ON TWO POPULATIONS OF SUNSPOT GROUPS

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Abstract: An analysis of the character of cycle to cycle variations of sunspot group distributions in observed area and in maximal area with the help of the principal component method permits to put forward the following hypothesis. Distribution variations are caused by a 80-year cycle change of a size ratio of two sunspot populations observed at the Sun. The first population corresponds to a "normal" distribution and dominates at the secular cycle maximum epoch and the second one contains a larger number of smaller sunspots and a smaller number of larger sunspot and gives an essential contribution at the minimum epoch. In favour of this fact linear relationships between the cycle mean characteristics of sunspots (T_0, $\bar{S}$, S_M, $\bar{H}$ etc.) evidence too. Within an 11-year cycle the population ratio variations are less regular.

A Concept of Two Populations and the Problem of the Accurate Determination of Their Characteristics

A study of the cycle-to-cycle variations of sunspot group area distributions with the help of the principal component method led us to the concept of two sunspot group populations [1]. It was in fact found that with an accuracy of 2—3% the distributions could be represented by the following expression:

$$f_i(S) \approx \xi_i \varphi_x(S) + \eta_i \varphi_y(S).$$

Here $\varphi_x(S)$ and $\varphi_y(S)$ are orthogonal. On applying an affine transformation the latter expression becomes

$$f_i(S) = \xi'_i \varphi_1(S) + \eta'_i \varphi_2(S),$$

where $\xi'_i > 0$, $\eta'_i > 0$, $\varphi_1(S) > 0$, $\varphi_2(S) > 0$, $\sum_s \varphi_1(S) = 1$ and $\sum_s \varphi_2(S) = 1$. Therefore, this representation is a linear combination of two non--orthogonal distributions. It was found that for all cycles considered the relationship

$$\xi'_i + \eta'_i = 1$$

is satisfied, i.e. that any considered distribution can be presented as a superposition of two basic distributions $\varphi_1(S)$ and $\varphi_2(S)$. The quantity ξ'_i then represents a part of the objects belonging to the distribution $\varphi_1(S)$ of population I in the considered sample. Clearly, a similar expression may be obtained for the other parameters, e.g., for the mean area $\bar{S}$:

$$\bar{S}_i = \xi_i \bar{S}_1 + (1 - \xi'_i)\, \bar{S}_2 \qquad (\bar{S}_1 > \bar{S}_2)\,.$$

This expression may be transformed to read

$$\bar{S}_i = \bar{S}_2 + \xi'_i(\bar{S}_1 - \bar{S}_2) = \alpha + \beta \xi'_i\,.$$

Hence, many characteristics of the sunspot groups must change according to a linear law and must be connected by a linear relationship, if the assumptions about the existence of two populations, which the portion ratio changes from cycle to cycle, is correct. However, we note that it is easier to confirm the linear relationship than to estimate the true values, say, of $\bar{S}_1$, $\bar{S}_2$ and ξ'_i.

Generally speaking, in order to find the true values of ξ'_i, it is necessary to know $\bar{S}_1$ and $\bar{S}_2$, or at least to know the exact values of ξ'_i for two cases. It is clear that the $\bar{S}_i$-values, the larger $\bar{S}_1$ and the smaller $\bar{S}_2$, must not occur. In other words we must know the limits of the $\bar{S}_i$-values in advance. It is possible to adopt a slightly different procedure. It is then necessary to find the limiting values of ξ'_i and η'_i which qugrantee non-negative values of $\varphi_1(S)$ and $\varphi_2(S)$, as well as of ξ'_i and η'_i for all i and S, i.e. to determine all permissible affine transformations $(\xi_i, \eta_i) \rightarrow (\xi'_i, \eta'_i)$. However, for practical purposes the values of ξ_i and η

and, therefore, also the values of ξ'_i and η'_i are determined as statistical estimates on the basis of the samples of a finite and not so large a size. Hence, the deterministic formulation of the problem is incorrect, because in some cases there is no solution (a set of such affine transformations is an empty one) and the stochastic formulation of the problem leads to an ambiguous solution, according to its nature. Provided we can add some other parameters to S, it is possible that the sets of permissible ξ_i- and η_i-values, determined on the basis of different parameteric data, do not overlap.

Apparently the question of the method of determining the optical totality of affine transformations for different sets, which guarantee compatible solutions, requires a special investigation. If we introduce a new arbitrary parameter q'_i, related to ξ'_i by a linear relationship with unknown coefficients, it is possible to obtain equations of the considered characteristic dependence on q'. In this case, it is necessary to speak of the quasi-populations A and B, determined for two bound and arbitrarily chosen values of q', but not of the true populations I and II.

The Linear Dependence of Sunspot Characteristics on q'

Let us consider the possibility of describing the cycle-to-cycle variations of the sunspot characteristics by a linear relationship of the kind

$$R_i = a_R + b_R q'_i .$$

To solve this problem the following data were used: the mean lifetimes T_0 of the sunspot groups [2], the sunspot group maximum area (S_M) distributions [3], the sunspot group maximum magnetic field (H_M) distributions [4], the sunspot group area (S) distributions for the years 1913—1954, kindly placed at our disposal by Kopecký and already used in papers [1, 5]. The representations of type [1] were found for the distributions of the values of S, S_M and H_M, and the corresponding values of ξ_i, η_i and the mean values $\bar{S}$, $\bar{S}_M$ and $\bar{H}_M$ were estimated. Using 10 parameter values for four cycles and normalizing every four to their r.m.s. deviations, we found the coefficients of the linear relations for them and the general values of q'_i:

$$q'_{15} = -0.663\,; \qquad q'_{16} = -0.253\,;$$

$$q'_{17} = +0.262\,; \qquad q'_{18} = +0.654\,.$$

Table 1 gives the values of the coefficients a (the 1st line), b (the 2nd line) and of the general part of the parameter dispersion, described by the investigated linear relation (the 3rd line).

The linear dependence of these 10 parameters on q' does not cause any doubt. The reverse problem to determine q', using the observed values of the parameters, may be solved simply, but with a smaller accuracy. Because of this we ought to unite the estimates of q', obtained with the help of some parameters. Ascribing equal weights to the observed values of different parameters, one may obtain, e.g., the following relation:

$$q' = 0.07354\ T_0 + 0.005334\ \bar{S} + 5.8780\ \xi_S - 0.2524\ \eta_S - 4.5462\,.$$

It was used to determine q' for each year from 1913 to 1945. The results of the computations are presented in Figure 1.

The Time Variations

The results of paper [1] demonstrate the run of parameter q' during the 80-year cycle. Only we ought to note that in [1], in Figure 4, this variation of q is presented with respect to q' with an accuracy upto the linear transformation. In Figure 2 the recomputed values of q' are given for cycles 12—18 according to the relation

$$q' = 0.07354\ T_0 + 0.004013\ S_M + 13.178\ \xi_{SM} - 1.7005\ \eta_{SM} - 5.3196\,.$$

Let us turn our attention to Figure 1. Although the general tendency of q' to increase is undoubtable, nevertheless, the scattering of the individual points is large. We should note that the values of q', determined for some year, using different parameters, show an even greater scatter. This fact apparently indicates that the concept of two popu-

Table 1

T_0	$\bar{S}$	ξ_S	η_S	$\bar{S}_M$	ξ_{SM}	η_{SM}	H_M	ξ_{HM}	η_{HM}
9.15	177	0.4995	0.0125	152	0.3089	0.0192	1184	0.4905	0.095
3.40	46.9	0.0425	−0.9906	62.3	0.0190	−0.1470	−384	0.1923	−0.936
0.987	0.994	0.984	0.991	0.997	0.918	1.000	0.991	0.992	0.954

lations is not valid, if applied to individual years. The dashed lines in Figure 1 correspond to the values of q' for each cycle found above. If the

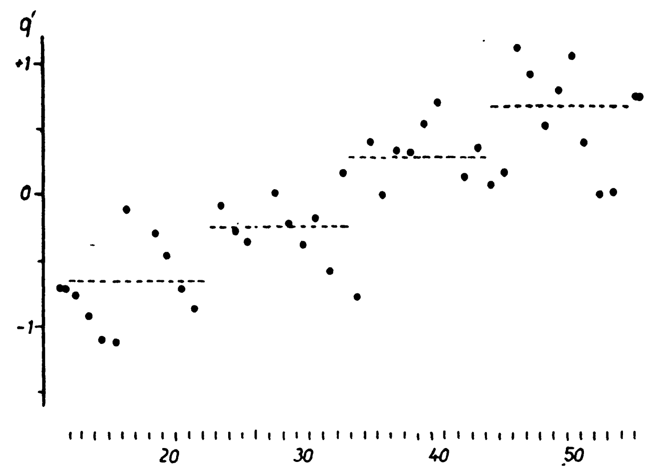

Fig. 1. The annual values of q' from 1913 to 1954.

values of the deviations of q' from these levels for the same phases of the 11-year cycle are averaged, the mean 11-year variation of q' is obtained (Fig. 3). We should note that this variation is not very reliable, because some values of $\Delta_{11}q'$, differ from zero at an insufficient confidence level. However, some regularities, which agree with the obser-

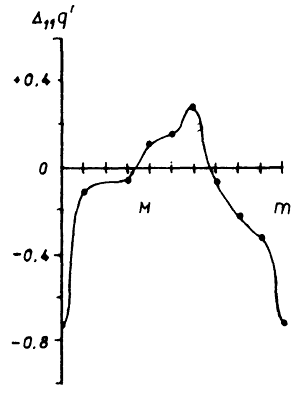

Fig. 3. The 11-year cyclic variation of $\Delta_{11}\ q'$.

vations, may be shown. In the years of minima $\Delta_{11}q'$ has large negative values, i.e. population II, which mainly consists of small sunspots, is dominating. Population I prevails in the maximum epoch, especially in the second year after the cycle maximum, where the proportion of very large sunspot groups increases. The large scatter of the annual values of q' and the small reliability of the cyclic curve $\Delta_{11}q'$ are the reasons which prompt the interest in the problem, whether q would change continuously from year to year, or uneven-

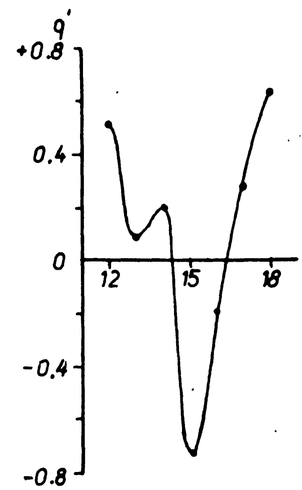

Fig. 2. The variations of q' during the 80-year cycle.

ly from cycle to cycle. For this purpose we have computed the regression coefficients

$$q' = c_1 + c_2 t$$

in each cycle, assigning weights proportional to the number of sunspot groups observed during the considered year to the individual values of q'. The computed values were equal to +0.0023, +0.0009, +0.0017 and −0.0031 which is less by more than an order of magnitude than required by the sequence q'_{15}, q'_{16}, q'_{17} and q'_{18}. On the contrary, if the regression coefficients were computed for the interval from one cycle maximum to the next, the agreement would be satisfactory. Therefore, the parameter q' varies from cycle to cycle in jumps. Hence, the concept of two populations is valid only in the application to the 11-year cycle on the whole. Apparently the fixed values of q' predetermine the character of cycle development more than the conditions under which populations I and II exist within the cycle. Unfortunately, we cannot say at present what the changes of q' depend on, whether they are determined by the 80-year cycle of the changes of the physical conditions in the convective zone [6], or by the situation created by the preceding cycle.

The Characteristics of the Quasi-Populations A and B

We shall determine conditionally the quasi-populations A and B as the totalities of sunspot groups for $q' = \pm 0.7071$. Therefore, the quasi-population A corresponds to the sunspot groups usual for the 80-year cycle maximum and the quasi-population B to those for the minimum. Figure 4 gives the sunspot group area distribution densities of both quasi-populations. The small sunspot groups prevail in the quasi-population B, but the large sunspot groups occur more frequently in the quasi-population A.

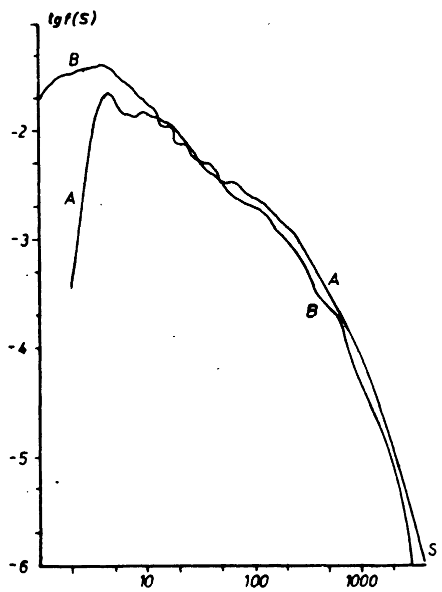

Fig. 4. The densities of the sunspot group area distribution in the quasi-populations A and B.

It is curious that there are "steps" on both the curves, i.e. parts corresponding to the stable stages of the sunspot group development [5]. However, these parts do not coincide in the quasi-populations A and B. The mean values of the physical characteristics, computed with the help of Table 1, are given in Table 2.

Table 2

	T_0	$\bar{S}$	$\bar{S}_M$	$\bar{H}_M$
A	11.56	208	196	913
B	6.75	145	108	1456

The relation $H_M = \psi(S_M)$ is interesting. To obtain it we shall apply the equiguaranteed quantile method [7]. Since the distributions of the S_M- and H_M-values have long tails, it is more advisable to construct the functions $N(R) = 1/[1 - F(R)]$. $N(R)$ represents the number of sunspot groups which have parameter values not exceeding R and fall in with the shape of the sunspot group, the parameter value of which is equal to R. Such curves for the quasi-populations A and B, constructed using S and H, are presented in Figure 5 and 6. Now, the stochastic dependence $H_M = \psi(S_M)$ can be obtained in the following way. We shall give some totality of the values of N each of which has its own corresponding values of S_M

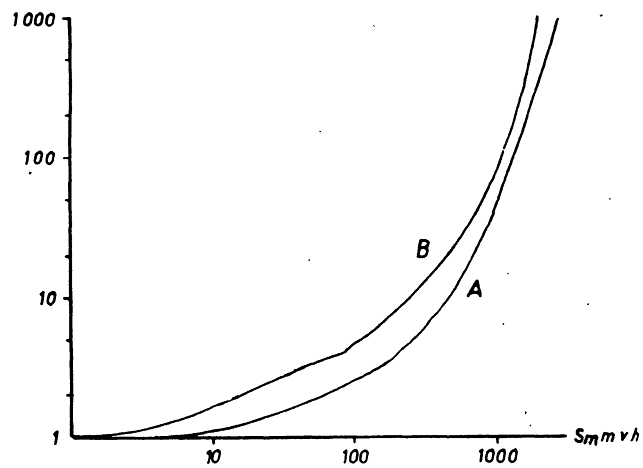

Fig. 5. The dependence $N(S_M)$ for the quasi-populations A and B.

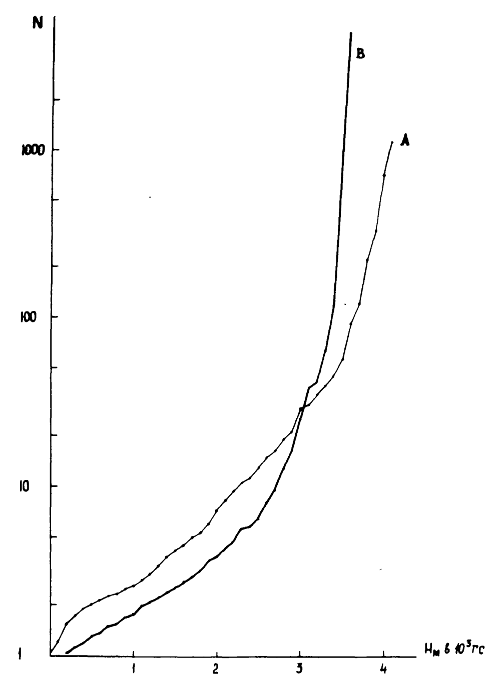

Fig. 6. The dependence $N(H_M)$ for the quasi-populations A and B.

and H_M, forming pairs. One is then able to construct $H_M = \psi(S_M)$ in all details. The obtained curves are presented in Figure 7. The curves A and B only differ in the logarithmic scale of H_M (the right-hand one).

tunately, only the Mount Wilson observations were used as the initial material in cycle 18. A series of magnetic field measurements was begun by the Potsdam Observatory and these can be used for verification. We suspect that during

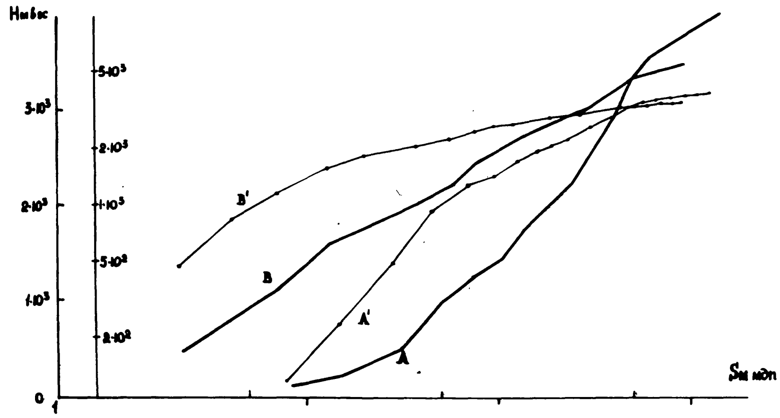

Fig. 7. The dependence $H_M(S_M)$ for the quasi-populations A and B.

We can see that upto $S_M \approx 10^3$ mvh the magnetic field intensity in the B sunspot quasi-population is larger than in the A quasi-population. The parts of the curves within 100 mvh$<S<$1000 mvh obey the approximate formula of the type $\lg H_M = a \lg S_M + b$ best of all. However, the magnetic field intensity in the quasi-population A is anomalously low. Actually, according to modern ideas, the magnetic field in the pores and nodes is not less than 10^3 Gauss. For the quasi-population B it corresponds to a sunspot group area of about 16 mvh (or about 8 mvh for one sunspot), which is quite acceptable, if effects of blurring are taken into account. But in the quasi-population A magnetic field intensities equal to 10^3 Gauss are only reached for areas of about 100 mvh, which is certainly very strange. It is impossible to explain so unusual a pattern of the dependence $H_M = \psi(S_M)$ by the presence of blurring effects. It seems to be unlikely that sunspot groups of this mean size could exist with so small magnetic fields. It is also difficult to make this fact conform to modern theoretical ideas. Unfortunately,

cycles 17 and 18 the magnetic field observations of small and middle-sized sunspot groups made at the Mount Wilson Observatory were burdened with extremely large systematic errors, unless one could prove that such small magnetic field intensities were quite real.

Evidently, the differences in the characteristics between the sunspot groups of populations I and II are not smaller than between the sunspot groups of the quasi-populations A and B. We are prepared to assume that such differences exist for other characteristics, not considered here. For example, by virtue of the relationship determined between the magnetic field intensity and the sunspot temperature, the sunspots of population II (the quasi-population B) should be cooler. For this population the stable stages of evolution are expressed more clearly. Apparently there are grounds to speak about a genetic difference between both sunspot population groups. Therefore, the problem arises to explain the causes of the differences in the conditions of origin and in the physical characteristic of the populations.

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