

Stochastic nonlinear self-oscillatory model of an accretion disk for the X-ray bursting of the microquasar GRS 1915+105

P.-J. Chen and Z.-Y. Wang 

School of Physics and Electronic Engineering, Hubei University of Arts and Science, Xiangyang 441053, China, (E-mail: wangzhiyun@hbuas.edu.cn)

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Abstract. The observed light curves of the microquasar GRS 1915+105 show three main characteristics, namely chaotic, stochastic and semi-stochastic (or nonstochastic). In order to probe the origin causing these different features, we propose a stochastic nonlinear self-oscillatory model of an accretion disk, and use its oscillating luminosities with different noise strengths and nonlinear factors to simulate the observational variability, and compare their correlation dimensions calculated by a nonlinear time series analysis. It is shown that when the stochastic noise strength is small and the nonlinear factor is large, the luminosity curves show chaotic behavior. On the contrary, it shows stochastic or semi-stochastic behavior when a stochastic factor dominates. The simulated data exhibits the same nonlinear and random characteristics as the observational sequences, and their corresponding correlation dimensions are also consistent. This indicates that the X-ray bursting of the microquasar should be related to a nonlinear self-oscillation of the accretion disk.

Key words: accretion disk – self-oscillation – X-ray bursting

1. Introduction

It is well known that the microquasar GRS 1915+105 is a prominent black hole system exhibiting a large variable X-ray emission on different time scales and patterns (Belloni *et al.* 1997; Chen *et al.* 1997; Paul *et al.* 1997). Based on a large collection of multi-epoch RXTE observations, Belloni *et al.* (2000) classified its observed light curves into 12 temporal states, of which four classes show chaos, three are random, while five others show a deviation from being random (Mukhopadhyay 2004; Harikrishnan 2010). That is, the 12 temporal classes could be grouped into three main types, namely chaotic behavior, stochastic behavior and semi-stochastic (or nonstochastic) behavior (Misra *et al.* 2004, 2006), respectively. Many studies suggest that the complexities of light curves may be caused by the noise fluctuations in a chaotic system involving nonlinear processes, whose evolution can be described by several ordinary differential equations. For example, Massaro *et al.* proposed two nonlinear mathematical

models, Fitzhugh-Nagumo (FhN) and Modified Hindmarsh-Rose (MHR) models, and used their solutions to reproduce several different classes of light curves of GRS 1915+105 (Massaro *et al.* 2014, 2020 a, 2020 b, 2020 c). The MHR model described well the transition from stable to bursting states and the occurrence of low-frequency quasi periodic oscillations as a consequence of a transition from an unstable to a stable equilibrium. However, the physical explanation of these nonlinear differential equations is not completely understood.

As an important theoretical model, the oscillations of an accretion disk can interpret a large number of the observational features of black hole binary systems (Kato 2016). In the last decade, the stochastic oscillation of an accretion disk has attracted great attention in astrophysical studies, and has succeeded in explaining some observations. For example, a generalized Langevin equation with white or color noise has been used to describe the vertical oscillations of accretion disks, and the luminosity of stochastic oscillations could reproduce the observations of optical intra-day variability of BLLac objects ((Harko *et al.* 2012, 2014; Leung *et al.* 2011, 2014; Long *et al.* 2016). Based on their model, we have discussed the stochastic resonance phenomenon in power spectral density curves, and considered that the resonance peak is an alternative explanation of the observed low-frequency quasi periodic oscillations (Wang *et al.* 2013 a, 2013 b, 2015, 2016). However, the light curves of this stochastic oscillating model only show stochastic behavior, not present periodicity and chaotic behavior. So Ou *et al.* (2014) developed a chaotic+stochastic oscillating accretion disk model, in which the chaotic factor was introduced into the stochastic oscillating disk. The stable response of this model is completely controlled by the external random and chaotic driving of the system, so it could simulate the light curves showing chaotic behavior of variability, and their correlated dimensions are consistent with that of the observational data.

The oscillations of an accretion disk surrounding a compact object are generally driven by magneto-hydrodynamic turbulence which is an intrinsically nonlinear process. The model for this process should be nonlinear and is expected to show qualitative changes in its behavior as a control parameter is varied. So in this paper, we improve the stochastic oscillation model mentioned above, and suggest a stochastic nonlinear self-oscillatory model of an accretion disk to simulate the light curves of GRS 1915+105. In contrast with linear stochastic oscillation model, self-oscillations of a constrained cycle can generate and maintain a regular mechanical periodicity without requiring a similar external periodicity to drive it (Jenkins 2013). Such constrained cycle oscillations usually occur in non-conservative dissipative systems and have been considered in the dynamics of Cepheids and RR Lyrae stars (Rudd, Rosenberg 1970; Buchler, Kovacs 1987; Das *et al.* 1996). Nakariakov *et al.* (2016) use a limit cycle solution corresponding to the self-oscillations to interpret the undamped quasi-harmonic kink oscillations of coronal loops, and successfully reproduce their observed properties. On some occasions, the complex variability of GRS 1915+105 is characterized by a long series of bursts expected by limit cycles (Castro-Tirado *et al.* 1992). In

addition, nonlinear effects in the model may lead to overstable pseudo-periodic, or even chaotic motions. So we will calculate the dissipated energy of this self-oscillating system to simulate the X-ray variability classes of GRS 1915+105, and probe the intrinsic relation between the simulated data with observational variability by comparing their correlated dimensions.

2. Stochastic nonlinear self-oscillatory model of accretion disk

For a thin accretion disk around a black hole in contact with an isotropic and homogeneous external heat bath, the vertical oscillation of unit mass can be described by a generalized Langevin equation, given by Leung *et al.* (2011)

$$\frac{d^2 z}{dt^2} + \mu \frac{dz}{dt} + \omega_0^2 z = \frac{dF(t)}{M_D}, \quad (1)$$

where z and M_D are the displacement of oscillation and the mass of the accretion disk, respectively. μ expresses a viscosity coefficient and $dF(t)$ is a stochastic excitation. The intrinsic angular frequency ω_0^2 can be written as (Titarchuk, Osherovich 2000)

$$\omega_0 = \frac{4.4\pi \times 10^3}{m} \left\{ \frac{2 - \gamma}{r_{in}^3} \frac{[1 - \gamma / ((\gamma + 1)r_{adj})]}{r_{adj}^\gamma r_{out}^{2-\gamma}} \right\}^{1/2} Hz, \quad (2)$$

where, $m = M_0/M_\odot$, M_0 and M_\odot are the masses of the central object and the sun. $r_{in} = R_{in}/3R_S$ is the innermost radius of the disk and $R_S = 2GM_0/c^2$ is the Schwarzschild radius. $r_{adj} = R_{adj}/R_{in}$ and $r_{out} = R_{out}/R_{in}$ are the adjustment radius and the outer radius of the disk, respectively. The index γ is either 3/5 or 3/4 (Shakura, Sunyaev 1973).

The accretion disk as a whole is subjected to viscous forces mainly from friction between the disk and the external heat bath. If the interaction of the disk with the surrounding moving medium is not stiff but slippery, this friction is a possible way to compensate for the dissipation losses (Nakariakov *et al.* 2016). Further, if we incorporate a cubic nonlinearity into the dissipation, then the damping term of the system may be expressed as $-\mu_0 \frac{dz}{dt} + \mu_0 a z^2 \frac{dz}{dt}$ (μ_0 and a are positive constants). The first term is the linear friction force, by which energy is transferred from the external heat bath to the disk, and the second one is the nonlinear dissipation force resulting in the energy loss of the system.

In addition, if we consider that the self-gravitational potential V_z of the disk is $\frac{1}{2}\omega_0^2 z^2 + \frac{1}{4}\omega_0^2 b z^4$, then the nonlinear gravitational term of a disk oscillation may be represented as $\omega_0^2(z + b z^3)$. We also assume random excitation $F(t) = DdW(t)$, where D refers to the noise strength and $dW(t)$, defining the normalized δ -correlated Gaussian noise with the zero mean and unit variance, may be represented by the formal derivative of a Wiener process, $W(t)$. Then

we replace the equation (1) of the disk motion with a stochastic nonlinear differential equation, which can be written as

$$\frac{d^2z}{dt^2} - \mu_0(1 - az^2)\frac{dz}{dt} + \omega_0^2(z + bz^3) = DdW(t). \quad (3)$$

Self-oscillatory motions described by equation (3) are essentially dissipative, with the dissipative losses compensated by the continuous energy supply of the external heat bath and the radiation luminosity of the disk, representing the energy lost by the disk due to viscous dissipation and the presence of the random force, is given by:

$$L(t) = \mu_0az^2\left(\frac{dz}{dt}\right)^2 - DdW(t)\frac{dz}{dt}. \quad (4)$$

Obviously, because equation (3) contains nonlinear terms and a random term, the luminosity variables can show different behavior, which should be similar to different temporal states of GRS 1915+105.

3. Light curves of observation sequences and simulated data

We show in this section that the numerical solutions of the nonlinear oscillation system of equations (3) and (4) reproduce the main features of three main variability types of GRS 1915+105. Based on the PUBLIC in the RXTE TOO archive (at heasarc.gsfc.nasa.gov), here we choose three representative observation sequences whose observation IDs are 20402-01-02-02, 10408-01-40-00 and 10408-01-08-00, respectively. Their continuous light curves in the energy range 2-30keV binned at 1s intervals are plotted in Figures 1(a),(c) and(e), which show the stochastic behavior, semi-stochastic behavior and chaotic behavior, respectively.

In order to simulate the light curves of three observational sequences, we perform numerical computations of equations (3) and (4) by means of a Runge-Kutta fourth order integration routine, and obtain luminosities with different parameters. For GRS1915+105, we set the same disk parameters as Titarchuk and Osherovich (2000), $r_{in} = 1$, $r_{out} = 10^4$ and $\gamma = 3/5$, $r_{in} = 3$, and adopted $m = 14$ (the mass range of this source is from $10M_{\odot}$ to $18M_{\odot}$ as estimated by Remillard and McClintock (2006)). Then the intrinsic oscillating frequency of the disk can be calculated, $\omega_0^2 \approx 1.5$. In the following we will calculate the self-oscillating luminosities of the accretion disk by adjusting the nonlinear term parameter b and the random strength D , with the other three kept fixed, $\mu_0 = 0.5$, $a = 1.0$, $\omega_0^2 = 1.5$.

Figures 1(b), (d) and(f) show the normalized simulated light curves for (b) $D = 0.3, b = 0.0$, (d) $D = 0.2, b = 0.2$, and (f) $D = 0.1, b = 0.23$, which are similar to the light curves of three observational sequences (see Figures 1(a),(c) and(e)). It is very clear that the presence of the noise and the nonlinear

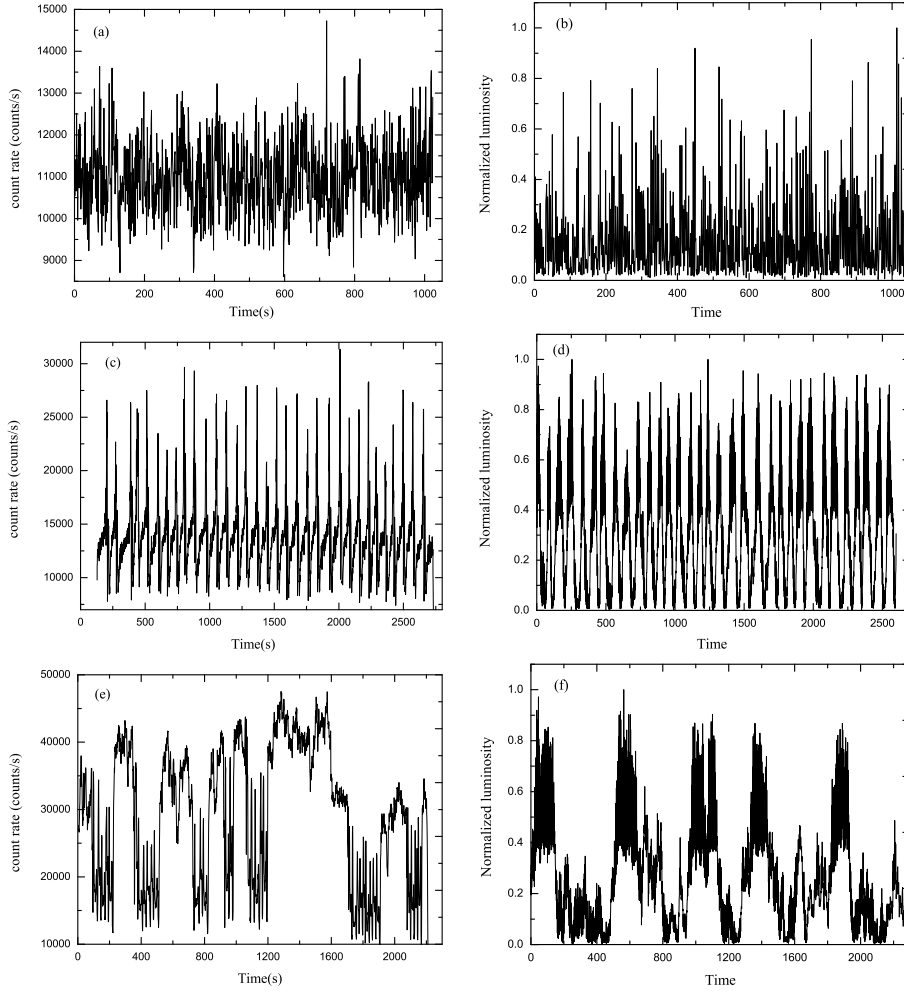


Figure 1. The light curves of the observational sequences and simulated data of GRS 1915+105. The cases (a), (c) and (e) are for the observational sequences 20402-01-02-02, 10408-01-40-00 and 10408-01-08-00, whereas (b), (d) and (f) correspond to the simulated data for $D=0.3$ $b=0.0$, $D=0.2$ $b=0.2$ and $D=0.1$ $b=0.23$, respectively.

term makes the self-oscillating luminosity curves of the accretion disk appear chaotic, random and semi-random. The nonlinear factor makes the system practically appear as chaotic, but noise always suppresses it. Therefore, when the noise fluctuations are much smaller than the variability, the nonlinear factor is dominant, and the chaotic nature can be detectable. Similarly, the random and semi-random appearances are due to the dominance of the noise in the system.

4. Correlation dimension D_2 of light curves

In order to quantitatively compare the similarity between the simulated luminosities and the observed data, we use a nonlinear time series analysis to compute their correlation dimension D_2 , which is often used as a discriminating parameter for a hypothesis testing to detect nontrivial structures in the time series.

4.1. Correlation dimension D_2

The conventional method for the calculation of D_2 is the delay embedding method known as the Grassberger-Proccacia (GP) algorithm (Grassberger & Procaccia 1983; Harikrishnan *et al.* 2006). In this technique, an embedding space of dimension M with delay vectors is constructed by splitting a discretely sampled scalar time series $s(t_i)$

$$\mathbf{x}_i = [s(t_i), s(t_i + \tau), \dots, s(t_i + (M - 1)\tau)], \quad (5)$$

where τ is the delay time computed by means of the mutual information method (Kraskov *et al.* 2004), assuming that the vectors are not correlated. The correlation function for the embedded time series is the number of points that are within a distance R from the center, averaged over all the centers, and may be written as

$$C_M(R) = \lim_{N_\nu \rightarrow \infty} \frac{1}{N_\nu(N_\nu - 1)} \sum_i^{N_\nu} \sum_{j, j \neq i}^{N_\nu} H(R - |\mathbf{x}_i - \mathbf{x}_j|), \quad (6)$$

where $N_\nu = N - (M - 1)\tau$ is the total number of reconstructed vectors, N is the size of the data set and H is the Heaviside step function. The correlation dimension $D_2(M)$ is defined as

$$D_2(M) = \lim_{R \rightarrow 0} \frac{d \log C_M(R)}{d \log R}. \quad (7)$$

In order to test the computational schemes presented here, we first apply the D_2 analysis on the standard chaotic time series and the pure noise from the standard Lorenz attractor and stochastic white noise. Their correlation dimension curves calculated by this method are shown in Figure 2. We can see, as

expected, that the $D_2(M)$ values for the Lorenz system saturate at $M = 2$ to $D_2^{sat} \approx 2$, which is close to the known value of 2.04. For the pure uncorrelated stochastic white noise, $D_2(M)$ varies approximately linearly with M ($D_2 \approx M$) and no such saturation exists. This is consistent with the results of Misra et al. (2006), and confirms that the random data and the low-dimensional chaotic system can clearly be distinguished in this scheme.

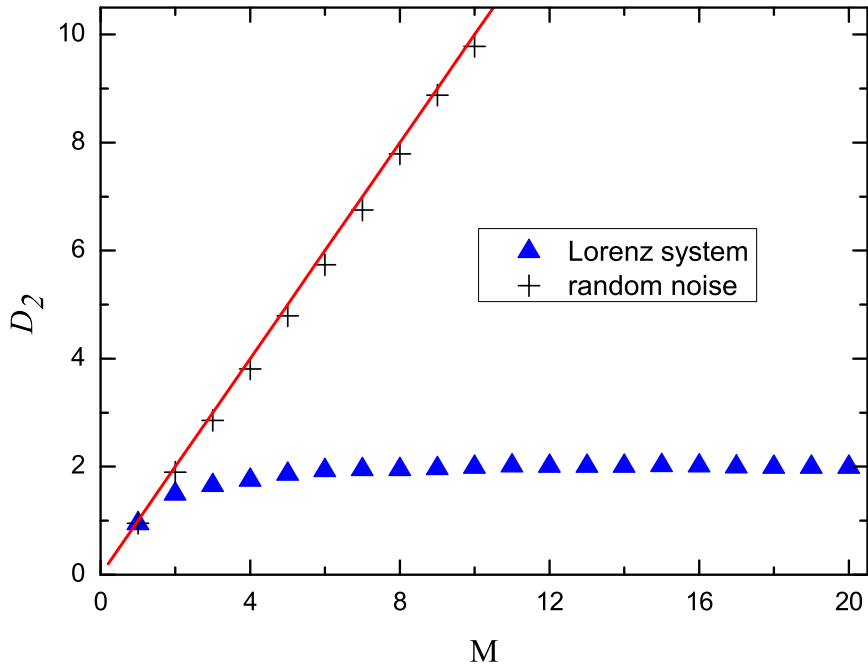


Figure 2. $D_2(M)$ curves for the random noise (crosses) and for a Lorenz system (triangles), the straight line represents a $D_2 = M$ curve.

4.2. Comparison of D_2 between the observed and simulated data

By the scheme mentioned above, we calculate the correlation dimension D_2 of the simulated and the observed data given in section 2. Their $D_2 - M$ curves are plotted in Figure 3.

In this figure, the solid line in each panel indicates the ideal random curve ($D_2 = M$) seen in Figure 2. Figure 3(a) shows the results of observational sequence 10408-01-08-00 and simulated data for $D = 0.1, b = 0.23$, both cases have the same chaotic behavior, namely, their curves show a clear deviation from

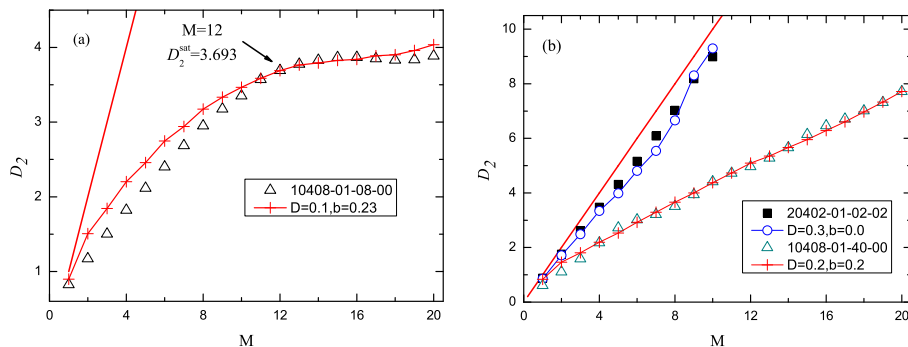


Figure 3. $D_2(M)$ curves for the observation sequences and simulated data.

this ideal random curve, and there is the same saturation correlation dimension of $D_2 = 3.693$ for $M = 12$.

For observational sequences 20402-01-02-02 and 10408-01-40-00, their curves show a different deviation from the ideal random noise (see Figure 3(b)), and D_2 varies approximately linearly with M and no such saturation exists. It can be seen that for the observational sequence 20402-01-02-02, its $D_2(M)$ curve is close to that of the ideal random noise, and is random dominated. While the other one deviates greatly from it and shows semi-stochastic behavior. As expected, the correlated dimensions of these observational variabilities are consistent with the results of the simulated luminosities for different model parameters D and b .

5. Summary and discussion

Many studies suggest that the variability of the micro quasar GRS 1915+105 may be chaotic in nature, this chaotic signature being suppressed in some cases due to the noise dominance when it appears as random or semi-random, but actually this may not be its fundamental characteristic. Therefore, in the present paper we have considered a stochastic nonlinear self-oscillatory equation to describe the vertical oscillations of an accretion disk around a black hole, and obtained the oscillating luminosities showing chaotic, random or semi-random behavior for different noise strengths and nonlinear parameters. Their properties are similar to those of the observed variability of GRS1915+105.

Our results show that the correlation dimensions between observational sequences and simulated data are identical approximately, which implies that the variability of the microquasar may be controlled by the nonlinear self-excited oscillation system; at the same time, it is also affected by the noise with varying intensity, resulting in a variety of variability types. So, as a possible astro-

physical application, this model may turn out to be very useful to understand complicated temporal behavior of X-ray bursts.

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