions we can observe only those whose carriers fall on the ideal line. This leads to the already-declared connection between the occurrence of a time arrow and the multiplicity of observed spatial coordinates (Saniga, 1996a,b; 1998a).

3. REGULAR SPACE-TIMES OF THE 2%2 TYPE

In order to make an abstract story more concrete we will restrict our considerations to the projective plane over the field of real numbers and focus our attention on the structure of space-time configurations that are generated by the following pencil of conics (Saniga, 1996a; 1998a):

$$Q^{\vartheta}_{\breve{x},\breve{x}}(c) \equiv \sum_{i\leq j} c_{ij}(\vartheta_{1,2})\breve{x}_i\breve{x}_j = \vartheta_1\breve{x}_1\breve{x}_2 + \vartheta_2\breve{x}_3^2 = 0. \tag{3}$$

The pencil is of the 2%2 type (Saniga, 1998a, Sec. 6) as it exhibits *two* base points ($\varrho \neq 0$), B_1: $\varrho\breve{x}_i = (0,\ 1,\ 0)$ and B_2: $\varrho\breve{x}_i = (1,\ 0,\ 0)$, and *two* singular conics, $\vartheta(\equiv \vartheta_2/\vartheta_1) = \pm\infty$, i.e. a double real line $\breve{x}_3 = 0$, and $\vartheta = 0$, i.e. a pair of real lines $\breve{x}_1 = 0$ and $\breve{x}_2 = 0$ whose point of intersection S: $\varrho\breve{x}_i = (0,\ 0,\ 1)$ is a singular point. These degenerates also separate the set of proper conics into two distinct families: $-\infty < \vartheta < 0$ and $0 < \vartheta < +\infty$.

Let us now affinize the above pencil by choosing the ideal line (dashed in Fig. 1) in such a way that it incorporates neither of the base points $\mathrm{B}_{1,2}$, nor the singular point S. It is obvious that the most general equation of a line meeting such constraints reads

$$\breve{x}_1 - m\breve{x}_2 - n\breve{x}_3 = 0 \tag{4}$$

if both m and n are *non*-zero (assumed, without any loss of generality, to be positive). Inserting this equation into eq. (3) yields

$$P(x) \equiv mx^2 + nx + \vartheta = 0, \tag{5}$$

where we put $x \equiv \breve{x}_2/\breve{x}_3$; this is a *quadratic* equation with roots

$$x_{\pm} = \frac{-n \pm \sqrt{n^2 - 4\vartheta m}}{2m}. \tag{6}$$

Figure 1: The structure of a temporal arrow (a) and the multiplicity of spatial coordinates (b) of a space-time configuration generated by the 'regularly' affinized pencil of conics defined by eq. (3). The symbols and notation are explained in the text.

From the last expression it follows that whereas one family of proper conics, $-\infty < \vartheta < 0$, consists, as illustrated in Fig. 1a, solely of hyperbolae (thick shaded area), the other acquires a very interesting structurization featuring the domain of the hyperbolae ($x_{\pm}$ distinct and real; shaded area)

$$0 < \vartheta < n^2/4m \tag{7}$$

as well as the region of the ellipses ($x_{\pm}$ distinct but complex; dotted area)

$$n^2/4m < \vartheta < +\infty, \tag{8}$$

the two being separated from each other by a *single* parabola ($x_{+} \equiv x_{-}$; the bold curve)

$$\vartheta = n^2/4m. \tag{9}$$

And this is really a very remarkable pattern for it is seen to reproduce strikingly well, at least at a qualitative level, the observed arrow of time after we postulate (Saniga, 1996a,b; 1998a-e) that *the events of the past (future) are represented by the hyperbolae (ellipses), and the parabola stands for a single moment of the present.*

In addition to being a mechanism that enables us, as we have just shown, to endow the temporal coordinate with a non-trivial internal structure, the above-described affinization of the projective plane also induces a very interesting sort of non-equivalence among the potential spatial dimensions, i.e. among the s-pencils. To see this we recall our definition of an s-pencil as a pencil of lines whose carrier (i.e. the point shared by all the lines of the pencil) falls on (one of) the degenerate(s) of a given pencil of conics; in the particular case of pencil (3) it is thus any pencil of lines with the carrier lying on one of the lines SB_1, SB_2, or $\mathrm{B}_1\mathrm{B}_2$. Since any affinization of a projective plane means singling out, or deleting from the latter one line, which is given a special status of the ideal line, or the 'line at infinity' (see, e.g. Klein, 1928, p. 3), then it is quite natural to assume that those s-pencils which incorporate this line will also play a special physical role in our theory. Namely, we have stipulated (Saniga, 1996a,b; 1998a) that *only such pencils* (henceforth referred to as s*-pencils) *generate the spatial dimensions we can observe.* The puzzling *three*-dimensionality of the physical space is then laid bare as soon as we realise that the above 'regular' affinization of pencil (3) is characterized by just *three* distinct s*-pencils, namely

$$\mathrm{L}_1(\varpi_1) = \breve{x}_1 + \varpi_1(\breve{x}_1 - m\breve{x}_2 - n\breve{x}_3) = 0, \tag{10}$$

$$\mathrm{L}_2(\varpi_2) = \breve{x}_2 + \varpi_2(\breve{x}_1 - m\breve{x}_2 - n\breve{x}_3) = 0, \tag{11}$$

and

$$\mathrm{L}_3(\varpi_3) = \breve{x}_3 + \varpi_3(\breve{x}_1 - m\breve{x}_2 - n\breve{x}_3) = 0, \tag{12}$$

where the parameters ϖ_i run through all the real numbers and infinity. This is illustrated in Fig. 1b, where these pencils are represented by circles centered at the corresponding carriers; we see, in particular, that two spatial coordinates (denoted by x^1 and x^2) are borne by the $\vartheta = 0$ degenerate, while the remaining one (x^3) is supported by the other, $\vartheta = \pm\infty$ singular conic.

As can easily be discerned from Fig. 1, this unique relation 'arrow of time – three-dimensionality of space' holds only in the case of the ideal line avoiding both the singular point S and (either of) the base points B_1, B_2. With the latter condition relaxed this property disappears, and we enter the land of 'singular' or 'pathological' space-times which, and perhaps to the great astonishment of most of the readers, represent observable aspects of Nature, too!

4. SINGULAR SPACE-TIMES OF THE 2%2 TYPE

4.1 *S-configurations*

Because the lines SB_1, SB_2 and $\mathrm{B}_1\mathrm{B}_2$ are linearly independent (i.e. do not form a pencil), it is straightforward to see that the minimum possible number of s*-pencils (and, so, observable spatial dimensions) is *two*. This is clearly the case when the ideal line passes via S (Fig. 2), or through one of the base points (Fig. 3).

We will first examine the former case, which is clearly a limiting case of eq. (4) for $n \to 0$. After taking the corresponding limits of eqs. (7)–(9) we see that, while the family of proper conics with negative ϑ's does not change its affine properties, this is not the case for the $\vartheta > 0$ family. Really, here both the parabola and the (whole domain of) hyperbolae disappear, which means that the family becomes composed of ellipses only and, on its own, generates no time arrow at all. This situation is illustrated in Fig. 2a; all the proper conics falling into the dotted (shaded) area have the dashed line as an exterior (a secant) line so that they indeed become ellipses (hyperbolae) after this line is shifted to 'infinity.' (Fig. 2b shows the same situation, with the only difference being that the two families have switched their roles, i.e. m has changed its sign from '+' to '−').

Although either family of propers is homogeneous in the above limit and, so, cannot give rise to any arrow *in itself*, they do induce a 'pathological' kind of arrow when regarded as *a union*: the arrow featuring both the past (hyperbolae; shaded areas) and future (ellipses; dotted areas), but lacking the present (*no* parabola). We

Figure 2: The same as in Fig. 1 for the case when the ideal line incorporates singular point S, but neither of the base points – i.e. when, in eqs. (4)–(9), n is set equal zero and m is positive (a) or negative (b). The symbols and notation are identical with those of Fig. 1.

have already encountered this sort of defective temporal dimension in the case of Galois fields of characteristic two (Saniga, 1998c), but have not yet addressed an important issue, namely what the perceptions of such arrows should look like.

It is intuitively obvious that out of the three levels characterizing ordinary temporal arrows, i.e. the past, present and future, it is the present that seems to be fully 'responsible' for what we experience as the 'flux' of time; it is a unique connecting link between the past and future, an agent mediating a smooth transition between the two domains (see Fig. 1a). Hence, its absence in the above-mentioned arrow implies that, when experienced, such time does *not* flow.

Thus, a typical S-space-time consists of *two*-dimensional space and time *standing still*. This is indeed a very peculiar space-time structure and, at first sight, it may seem only to belong to the realm of pure academic interest. In the present case, however, appearances and reality differ perhaps more greatly than in any other area of human experience, for the theory dealt with can be linked with the phenomenological world in a way that has had absolutely no previous existence of any kind, of which science has not allowed us to form even the remotest conception. What the term 'phenomenological' world means here is nothing but the already-mentioned realm of pathological space-times as experienced in altered states of consciousness. It is a world where there is so much unsettled and uncertain that seemingly small matters can take an outsize significance and where, thus, the strength of our formalism can be shown in full.

The narratives given below contain quite an impressive and detailed account of 'distorted' space-times whose properties bear a very strong similarity to those of S-configurations. The narratives are due to mentally ill patients who try to describe their state(s) in temporo-spatial terms. The first two excerpts, taken from a very interesting paper by Tellenbach (1956a), represent the most spectacular examples of 'observed' S-configurations we were able to find in the available literature. Thus, the space-time perceived by the patient aliased as Th. H. clearly possesses all the attributes of an S-configuration (Tellenbach, 1956a, p. 13):

"I sure do notice the passing of time but can't experience it. I know that tomorrow will be another day again but don't feel it approaching. I can estimate the past in terms of years but I don't have any connection to it anymore. The time standstill is infinite, I live in a constant eternity. I see the clocks turn but for me time does not flow...

Everything lies in one line, there are no differences of depth anymore... Everything is like a firm plane."

Almost identical is the description given by another patient, denoted as M. B., who pays a little bit more attention to spatial characteristics (*ibid*, p. 14):

"Everything is very different in my case, time is passing very slowly. Nights last so long, one hour is as long as usually a whole day ..." Sometimes time had totally stood still, it would have been horrifying.

Even space had changed: "Everything is so empty and dark, everything is so far away from me... I don't see space as usual, I see everything as if it were just a background. It all seems to me like a wall, everything is flat. Everything presses down, everything looks away from me and laughs."

The fact that the time standing still does contain the domains of both the past and future, as implied by our theory, is evident from the following experience (Fischer, 1929, p. 560):

"Time stands still, you drift even between past and future. Everything is so boringly habitual. Before, everything was connected backward and forward – and it was good that way. And this connection has been cancelled now. This is a boring and extended time without an end, therefore, I cannot plan anything. This way there are many mysteries!"

while the $3 \rightarrow 2$ reduction in the number of spatial dimensions is depicted in detail in Tellenbach (1956b, p. 290):

"...My look is always directed to the inside. I notice the details of things only willingly. Otherwise the object becomes blurred and moves off like a foggy shape into the distance. Objects that are faraway cannot be noticed anymore... I can barely recognize the spatial sequence of things. I have got an impression that space shrinks to a plane."

4.2 *B-configurations*

As already mentioned, other configurations featuring two spatial dimensions emerge in the case when the ideal line passes via one of the base points, avoiding point S (see Fig. 3). It is obvious that a line selected in this way is a secant to every proper conic of (3), because it is different from the line B_1S/B_2S, their common tangent line at B_1/B_2. Hence, the corresponding affine image of the pencil (i.e. the image obtained after the dashed line of Fig. 3 has been 'shifted to infinity') will exhibit *hyperbolae* only and the corresponding temporal dimension will thus consist of the *pure past*.

This is another group of pathological space-times well documented in the psychiatric literature. The examples below concern uniquely the properties of time. The first narrative, taken from an extremely inspiring and very readable paper by Fischer (1929), makes it explicit that the temporal perceived is devoid of the notions of both future and present (Fischer, 1929, p. 563 – patient *Ku*):

"I stop still, I am being thrown back into the past by words that are being said in the hall. But

Figure 3: The same as in the previous two figures, but for the ideal line that hits one of the base points, passing, however, past point S.

this all is self evident, it must be that way! There is no present anymore, there is only this stated being related to the past, which is more than a feeling, it goes through and through. There are all sorts of plans against me in the air of this hall. But I don't listen to them, I let my mind rest so that it doesn't corrode.

Is there any future at all? Before, the future existed for me but now it is shrinking more and more. The past is so very obtrusive, it throws itself over me; it pulls me back... By this I want to say that there is no future and I am thrown back. I live much faster than before. This is the contact with the old things. Strange thoughts enter my mind and drive me off into the past..."

Another patient examined by Fischer (*Ze*) conveys his impressions in a rather succinct manner (Fischer, 1929, p. 561):

"It pulls me back, well, where to? To where it comes from, there, where it was before. It enters the past. It is that kind of a feeling as if you had to fall back. This is the disappearing, the vanishing of things. The time slips into the past, the walls are fallen apart. Everything was so solid before. It is as if it were so close as to be grabbed, as if you had to pull it back again: Is that the time? Shifted way back!"

However, from the references we have at our disposal, by far the most illustrative examples of this type of 'time pathology' can be found in an article by Kloos (1938). The paper contains a whole section devoted to the 'futureless' temporal, from which we pick up the following account (Kloos, 1938, p. 229):

"I cannot anticipate anything at all, as if there were no future anymore. I used to think about what I would do next week or what I would buy myself in the summer, I cannot do that now. I always believe, it all stops now and there is altogether no tomorrow anymore. I feel like a doll, which is without life, to which a new life first has to be given. I cannot think of the future - I cannot anticipate." (And what about the past?) ,,Actually, I think about it very intensively. It is very painful for me to think that those wonderful moments are gone..." (And the present?) ,,I don't think all too much about the present. I just take it as it comes, the day goes by. I always study where this all comes from."

A special class of B-configurations is obtained in the case when the ideal line coincides with the line joining the base points, B_1B_2. While the structure of time

Figure 4: The same as in the previous figures, but for the ideal line joining the base points (a), or one of the base points and S (b, c).

clearly remains unchanged (the pure past), space itself undergoes a dramatic change, acquiring *infinitely* many dimensions; really, since the ideal line is identical with B_1B_2 and the latter is a ($\vartheta = \infty$) degenerate of pencil (3), each pencil of lines with the carrier falling on it thus becomes an s*-pencil. This is visualised in Fig. 4a, where the ∞-dimensionality of space is demonstrated by a couple of lines parallel to the ideal line.

4.3 *BS-configurations*

To make our classification of the singular 2%2 space-times complete, it still remains to examine the mode when the ideal line coincides with either B_1S or B_2S (Fig. 4b, c). Thus, as the lines B_1S and B_2S represent, when taken together, the other (i.e. $\vartheta = 0$) degenerate conic of the pencil (3), it is also in this mode where the dimensionality of space is saturated, i.e. *infinite*. On the other hand, since both the above-mentioned lines are common tangents to all non-singular conics of (3), these conics become *parabolae* when the pencil is affinized correspondingly, and the temporal will thus feature the *pure present.*

Rephrased in a more layman-friendly language, in a BS-state we are able to completely free ourselves from the 'bounds of time,' to break the barrier between the present and past and/or future, because all the events are laid out before us at once, all simultaneously. Hence, this state is simply incompatible with any notion of succession as passage, being also perfect in the sense that every single potential spatial dimension is 'perceived.' A particularly impressive description of such a state is one we found in Huber (1955, pp. 45–46), being based on the author's own experience:

"I woke up in a whole different world in which the puzzle of the world was solved extremely easily in form of a different space. I was amazed at the wonder of this different space and this amazement concealed my judgement, this space is totally distinct from the one we all know. It had different dimensions, everything contained everything else. I was this space and this space was me. The outer space was a part of this space, I was in the outer space and the outer space was in me...

Anyway, I didn't experience time, time of the outer space and eons until the second phase of this dream. In the cosmic flow of time you saw worlds coming to existence, blooming like flowers, actually existing and then disappearing. It was an endless game. If you looked back into the past, you saw eons, if you looked forward into the future there were eons stretching into the eternity and this eternity was contained in the point of the present. One was situated in a state of being in which the 'will-be' and the 'vanishing' were already included, and this 'being' was my consciousness. It contained it all. This 'being-contained' was presented very vividly in a geometric way in the form of circles of different size which again were all part of a unity since all of the circles formed exactly one circle. The biggest circle was part of the smallest one and vice versa. As far as the differences of size are concerned, I could not give any accurate information later on..."

A sort of transition from the ordinary state (see Sec. 3) to a BS-state, without, however, attaining the latter, is presented in a remarkably detailed way in Ciompi (1961, pp. 104–107):

"The patient elevates herself above normal boundaries of time without totally surmounting them. The distinction of the present and the future is not cancelled out as the patient still speaks about both dimensions, yet the line between the actual present and the only maybe-possible and unreal future becomes swaying and possible to cross. Both dimensions incapsulate and overlap each other without a steady transition. The future fuses with the present and vice versa and experiencing acquires a flickering twilight character which is radically distinguished from how a healthy person anticipates the future in day-dreams and the like...

The edge between the present and the past is swaying as well. At the same time and in a totally different way, the past is included in and fuses with the events of the present as well as usually the present is a part of the past. There is a kind of condensation of time; the present is not distinguished amidst the continuous-steady flow of the past any more, but at the same time the present is not filled with something past as it usually is with normal people; in this case it overlaps...

The three temporal levels of past, present and future therefore seemed to overlap in the psychotic experience of the patient in an extremely peculiar simultaneousness without really invalidating the distinction of past, present and future."

5. THE TYPE 2%2 SPACE-TIMES OVER FINITE FIELDS

The classification of the 'pathological' space-time configurations given in the preceding sections relies on the fact that we change the position of the ideal line in the projective plane whereas the ground field of the latter remains intact, being identical to that of the real numbers. In this section we shall consider a complementary case.

That is, we shall keep the ideal line in a fixed position, and replace the real numbers by some other field(s). At a formal level, this case has already been dealt with in detail in Saniga (1998b,c), where the interested reader is referred to for the corresponding mathematical background. Here, we shall introduce only those elements of the formalism that are necessary for grasping the idea behind, focusing on the case when the ideal line is in a general position, i.e. when it incorporates neither the base point ($B_{1,2}$), nor the singular point S (see Fig. 1).

To this end, it is necessary to generalize our concepts of the past, present and future, given in Sect. 3 for the case of the field of real numbers R, to an arbitrary ground field. This is quite easy if we take into account factorization properties of the polynomial $P(x)$ in eqn. (5). Obviously, if the ideal line is in general position, then both m and n in eqn. (4) are non-zero. As these two parameters are now kept fixed, the factorization properties of the polynomial $P(x)$ depend solely on the value of the parameter ϑ. From eqns. (6)–(9) it follows that if this polynomial splits into two distinct factors, two coinciding factors, or does not factor at all, then the conic Q^{ϑ}, eqn. (3), is (stands for) a hyperbola (an event of the past), parabola (the present moment), or an ellipse (an event of the future), respectively. And this is the definition of the three temporal domains that can clearly be extended to any field.

Let us now take an event of the R-*future*, for example the one defined by $\vartheta = \vartheta_0 = 6$ for $m = 1$ and $n = 2$; then

$$P(x) = x^2 + 2x + 6. \tag{13}$$

Next, switch to a finite (Galois) field, say to F_7 – the field of residues modulo 7. Since in this field $+2 \equiv -5$, the above polynomial factors as

$$P(x) = x^2 + 2x + 6 = x^2 - 5x + 6 = (x-2)(x-3) \tag{14}$$

which implies that the event in question belongs to the F_7-*past*! Moreover, as

$$\tilde{\vartheta}_0 = \vartheta + 7\ell, \quad \ell \in N, \tag{15}$$

represents one and the same conic in F_7, we see that this past moment of the F_7-temporal corresponds to an *infinite* sequence of events of the R-time: the events with non-negative values of ℓ being in the R-future, while those for which ℓ is negative falling into the R-past. Hence, what we perceive as an ordered sequence of events in our ordinary state, i.e. the state backed by the reals (Saniga, 1998a), will appear as a disordered, chaotic set in an 'extraordinary' state that corresponds to a modular field (Saniga, 1998b).

This is another fascinating aspect of our theory that finds its counterpart in the realm of psychopathology. Christopher Mayhew, a (former) member of the British parliament, volunteered to participate in a series of laboratory experiments with the drug called mescaline. He reported his experience in the article "An Excursion Out of Time" in the London *Observer* on October 28, 1956, and the way how he perceived time under influence of this drug conforms nicely to the above-outlined pattern (Ebin, 1961, p. 295):

"For half an hour nothing happened. Then I began feeling sick; and various nerves and muscles started twitching unpleasantly. Then, as this wore off, my body became more or less anaesthetised, and I became 'de-personalized,' i.e., I felt completely detached from my body and the world...

This experience alone would have fully justified the entire experiment for me..., but at about 1.30 all interest in these visual phenomena was abruptly swept aside when I found that *time* was behaving even more strangely than colour. Though perfectly rational and wideawake...I was not experiencing events in the normal sequence of time. I was experiencing the events of 3.30 before the events of 3.0; the events of 2.0 after the events of 2.45, and so on. Several events I experienced with an equal degree of reality more than once.

I am not suggesting, of course, that the events of 3.30 *happened* before the events of 3.0, or that any event *happened* more than once. All I am saying is that I experienced them, not in the familiar sequence of clock time, but in a different, apparently capricious sequence which was outside my control.

By 'I' in this context I mean, of course, my disembodied self, and by 'experienced' I mean learned by a special kind of awareness which seemed to comprehend yet be different from seeing, hearing, etc...I count this experience, which occurred when, as I say, I was wide awake and intelligent, sitting in my own armchair at home, as the most astounding and thought-provoking of my life."

6. DISCUSSION AND CONCLUSION

The findings and results of the previous sections provide us with further evidence, much stronger and provoking than that already given in Saniga (1998b,c), that *not only* are the manifestations of 'mental' or 'psychological' space-times so diverse and unusual that they totally fail to conform to the generally adopted picture of the macroscopic physical world as being locally Minkowskian (see, for example, Misner *et al.*, 1973), *but* that there also does exist a unique mathematical framework which, at least qualitatively, underlies and unifies their seemingly bizzare properties. Hence, any attempt to disregard these 'psychopathological' space-time constructs as pure hallucinatory phenomena would simultaneously cast a doubtful eye on the very role of mathematics in our understanding of Nature. To the contrary, it is just mathematics (here projective geometry) that plainly tells us that it is far more natural to expect all these 'peculiar' states to be simply as real as our ordinary ('waking') state – this being one of the principal messages the paper should convey to the reader. We are, however, fully aware that this point of view is very likely to meet with scepticism, and even with fierce opposition, from the side of 'hard-line' scientists, because (St. John, 1977, pp. 161–2):

"...the traditional scientific mind refuses to recognize even the possibility of what I am concerned with – the actual nature and quality of the subjective, mystical experience. Materialist scientists of course have no difficulty in recognizing intangible phenomena such as magnetism or cosmic rays passing through a cloud chamber, because they can measure them. Anything that cannot be measured and enumerated doesn't, for the practical purposes of science, exist...By not being able to observe beyond the physical correlates, to reach the internal mental states themselves, science fails to get to grips with what really matters, to recognize that for the individual experiencing them these mental states are much more than a by-product and quite, quite different from a series of (maybe related) biochemical and neurophysiological happenings..."

Yet, we are firmly convinced that anything that shows a definite mathematical structure, whatever bizzare and contraintuitive it may appear, deserves effort and ingenuity to be thoroughly explored and examined. Moreover, since the nature of time is, in our opinion, very intimately tied up with that of consciousness, and the latter seems to be a basic characteristic of the world, the above-discussed geometrical patterns of psycho-space-times may well turn out to be a decisive clue to breaking the enigma of time. If nothing else, they should at least stir up some controversy, and perhaps encourage some research along what seems to be an extremely promising line of scientific inquiry.

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