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# Studying coherent scattering in the CP stars atmospheres

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**Abstract.** Chemically peculiar stars form a very interesting class of stars which frequently show variability. The variability is probably caused by the uneven surface distribution of chemical elements. Some elements are overabundant and some elements are underabundant compared to the solar chemical composition. In the case of chemically overabundant composition some of the rare photon-atom processes can be more important than in the atmospheres of stars with solar chemical composition. We study the importance of Rayleigh scattering by helium.

**Key words:** stars: chemically peculiar – stars: atmospheres – atomic processes – scattering

## 1. Introduction

Chemically peculiar (CP) stars have a complicated structure of their atmospheres. We call them "peculiar" because they differ from other stars whose chemical composition is very similar to the solar one. Several types of CP stars among A-type stars exist (see Preston, 1974). The B-type CP stars are less manifold and are typically divided to helium over- and under-abundant stars.

We study the effect of Rayleigh scattering on the atmospheres of CP stars. Rayleigh scattering is important in cool stellar atmospheres. It is much less important in hot stars, but may be more significant in CP stars.

## 2. Rayleigh scattering

Rayleigh scattering is a process of interaction of photon with a bound electron. The effect of Rayleigh scattering in the atmosphere of our planet is very well known, because it causes the blue colour of the sky.

The best and most accurate way to calculate the Rayleigh scattering cross section is to use quantum mechanics – using the Kramers-Heisenberg equation in the form:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{e^4\omega^4}{16c^4\pi^2\epsilon_0^2\hbar^2}\left|\sum_j\left(\frac{(\varepsilon_{\mathbf{k}_s}\cdot\mathbf{D}_{1j})\,(\varepsilon_{\mathbf{k}}\cdot\mathbf{D}_{j1})}{\omega_j-\omega}+\frac{(\varepsilon_{\mathbf{k}}\cdot\mathbf{D}_{1j})\,(\varepsilon_{\mathbf{k}_s}\cdot\mathbf{D}_{j1})}{\omega_j+\omega}\right)\right|^2, \tag{1}$$

where $\varepsilon_{\mathbf{k}}$ and $\varepsilon_{\mathbf{k}_s}$ denote polarization vectors of incoming and scattered photons, respectively, $\mathbf{D}_{ij} = \langle i\,|\mathbf{D}|\,j\rangle$, $\mathbf{D} = \sum_k \mathbf{r}_k$, where $\mathbf{r}_k$ is a position of the $k$-th electron, $\omega_j = (E_j - E_i)/(h\nu)$, where $E_i$ is the energy of the initial electron state and $E_j$ is the energy of the virtual state. This equation describes coherent scattering of photons on an arbitrary atom. The equation accounts for polarization of radiation, which we will, however, neglect. Rewriting in terms of frequencies instead of $\omega$ we get

$$\sigma = \mathcal{K}(2\pi\nu)^4\frac{8}{3}\pi\left|\sum_j\left|\left\langle n_0, l_0\left|\hat{R}\right|n, l\right\rangle\right|^2\frac{2\nu_j}{\nu_j^2-\nu^2}\right|^2. \tag{2}$$

where $\mathcal{K}$ is a constant from Eq. 1 and $8/3\,\pi$ comes from the integration over angles. One can calculate the cross section numerically, but a simpler way is to rewrite Eq. 2 for frequencies much lower than the frequencies corresponding to the line transitions of the given ion. This is called the low frequency limit and it has a polynomial dependence:

$$\sigma = \nu^4(a_0 + a_1\nu^2 + a_2\nu^4 + \dots + a_k\nu^{2k} + \dots),\ k\in\mathbb{N} \tag{3}$$

where $a_i$ represent the corresponding coefficients in the series. Using this limit we get the well known dependence of the cross section on frequency (we will neglect higher powers than four). The plot of the cross section for hydrogen computed using Eq. 1 is shown in Hubeny & Mihalas (2014, Fig. 6.3). The analytical formula for hydrogen can be found in Lee & Kim (2004). It is significantly more complicated to calculate cross sections for heavier elements.

Rayleigh scattering cross sections can be found in several publications. For example, Colgan et al. (2016) contains cross sections for the neutral chemical elements. Here we list computed cross section for neutral hydrogen

$$\frac{\sigma_{\mathrm{H\ I}}(\nu)}{1\,\mathrm{cm}^2} = 7.18\times10^{-87}\cdot\nu^4 + 1.96\times10^{-117}\cdot\nu^6 + 4.27\times10^{-148}\cdot\nu^8,$$

and singly ionized helium

$$\frac{\sigma_{\mathrm{He\ II}}(\nu)}{1\,\mathrm{cm}^2} = 2.80\times10^{-89}\cdot\nu^4 + 4.79\times10^{-121}\cdot\nu^6 + 6.52\times10^{-153}\cdot\nu^8.$$

These cross-sections are plotted in comparison with the Thomson scattering cross section in Fig. 1.

## 3. Calculation of models

The procedure of model calculation was the following: it was necessary to choose a model atmosphere as input. For stars with solar chemical composition we used an existing model atmosphere grid on the TLUSTY web pages (Lanz & Hubeny, 2007). For non-solar chemical composition we computed stellar atmosphere models from scratch using the TLUSTY code (Hubeny, 1988; Hubeny & Lanz, 1995; Lanz & Hubeny, 2003, 2007).

In the sebsequent step it was possible to calculate the emergent spectrum from the stellar atmosphere model with Rayleigh scattering included. To this end we used the SYNSPEC code. This code does not calculate the model atmosphere but it calculates emergent synthetic spectra. We calculated synthetic spectra with and without Rayleigh scattering included. These spectra were convolved with the Gaussian function, since we were interested only in continuum radiation, not in lines. The convolution made the spectrum smoother. The Gaussian

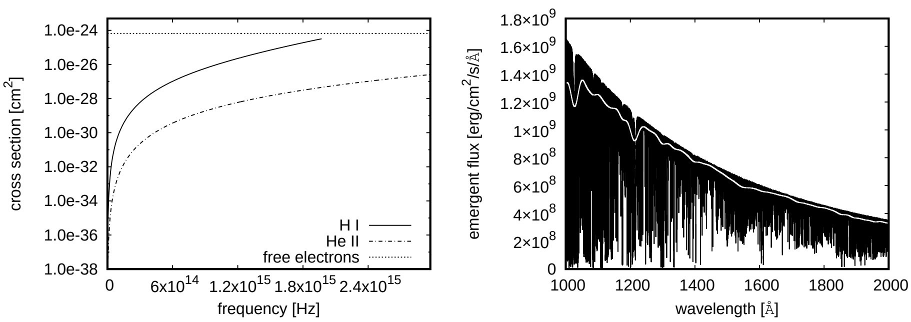


**Figure 1.** *Left*: Cross sections for neutral hydrogen and singly ionized helium. *Right*: Comparison of synthetic spectrum computed with the SYNSPEC code and the same synthetic spectrum after convolution with the Gaussian function.

function used to convolve the spectra was

$$(H * g)(\lambda) = \frac{1}{\sqrt{2\pi}\sigma} \int d\lambda'\; H(\lambda') \exp\left(-\frac{(\lambda-\lambda')^2}{2\sigma^2}\right), \tag{4}$$

where $\lambda, \lambda'$ is the wavelength, $H(\lambda')$ is the flux and $\sigma$ is a constant characterizing a half-width of the Gaussian curve. We choose to plot differences in magnitudes using the equation

$$\Delta m = -2.5 \log \frac{H_{\rm RS}}{H_{\rm NO}}, \tag{5}$$

where $H_{\rm RS}$ – flux with Rayleigh scattering included and $H_{\rm NO}$ – flux without Rayleigh scattering included.

## 4. Computed atmosphere models

We computed several series of atmosphere models. These models differ in helium abundance and in the effective temperature.

### 4.1. Stars with solar chemical composition

There were no significant differences between the measured and synthetic spectral energy distribution for stars with normal chemical composition. Consequently, we expected only very small changes caused by Rayleigh scattering by helium. Results confirmed our expectations and changes expressed in the form of Eq. 5 are about $10^{-4}$ mag large.

### 4.2. Helium overabundant stars

We were particularly interested in helium overabundant stars because for several stars of this type the light curves and the processes contributing to the measured spectra have not been fully explained yet.

We computed models in the temperature range 19–30 kK, with a surface gravity $\log(g) = 4$, and for helium ten times more abundant than hydrogen. The differences of the fluxes are of the order of $10^{-3}$ mag, see Fig. 2. These changes become one order of magnitude smaller with increasing effective temperature.

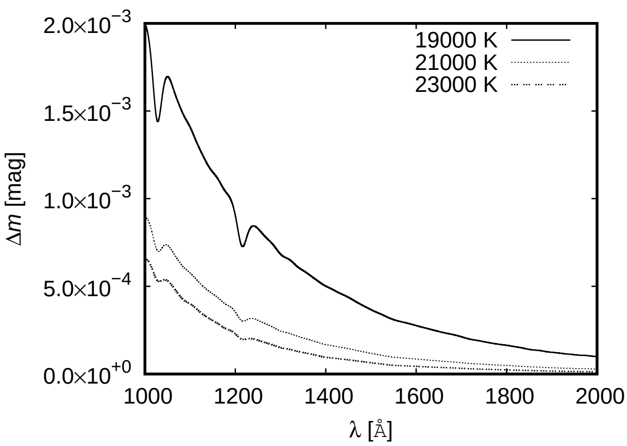


**Figure 2.** Calculated differences for helium rich B-type stars

### 4.3. Other objects

We computed several series of models for white dwarfs with $\log(g) = 8$ and subdwarfs with $\log(g) = 5.5$. The differences in the form of Eq. 5 are about $10^{-4}$ mag.

## 5. Conclusions

We computed several series of model atmospheres and we studied the importance of Rayleigh scattering in these cases. We obtained atmosphere models either from the existing grid or we computed models ourselves from scratch. We used these models as an input for the synthetic spectra calculation using the SYNSPEC code. The spectra were calculated with and without Rayleigh scattering included and were compared for the given temperature. We did not recalculate the temperature structure.

We can see that the largest differences in the flux are for helium stars for Rayleigh scattering by neutral helium. These changes are about $10^{-3}$ mag, which is not a very large value, however, it is possible to measure these changes with highly accurate instruments. A more detailed description of the results for hydrogen and singly ionized helium can be found in Fišák et al. (2016).

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