A FINE STRUCTURE OF FINITE PROJECTIVE RING LINES

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Abstract: I will introduce and discuss novel, and rather unexpected, properties of the fine structure of the projective lines defined over finite rings, which emerged as a by-product of our recent applications of these remarkable geometries in quantum physics [1–9]. The corner-stone concept of the talk will be a free cyclic submodule (fcs) over a finite ring and the particular focus will be on those rings which give rise to fcs's not containing any admissible pairs. I will first show that one can fine-tune the neighbour relation by taking into account the cardinality of shared pairs of the fcs's representing given points. This will be illustrated by examples of projective lines defined over local rings; here, in the case of rings of type 8/4 we find two different kinds of a projective line, and as many as four kinds for the 16/8 type. The difference between the individual kinds of lines will be shown to be intimately related with the number of pairs not lying on any fcs generated by an admissible pair – so-called "outliers". I will then proceed to demonstrate that there exist finite rings (some of) whose outliers even generate fcs's. The smallest case when this occurs is the non-commutative ring of type 8/6, but also some other examples – several non-commutative rings of order sixteen – will be given to illustrate the phenomenon. The talk will be finished by musing about possible physical implications of these intriguing features in quantum (information) theory.

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