PROJECT SUMMARY

In recent years, a number of finite geometries have appeared within various contexts of quantum information theory. Amongst the most prominent, one can rank projective lines over modular rings underpinning structure of the generalized Pauli groups of single qudits, symplectic and orthogonal polar spaces describing the commutation properties of the elements of the Pauli groups of multiple qubits and/or certain generalized polygons providing us with a novel look at the nature of still mysterious black-hole—qubit correspondence (BHQC). The project proposed aims, on the one hand, at a deeper understanding of specific already-discovered finite-geometrical settings of quantum theory and, on the other hand, at a quest for other finite geometries relevant for physics in a broader context.

As per the former objective, we plan, in particular, to reveal finer geometric traits of the structure of the three-qubit and four-qubit Pauli groups. In the three-qubit case, we shall analyze the complements of each type of geometric hyperplane of the split Cayley hexagon of order two, which is a prominent subgeometry of the associated symplectic polar space. As these complements are regular cubic graphs, we shall look for those that are bipartite and, so, isomorphic to the incidence graphs of certain symmetric configurations. It is properties of, and interrelations between, these particular configurations that may help unveil hitherto unnoticed geometric relations between elements (and sets thereof) of the three-qubit Pauli group. In the four-qubit case, we shall mainly be concerned with the hyperbolic quadric that is the locus of symmetric elements of the group. This quadric is well-known to admit a graph automorphism of order three, known as triality. We shall employ this triality transformation to ascertain novel relations between various sets of group elements that are of great physical importance. Concerning the latter objective, we intend to study finite projective ring lines that feature also "non-unimodular" points, as these are suspected to underlie some "ugly-looking" Pauli groups. Here, we shall primarily focus on non-unimodular parts of the lines, in particular on those that are homomorphic to some distinguished classical cases. Also, we aspire to deal with various finite generalized polygons and their products with selected distinguished point-line incidence structures, having in mind their possible role in the BHQC.

Successful accomplishment of the project should substantially deepen our understanding of the role of finite geometry in quantum physics and yield a solid mathematical background for evaluating and extending the theory into other areas of physics as well.