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Multivariate analysis of globular cluster system scaling relations: The roles of galaxy structure and evolution

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Abstract. In this work, we present improved scaling relations for globular cluster systems (GCS) and examine how type, elliptic shape (ϵ), systemic velocity ($V_{\rm sys}$), Sérsic index (n), dynamic index ($R_e \times \sigma$), light profile slope (α, γ) and age affect the number of globular clusters ($N_{\rm GCs}$). To get a better understanding of the formation and development mechanisms of globular clusters, we separate the influence of the structural and dynamic parameter using several source data from SLUGGS, ACSVCS, and Forbes et al. (2018). Our findings support the presence of a precise co-evolution scenario where the halo's age, shape, and depth all have distinct effects on globular cluster abundance.

Key words: globular clusters – number of globular clusters – age – galaxy formation – dynamical parameters

1. Introduction

Among the earliest stellar systems, globular clusters are essential to comprehending the history and evolution of galaxies, De Bórtoli et al. (2022). These systems have endured over several cosmic epochs, offering important insights into the early universe's star formation circumstances, Al-Mashhadani & Abdullah (2020). The stars within GC are in a compact structure distinguished by very high stellar density, low metallicities and relatively uniform ages. These GCs are important for studying the formation and history of their host systems because they exist in a wide range of environments, from dwarf satellites to massive elliptical galaxies Gieles et al. (2025).

The number of globular cluster ($N_{\rm GC}$) along with the properties of the host galaxy, has revealed scaling relations which enable us to comprehend how a galaxy's mass is obtained and evolves over time, Le & Cooper (2025). The previous studies (e.g., Harris & Van den Bergh (1981); Ashman & Zepf (1992)) well established a crucial relation between the number of GCs and the host galaxy's luminosity, stellar mass, and DM halo mass.

Recently, the new datasets from space based imaging and wide field ground-based surveys have polished these relations. The strong correlation between the galactic halo mass and the total mass of the globular cluster system are confirmed by Hudson et al. (2015) and Harris et al. (2015).

A primary motivation for these studies is the scaling relation that lies in the contradictory aspect of GCs: they are both outgrowth of hierarchical mergers and in situ remnants of ancient stellar population. Their spatial distribution, abundance and structural characteristics reflect the local of star formation condition, also the expanding halo growth history of the host system, Park et al. (2022).

The improvement of simulation techniques and the emergence of cosmological models, such as E-MOSAICS (Kruijssen et al., 2019), provided enhanced prospects to interpret the scaling relation within a physical framework that links them to the temporal and environmental evolution of galaxies and their mass sequences.

This paper expands a field of analyses to incorporate structural and dynamic variables, which includ the dynamic proxy, Sérsic index and light profile slope. We examine the effects of these parameters, individually and together, on $N_{\rm GC}$ to uncover secondary factors contributing to the diversity of GC populations. The paper is divided into the following : Section 2 presents data, and the methodology . Section 3 explores relation between the $N_{\rm GC}$ and the type and examines additional correlations with ellipticity, the product of the effective radius and velocity dispersion ($R_e \times \sigma$), and stellar age. Finally, we discuss the implications of our findings for GC and galaxy formation models.

2. Data and methodology

To understand how the GC scaling relation vary among galaxy types, we assembled a carefully selected sample. This sample covers a comprehensive range for systems., from early to the late type, with star masses $\log(M_*/M_\odot)$ ranging from approximately 10.2 to 11.8. The fundamental was to obtain the right measurements, for both directly observed and to accurately calculated. Which aim to get the number, shapes, and the internal motions for GCs.

SLUGGS data (SAGES Leacy Unifying Globulars and GalaxieS Survey), Brodie et al. (2014) form the core of study's dataset. Which supplying a wide range of the spectral and optical observations detailed for GCs, in nearby-early types galaxies. The SLUGGS data used in the work covers a much wider range of effective radii, enabling us to estimate masses of the galaxies at the edges, regions critical to understand how GCs form. For smaller radii in cluster environments, we included data from the ACS Virgo Cluster Survey, (ACSVCS) Jordán et al. (2009).

Globular clusters and their structure in the Virgo Cluster are precisely measured thanks to the Hubble Space Telescope mission. Additionally, we have integrated systems from the Forbes et al. (2018) catalog. This assemblage connects galactic properties like star masses and dark matter halos to globular cluster data, such as the masses of globular clusters and $N_{\rm GC}$. It allows us to understand how the environment affects the efficiency of globular cluster formation since it covers a broad range of environments, from dense cluster to isolate galaxies. When combined, these data provide us a comprehensive and detailed understanding of the relationship between globular clusters and the characteristics of their host galaxy.

Linear fits were performed using the ordinary least squares (OLS) method after verifying the approximate normal distribution of the residuals. The strength of the correlation was reported using the Pearson correlation coefficient (r), with two-tailed P values to test the zero hypothesis for no correlation.

3. Results and analysis

3.1. Number of GC and galaxy types

The distribution of the number of globular clusters ($N_{\rm GCs}$) among various morphological types of galaxies in the sample under analysis is shown in Figure 1. In contrast to later galaxies, such as spiral (Sa) galaxies, early galaxies, especially lenticular (S0) and elliptical (E) galaxies, tend to contain richer globular clusters, as the figure illustrates.

This distribution offers many significant insights. With a distinct tail that extends into systems with hundreds of globular clusters, E-type galaxies have the largest range. This notable variation is presumably due to the varied formation history of E-type galaxies, whose globular cluster formation efficiency was likely significantly increased by huge mergers and abundant early star formation, Abdullah & Kroupa (2021); De Lucia et al. (2024).

The S0-type galaxies, on the other hand, have a considerably denser distribution, indicating a more uniform evolutionary path that may be controlled by passive aging and small accretion events, even if they still occupy the upper end of the distribution.

The inclusion of the composite E/S0 galaxies category has enable us to link the two classes, indicating a gradation in structural properties and formation history, which is reflected in their stock of GCs Meantime, Sa-type spiral galaxies usually occupy the lower end of the spectrum.

3.2. The number of globular clusters and ellipticity (ϵ)

There is a definite inverse relationship between the number of globular cluster systems and the elliptical shape of the galaxy (Figure 2). The falling curve shows that richer GC systems are often found in more spherical galaxies (low ϵ value), while fewer GC systems are associated with more elongated galaxies (higher ϵ value). The history of galaxy formation and structure evolution is implicitly reflected in this inverse relationship. Intense early star formation and the violent dynamical conditions are frequently linked to the elliptical galaxies, for the dense

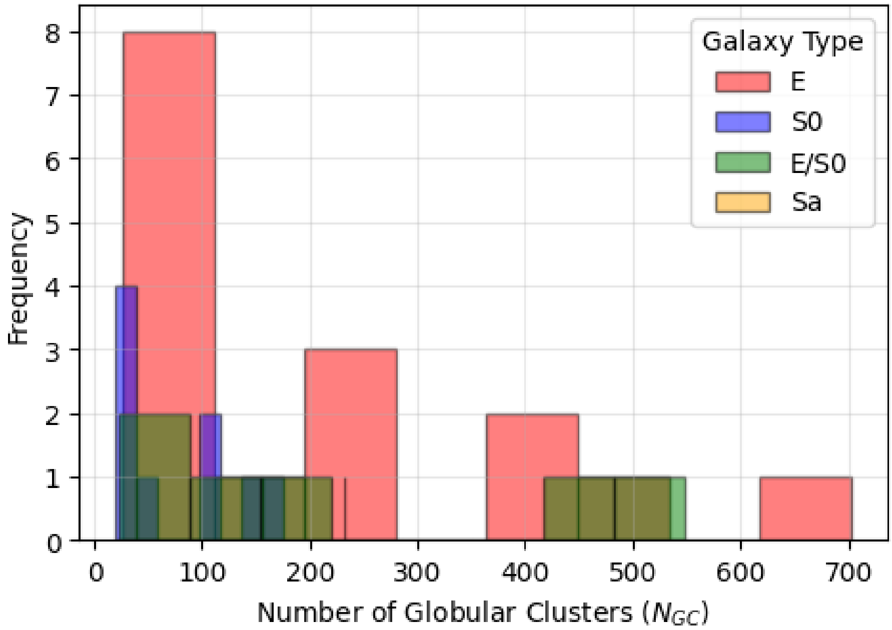

Figure 1. Frequency distribution of the number of GCs over morphological classes, (E= elliptical, $S0$= lenticular, Sa =spiral type a). This displays a clear bias for earlier – type galaxies and emphasizes the tendency of E and S0 galaxies to host richer GCs compared to their later complement.

central bulge (and lower elongation) provide a favorable environment for the formation of numerous massive GCs during bursts of star formation, Valenzuela et al. (2025); Berkheimer et al. (2026). On the other hand, the more elongated shapes may be a sign of rotational effects, a different merger history, or a less intensive star forming environment, that would lead to less effective cluster formation or retention.

The best linear fit can be expressed as follows:

$$N_{\rm GC} = (-392.96 \pm 141.38) \times \epsilon + (299.93 \pm 51.81), \quad (1)$$

with Pearson correlation coefficient (r)=-0.453, p- value (significance)= 0.0093 and coefficient of determination (R^2)= 0.205.

3.3. The number of globular clusters and systemic velocity ($V_{\rm sys}$)

Figure 3 illustrates the relationship between the number of globular clusters in a galaxy and the system velocity ($V_{\rm sys}$), a measure of the galaxy's radial velocity relative to the observer. Unlike some other internal physical measurements

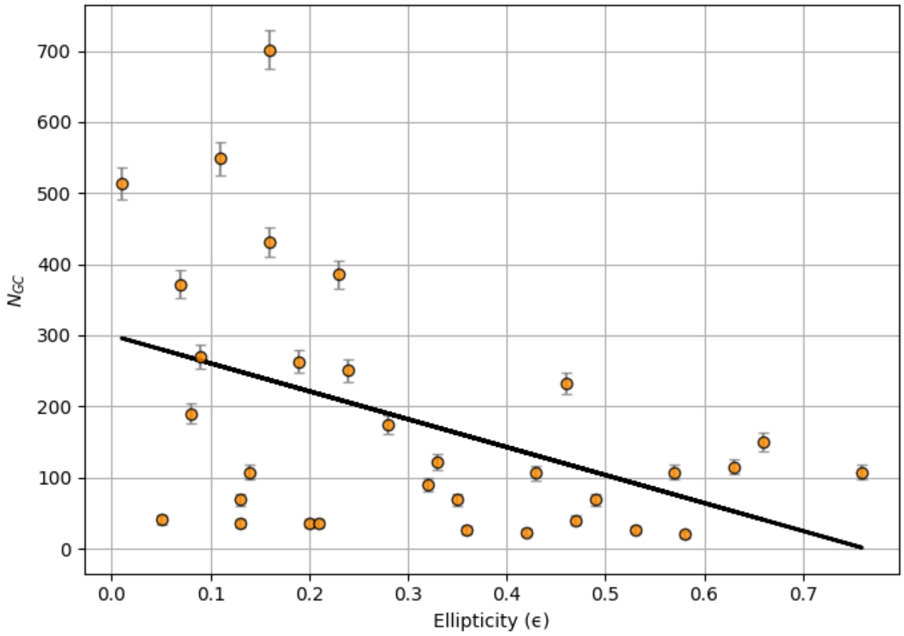

Figure 2. The relationship between the number of globular clusters and the ellipticity of the galaxy. Vertical error bars represent Poisson uncertainties ($\sqrt{N_{\rm GC}}$).This inverse relationship shows that galaxies with a lower ellipticity (more spherical) tend to contain a larger number of globular clusters.

analyzed in this study, the data here show a wide dispersion without a clear linear or statistically significant trend. This wide dispersion of points around the general trend line suggests that the system velocity, in itself, is not a major factor in determining globular cluster abundance. The linear fit is given by:

$$N_{\rm GC} = (-0.0268 \pm 0.0708) \times V_{\rm sys} + (210.84 \pm 89.47), \tag{2}$$

with r=-0.069, p- value $= 0.707$ and $R^2 = 0.0048$.

From an astrophysical perspective, this conclusion is not surprising. The $V_{\rm sys}$ primarily reflects the relative motion of a galaxy within the local or cluster cosmic framework and is strongly influenced by the galaxy's position within the large-scale cosmic web and its own motion, Issa & Abdullah (2024); Dolfi et al. (2021). Therefore, this velocity is not directly related to the internal processes of formation and evolution that govern the efficiency of globular cluster formation and survival, such as the depth of the gravitational well, the history of mergers, or the density of the interstellar medium in the early cosmic ages.

The weak correlation shown in Figure 3 confirms a key hypothesis that the richness of a globular cluster system is a local record of the internal structural and dynamic evolution of the host galaxy. The systemic velocity is still an external quantity that does not directly display the conditions that generated the cluster system, even if properties like mass, centripetal dispersion velocity, or appearance shape may show closer relationships with the host galaxy.

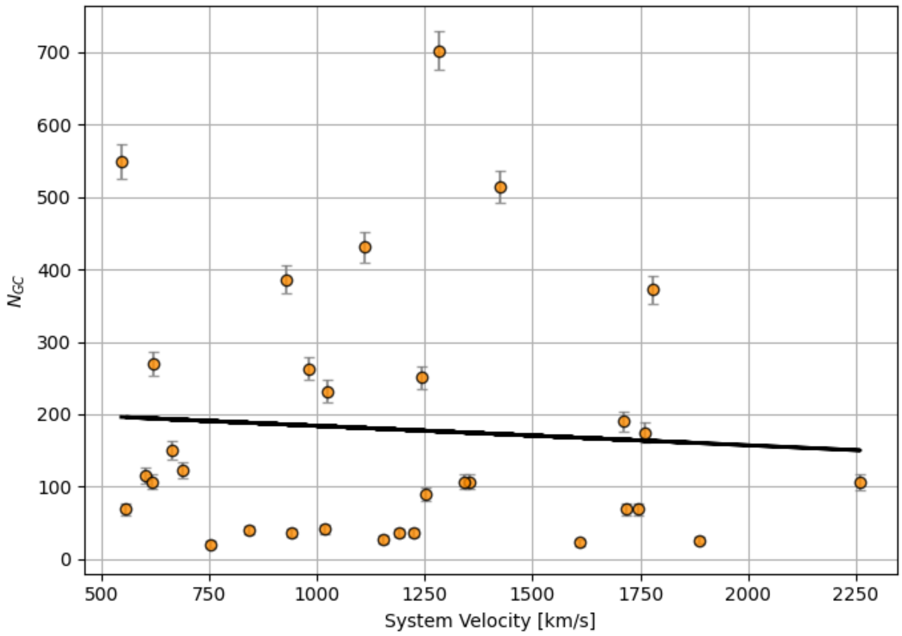

Figure 3. The total number of globular star clusters in proportion to the system velocity ($V_{\rm sys}$). The large dispersion indicates that local forces predominate.Vertical error bars represent Poisson uncertainties ($\sqrt{N_{\rm GC}}$).

3.4. The number of globular clusters and the dynamical mass proxy ($R_e \times \sigma$)

The total number of globular clusters is shown as a function of the dynamic mass proxy ($R_e \times \sigma$) in Figure 4. The assumption that the depth of gravitational potential, not just merely stellar mass, determines the capacity to form and maintain GCs is supported by the dispersion plot, which shows a strong and obvious association.

The following is an expression for the best-fit linear relation:

$$N_{\rm GC} = (0.0665 \pm 0.0215) \times R_e \cdot \sigma + (89.50 \pm 39.93), \tag{3}$$

with r=0.492, p- value = 0.0042 and R^2= 0.242.

Dynamic mass is resolved by multiplying the effective radius (R_e) by the velocity dispersion (σ). The assumption that gravitational efforts are more effective in producing and sustaining GCs is supported by our results, which show a rather high correlation with $N_{\rm GC}$. This supports the theoretical prediction that the development of GCs is more closely associated with the halo binding energy than with baryonic mass indices alone, Kruijssen et al. (2019).

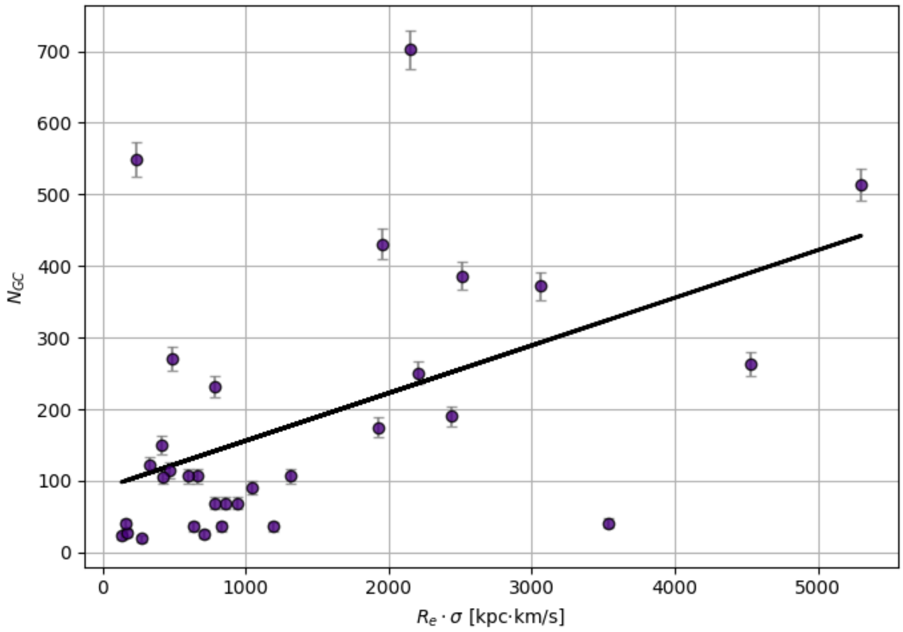

Figure 4. The total number of globular clusters in relation with the dynamic mass proxy ($R_e \times \sigma$).Vertical error bars represent Poisson uncertainties ($\sqrt{N_{\rm GC}}$).

3.5. The number of globular cluster and Sérsic index (n)

The relation between the $N_{\rm GC}$ and the Sérsic index (n), a structural parameter that determines a galaxy's concentration of light profile, is shown in Figure 5. The best linear relationship is

$$N_{\rm GC} = (33.45 \pm 18.80) \times n + (15.07 \pm 96.96), \tag{4}$$

with r=0.4667, p- value = 0.0071 and R^2= 0.217.

A high number of globular clusters are often found in galaxies with higher concentrations toward centers, as indicated by a larger value of n. This is perhaps due to earlier phases of central bulge development and a stronger central gravitational field.

As n increases, we observe a slight rise in N_{GC}. This trend implies that there are more GCs in galaxies with higher center star densities. This might point out to early bulge formation conditions that facilitate the production of huge clusters, such as high gas density and quick dynamic times, Lim et al. (2025).

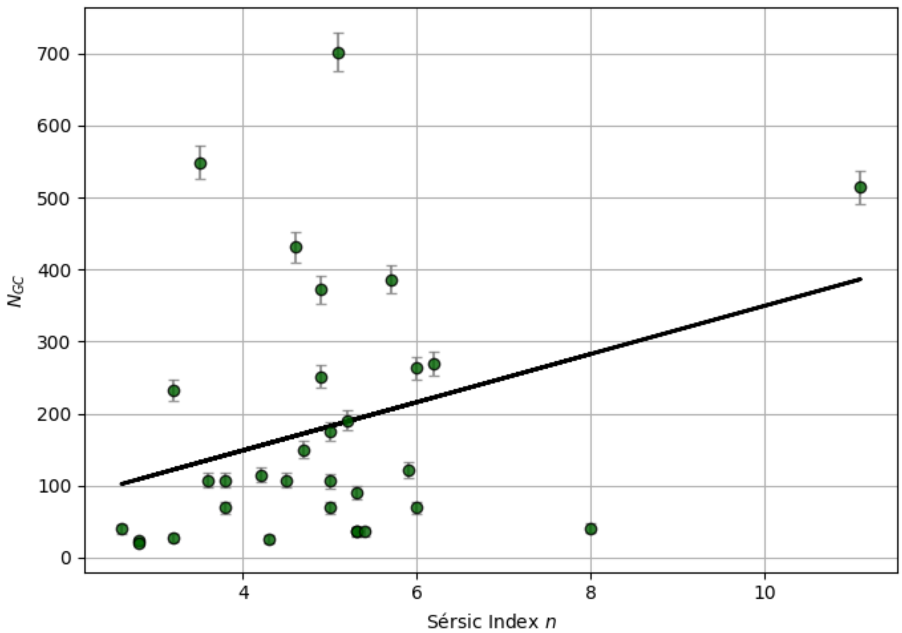

Figure 5. Total number of globular cluster in relation to the Sérsic Index (n).Vertical error bars represent Poisson uncertainties ($\sqrt{N_{\mathrm{GC}}}$). Richer cluster systems are typically found in galaxies with more concentrated light distributions.

3.6. The number of globular clusters and light profile slope (α)

We model the galaxy surface brightness profile $I(R)$ as a broken power law :

$$I(R) \propto \begin{cases} R^{-\gamma}, & \text{for } R \leq R_{\mathrm{break}}, \\ R^{-\alpha}, & \text{for } R > R_{\mathrm{break}}, \end{cases} \tag{5}$$

where γ is the inner slope and α is the outer slope, taken from the SLUGGS catalog, Brodie et al. (2014).

The relation between $N_{\rm GC}$ and the outward slope (α) of the galactic surface brightness profile is shown in Figure 6. Flatter outer slopes indicate more extended, denser stellar halos, which are associated with more globular clusters, likely as a result of past accretion processes. The linear fit reveals a negative trend:

$$N_{\rm GC} = (-471.19 \pm 154.00) \times \alpha + (259.95 \pm 38.20), \tag{6}$$

with r=-0.488, p- value = 0.0046 and R^2= 0.238.

The outward slope (α) of the galactic surface brightness profile is only weakly correlated with $N_{\rm GC}$. However, systems with steep outer profiles tend to have a greater number of globular clusters. This suggests that the extended stellar halos that retain information about accretion satellites may be GCs, Brodie et al. (2014); Elmegreen & Efremov (1997).

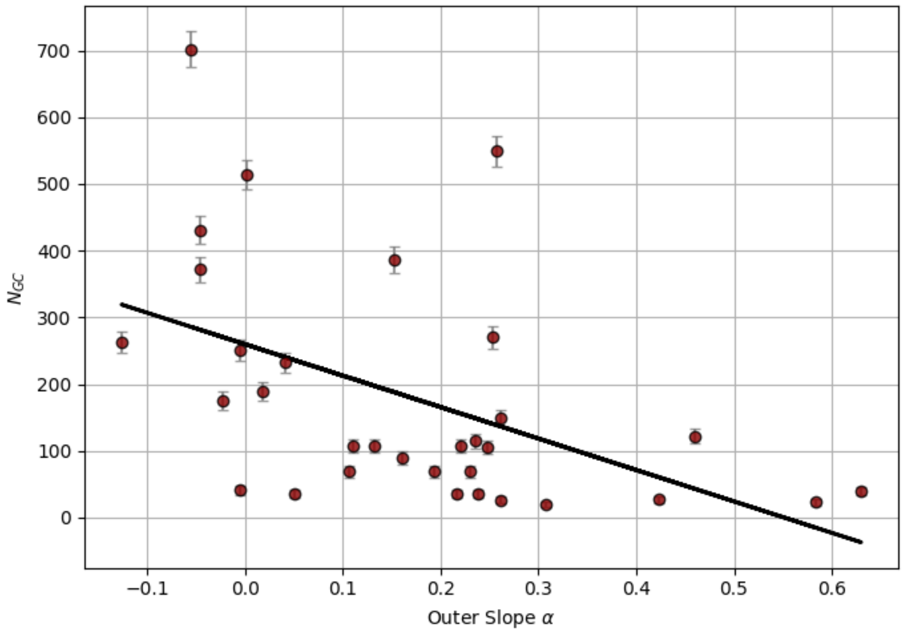

Figure 6. Total number of globular clusters versus the outer light profile of the galaxy (α). The weak negative trend indicates that flatter outer halos are associated with richer globular cluster systems.

3.7. The number of globular clusters and inner light profile slope (γ)

The number of GCs is plotted against the inner light slope (γ) (Figure 7). The inverse trend is expressed by the following equation:

$$N_{\rm GC} = (-324.78 \pm 62.4) \times \gamma + (1138.86 \pm 175.76) \quad (7)$$

With r=-0.689, p- value $= 1.43 \times 10^{-5}$ and R^2= 0.474.

The light concentration, as indicated by the internal slope, shows a limited but significant correlation with the $N_{\rm GC}$, with low γ values tending to have a greater number of GCs and, consequently, flatter inner profiles. This may arise from dynamic heating resulting from early merger, linking GCs to ancient structural assembly events, Forbes et al. (2020).

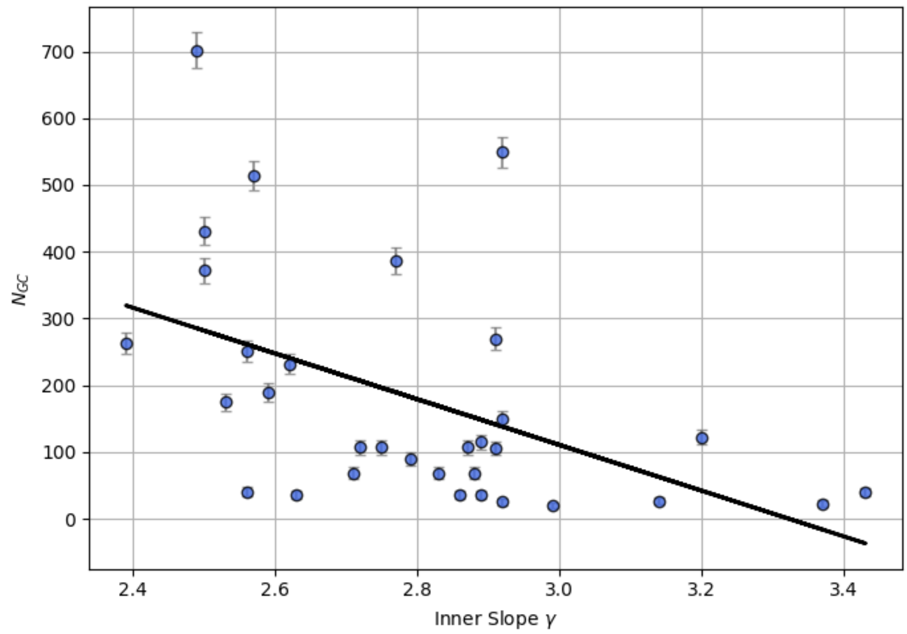

Figure 7. Number of GCs versus the inner slope (γ) of the surface brightness profile. Vertical error bars represent Poisson uncertainties ($\sqrt{N_{\rm GC}}$).The weak inverse trend suggests that flatter central profiles may be associated with early cluster formation resulting from a merger.

3.8. The number of globular clusters and stellar age

Figure 8 illustrates the relationship between the total number of globular clusters and the mean stellar population age of galaxies. The resulting linear relationship is given by

$$, N_{\rm GC} = (23.51 \pm 8.24) \times \text{age} - (95.84 \pm 95.88), \tag{8}$$

with r=0.4752, p- value = 0.0068 and R^2= 0.2258.

This fit reveals a gradual positive trend: older galaxies tend to contain more globular clusters. This finding supports the hypothesis that globular cluster formation was more efficient in the early universe, when the interstellar medium was denser and star-forming conditions were more extreme. The slight but consistent increase in the $N_{\rm GC}$ value with age, particularly for galaxies older than 10 Gyr, suggests a fossil imprint of early cosmic conditions. The GC formation might have resulted from more frequent or dominant star formation bursts in systems that formed earlier, likely in high density regions of the cosmic web.

The correlations discussed in this work will need to be explained by the theory of galaxy formation and evolution. Therefore, a useful window into the time and environment of early galaxy formation epochs is provided by the correlation between the ages and $N_{\rm GC}$. The positive correlation between age and $N_{\rm GC}$ is qualitatively consistent with the downsizing scenario. More massive early-type galaxies formed their stars earlier and also assembled more GCs (see Yan et al. (2021); Thomas et al. (2005); Thomas et al. (2010)). However, our partial correlation analysis suggests that age contributes beyond mass alone, indicating a direct link between the formation epoch and the GC formation efficiency.

4. Conclusion

This study clarifies the multitudinous relationships that control the richness of GCs in nearby galaxies. Our expanded parameter, for incorporates dynamic proxies ($R_e \times \sigma$), structural indices (n, α, γ), and the stellar population age, that reveals a more complex story of the GC formation and survival. Although, the established relation for total $N_{\rm GC}$ and the stellar mass remains robust.

Our results has confirmed notion that a galaxy's ability to form, and maintain the GCs is mostly dependent on the gravitational potential rather than just on the baryonic mass. This dynamic correlation is emphasized by a very close relationship for $N_{\rm GC}$ and the product of the effective radius, and velocity dispersion. Also, the supports theoretical models that link halo binding energy to abundance of the GCs. Concurrently, the connections with light profile parameters are observed, particularly the Sérsic index and surface brightness gradients which indicate that internal structure and accumulation history leave distinct imprints on globular clusters.

Perhaps much more interesting is the relation between the number of globular clusters and their age. The consistent rise in the globular cluster number

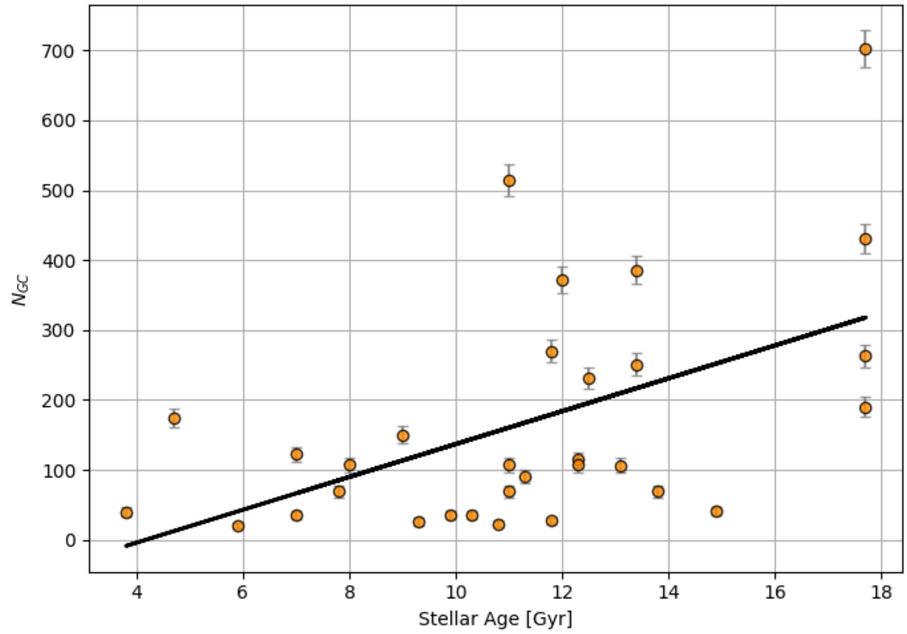

Figure 8. The total number of globular clusters with the age of the stellar population.Vertical error bars represent Poisson uncertainties ($\sqrt{N_{\mathrm{GC}}}$). Globular clusters are more common in older galaxies.

with age points to previous systems that most likely developed in denser cosmic environments, creating the perfect circumstances for the production of enormous clusters that are no longer present in the universe.

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