Collisions as a possible case of TNOs migrations

E.M. Pittich and N.A. Solovaya

Astronomical Institute of the Slovak Academy of Sciences, Interplanetary Matter Division, Dubravská cesta 9, 845 04 Bratislava, The Slovak Republic

Received: August 25, 2009; Accepted: October 17, 2009

Abstract. We present the results of the investigation of a possible mechanism which can cause migration of objects from the Kuiper belt, mutual collisions of the objects or collisions with other populations of small solar system bodies.

Analytical theories about the existence of periodical solutions in the outer variant of an averaged restricted three-body problem (the Sun–Jupiter–small body) established the existence of the periodic solutions corresponding to Keplerian osculating orbits in which the eccentricity, inclination, and semi-major axis have only periodical perturbations. The node exhibits the secular motion. Therefore, only additional forces can have influence on the motion of the Kuiper belt objects and change their orbital elements, the eccentricity, inclination and semi-major axis, such that they will have secular perturbations.

In our study we used model orbits of the Classical objects of the Kuiper belt with different values of the semi-major axis and studied their orbital behaviour after the collision with bodies in orbits similar to the Kreutz cometary orbits. The change of the tangential component of their orbital velocity lies within the interval of $\pm 3$ km s$^{-1}$. Depending on its direction, the objects will migrate to the inner or outer part of the solar system.

Key words: TNOs – Kuiper belt – collisions – Classical object – Kreutz comets – migration

1. Introduction

Within the last decade numerous small bodies located in the region of the solar system beyond the planets extending from the orbit of Neptune (at 30 AU) to approximately 50 AU from the Sun, the so-called Kuiper belt, were discovered. Astronomers classified them into several populations according to their physical and dynamical parameters. One of them, named the Classical objects, contains bodies of sizes from 100 km to 1000 km and less. A population is characterized by orbits of its bodies. Their orbits are much like the orbits of the planets, nearly circular with the orbital eccentricity being less than 0.1, and with the relatively low inclination up to about 10$^\circ$ (e. g. Bernstein et al., 2004; Morbidelli, 2004; Gomes et al., 2005; Delsanti and Jewitt, 2006).

Over the lifetime of the solar system, bodies in the Kuiper belt are subject to various physical processes, including collisions (e.g. Stern, 2003; Levison et al., 2008). Observational evidence for collisions in the Kuiper belt objects is quite good. Mutual collisions of objects in the Kuiper belt could have a variety
of effects on the belt’s structure. The current root mean square velocity in the Kuiper belt is about 1.5 km s$^{-1}$. Such velocity is large enough to lead to a catastrophic disruption of objects and/or to a change of their orbital velocity. (Gurnett et al., 1997; Humes et al., 1980; Jewit and Luu, 2000; Leinhardt et al., 2007). The effect of the mutual collision depends on masses of collisional bodies, a collisional velocity and its direction.

The question arises, if mutual collisions between Kuiper belt objects, or between a Kuiper belt object and other populations of small solar system bodies are extended to inner/outer regions of the solar system. Therefore, the Kuiper belt is believed to be the main source of the Centaurs and the Jupiter family comets (Delsanty and Jewitt, 2006).

Known objects "Centaurs" orbiting between Jupiter and Neptune, including 95P Chiron (2060 Chiron) and 5145 Pholus, do not have stable orbits. Their curious orbits are unstable on time scales of million years. It is believed that the "Centaurs" may be objects which have escaped from the Kuper belt (e.g. Barucci et al., 2002); A majority of these objects migrate toward the external part of the solar system. Some of them are ejected from the solar system with a great eccentricity, a few of them can become Earth and Mars crossers, and many objects make numerous close approaches to Jupiter and Saturn (López García and Correa, 2006).

Some of the Centaurs show cometary activity. 95P Chiron (Luu and Jewitt, 1990, Meech and Belton, 1990, Williams, 2003), 166P Neat (P/2004 A1 Loneos) (Mazzotta Epifani et al., 2006), and 174P Echeclus (60558) (Choi et al., 2006) have a detectable coma, indicating that they are cometary bodies, with the size more commensurate with a large asteroid.

With the discovery of large cubewanos, the Kuiper belt seems to be a thick disk or torus. It now appears that the distribution of orbit inclination peaks around $4^\circ - 40^\circ$, giving rise to a division into so-called cold and hot groups (Bernstein et al., 2004).

Dynamics and structure of the Kuiper belt have been studied by many authors, e.g. Jewitt (1999), Duncan et al. (1995), Weissman (1995), Morbidelli and Valsecchi (1997), Malhotra et al. (2000). Papers relevant to observational and dynamical properties of the Kuiper belt can be found on Jewitt's www-page (2009).

Torbett (1989), by the numerical integration of test particles in this region, including perturbative effects of the four giant planets, found evidence for chaotic motion with the inverse Lyapunov exponent of the order of Myr for moderately eccentricities, moderate inclined orbits with perihelia between and 45 AU. Except for a few cases, authors were unable to follow the orbits or long enough to establish whether or not most chaotic trajectories in this group led to Neptune crossing.

The exact mechanism for the instability remains unclear from numerical simulations. Orlov (1950) studied the question about the existence of periodical solutions in the outer variant of the restricted three-body problem in which the
perturbating function was substituted by its averaging meaning. This problem is equivalent to the task about the motion of a point-like particle under influence of the Newton’s attraction of a linear uniform ring. It is shown that this task gives good results when the perturbing body revolves around the main body faster than the perturbed body. Using the method of Poincare for the case of the outer variant there was established the existence of the periodic solutions corresponding to Keplerian osculating orbits with small eccentricities in which the eccentricity, inclination and semi-major axis change periodically with time. The node exhibits the secular motion.

Two of questions posed in the paper by Leinhardt et al. (2007) paper are: do mutual collisions between Kuiper belt objects change structure of the Kuiper belt? Can mutual collisions between Kuiper belt objects be extended to extra-solar disks to explain dust in the outer regions of these disks?

In our work we offer one possible mechanism which can cause the migration of objects from the Kuiper belt – collisions with small bodies. A target object, receiving impulse in the tangential component of orbital velocity near the aphelion, can be transferred to the inner or outer part of the solar system, depending on the direction and magnitude of the impulse.

2. Changes of orbital velocities by collisions

We present one of possible dynamical mechanisms of the migration of small bodies from the Kuiper belt, which have low inclinations and eccentricities, i.e. the dynamically cold population. It is mutual collisions between Kuiper belt objects and between comets moving on orbits similar to the Kreutz comets ones.

The orbital velocity of comets with near-parabolic orbits in the region of the Kuiper belt is $7.7 - 6.0 \text{ km s}^{-1}$ (heliocentric distance 30–50 AU). The current root mean square velocity in the Kuiper belt is smaller, about $1.5 \text{ km s}^{-1}$. But such velocity is large enough to give rise to catastrophic disruption of objects and/or to change their orbital velocity too.

In the moment of a collision the velocity of the small body will be changed by a finite quantity, but its position remains unchanged. For simplification we suppose that the center of masses of body coincides with the geometric center. Then one can employ the following formula from theoretical mechanics (e.g. Vilke, 2003)

$$
\Delta v_1 = \pm \frac{(1 + k)(v_1 - v_2)}{(m_1^{-1} + m_2^{-1}) m_1},
$$

where $k$ is the coefficient of restitution ($k = 1$ for elastic and $k = 0$ for inelastic collision), $v_1$ is the orbital velocity of a target small body, and $v_2$ is the velocity of an impactor. This formula is valid for the impact when cosine between the
velocity directions at the collision is equal 1. In this case the change of the momentum of motion is maximum.

\[ \Delta v_1 \leq 1 \]

In Figure 1, the dependence of the change of the orbital velocity \( \Delta v_1 \) of a target body with mass \( m_1 \) on the coefficient of restitution for various heliocentric distances \( r \) and corresponding orbital velocities at the collision – circular of a target object, parabolic of an impactor. The mass of an impactor \( m_2 = m_1 = 10^{12} \) kg.

It is well known from Whipple’s time that masses of comets are small, generally not exceeding \( 10^{-6} \) mass of the Earth (~ \( 6 \times 10^{18} \) kg). The collision would be effective for the change of the orbital velocity if the momenta of motion of collided bodies were comparable.

For an illustration of collisional possibilities we took the mass of a target body \( m_1 = 10^{12} \) kg and the mass of an impactor \( m_2 \) from the interval \( 10^{10} - 10^{12} \) kg. The coefficient of restitution \( k \) varied from 0 to 1. The orbital velocity of a target body was taken to be 4.71 km s\(^{-1}\), which are values corresponding to the
orbital velocity on a circular orbit in the heliocentric distances 40 AU. For the orbital velocity of an impactor we took 6.66 km s\(^{-1}\), which corresponds to its orbital velocity on a parabolic orbit at the same heliocentric distance.

The dependence of the changes of the orbital velocity \(\Delta v_1\) of a target body on the coefficient of restitution for equal masses of collided bodies, \(m_1 = m_2 = 10^{12}\) kg and selected heliocentric distances \(r\) is shown in Figure 1. There is not large dispersion of the orbital velocity changes of the target body with the selected heliocentric distances. They grow from \(\pm 0.88\) up to \(\pm 1.13\) km s\(^{-1}\) for inelastic collision to \(\pm 1.75\) up to \(\pm 2.25\) km s\(^{-1}\) for elastic ones.

Figure 2. The positive orbital velocity changes \(\Delta v_1\) of a target body with the mass \(m_1 = 10^{12}\) kg in dependence on the coefficient of restitution \(k\) and the impactor’s mass \(m_2\), for the collision in the heliocentric distance 40 AU, are presented in the next two figures. The parameters of the collision are similar as in Figure 1. Only the mass of the impactor \(m_2\) varies within the interval from \(10^{10}\) up to \(10^{12}\) kg.

Figure 2 shows a variant of the collision when the impact increases the orbital velocity of a target body. In this case the collision point on the former circular orbit is the aphelium of a new orbit. And vice versa, in the case when
the direction of impactor’s velocity is opposite, Figure 3, the impact decreases
the orbital velocity of a target body. In this case the collision point become the
perihelion of a new orbit.

\[ \Delta v \]

\[ v_1 \]

\[ km s^{-1} \]

\[ -1.0 \]

\[ -1.5 \]

\[ -2.0 \]

\[ 0.0 \]

\[ 0.2 \]

\[ 0.4 \]

\[ 0.6 \]

\[ 0.8 \]

\[ k \]

\[ 0 \]

\[ 2 \times 10^{11} \]

\[ 4 \times 10^{11} \]

\[ 6 \times 10^{11} \]

\[ 8 \times 10^{11} \]

\[ m_2 [kg] \]

\( \Delta v_1 \)

\( [km s^{-1}] \)

Figure 3. The negative orbital velocity changes \( \Delta v_1 \) of a target body with the mass
\( m_1 = 10^{12} \) kg in dependence on the coefficient of restitution \( k \) and the impactor’s mass
\( m_2 \) from the interval of \( m_2 = 10^{10} \) kg to \( m_2 = 10^{12} \) kg.

The change of the orbital velocity \( \Delta v_1 \) of the target body is directly propor-
tional to the value of the coefficient of the restitution \( k \) and to the mass \( m_2 \) of
the impactor. For example, if \( m_2 = 10^{10} \) kg, \( \Delta v_1 = \pm 0.02 \) km s\(^{-1} \) for \( k = 0 \) and
\( \Delta v_1 = \pm 0.04 \) km s\(^{-1} \) for \( k = 1 \). If \( m_2 = 10^{11} \) kg, \( \Delta v_1 = \pm 0.18 \) km s\(^{-1} \) for \( k = 0 \) and
\( \Delta v_1 = \pm 0.35 \) km s\(^{-1} \) for \( k = 1 \), and if \( m_2 = 10^{12} \) kg, \( \Delta v_1 = \pm 0.98 \) km s\(^{-1} \) for \( k = 0 \) and \( \Delta v_1 = \pm 1.95 \) km s\(^{-1} \) for \( k = 1 \), respectively.

Figure 4 shows a schematic distribution of an initial circular orbit with the
semimajor axis \( a = 40 \) AU in the ecliptic plane and its model variants for
changes in the orbital velocity of about \( \Delta v_1 \) from the interval \( 0 \pm 3 \) km s\(^{-1} \) in
the heliocentric coordinate system. The orbits of great planets are projected to
the ecliptic plane. Similar model orbits for the initial circular one with \( a = 50 \)
AU are plotted in Figure 5.

The circular orbit with \( a = 40 \) AU after obtained impulse which changes its
orbital velocity \( \Delta v_1 \) about \(-0.5 \) to \(-0.6 \) km s\(^{-1} \) is transformed to an elliptical
Collisions as a possible case of TNOs migrations

Figure 4. The possible changes of perihelion distance $q$ and aphelion distance $Q$ of a circular orbit with semimajor axis $a = 40$ AU as a function of its orbital velocity changes $\Delta v_1$.

orbit with the perihelion near inside the Neptune’s orbit. A body on such orbit is exposed to the gravitational influence of the planet. For the circular orbit with $a = 50$ AU a similar situation occurs for the impulse which change $\Delta v_1$ about $-0.7$ to $-0.8$ km s$^{-1}$. Such increments of the orbital velocity are real for collisional events in the Kuiper belt. Positive values of the increment of the component of orbital velocity $\Delta v_1$ change the initial orbit in such a way that bodies are withdrawn from the Kuiper belt to the outer part of the solar system.

These processes change the composition and structure of the Kuiper belt. Some of its objects can migrate inside the central part of the solar system, other to the outer region of the Kuiper belt or to more distant regions.
Figure 5. The possible changes of perihelion distance $q$ and aphelion distance $Q$ of a circular orbit with semimajor axis $a = 50$ AU as a function of its orbital velocity changes $\Delta v_1$.

For numerical computations in our task we took a dynamic model of the solar system with all planets. For small bodies with mass $m_1 = 10^{12}$ kg we selected model initial circular orbits with the semi-major axes 30, 40, and 50 AU.

We performed the numerical integration of the equation of motion of the selected circular orbits and their model variants, for the period of 100 k.y., using Everhart’s (1985) integrator. The starting epoch of the numerical integration is January 1, 2001. The model variants of the initial orbits are the initial orbits whose component of the orbital velocity was changed by a value of $\Delta v_1$ equal $\pm 1$, $\pm 2$, and $\pm 3$ km s$^{-1}$. The results, evolution of orbital elements of the initial
Figure 6. Evolution of the eccentricity $e$, inclination $i$, perihelion distance $q$, and aphelion distance $Q$ of the circular orbit with the semi-major axis $a = 30$ AU and its model orbits defined by the change of the tangential component of the orbital velocity $\Delta v_1$ of the circular orbit. The model orbit with $\Delta v_1 = +3$ is hyperbolic.
Figure 7. Evolution of the eccentricity $e$, inclination $i$, perihelion distance $q$, and aphelion distance $Q$ of the circular orbit with the semi-major axis $a = 40$ AU and its model orbits defined by the change of the tangential component of the orbital velocity $\Delta v_1$ of the circular orbit. The model orbit with $\Delta v_1 = +2$ and $+3$ km s$^{-1}$ is hyperbolic.
Collisions as a possible case of TNOs migrations

Figure 8. Evolution of the eccentricity $e$, inclination $i$, perihelion distance $q$, and aphelion distance $Q$ of the circular orbit with the semi-major axis $a = 50$ AU and its model orbits defined by the change of the tangential component of the orbital velocity $\Delta v_t$ of the circular orbit. The model orbit with $dv = +2$ and $+3$ km s$^{-1}$ is hyperbolic.
and their variant orbits within a period of 100 k. y., are presented in Figures 6–8.

Depending on the value and direction of an added velocity $\Delta v_1$ to the initial orbital velocity a body can migrate to the different part of the solar system, or can leave the latter on a hyperbolic orbit. It can approach planets and planetary perturbations can change its inclination within a very large interval, from $0^\circ$ up to $60^\circ$. This phenomenon is very important for the evolution of the space structure of the Kuiper belt and for mutual collisions of Kuiper belt bodies.

### 3. Conclusion

Mutual collisions of Kuiper belt bodies or their collisions with small bodies on high eccentric orbits, e.g. long- and a-periodic comets on low inclination orbits could lead to a variety of events that would take place over a period of many years. Large collisions in the region beyond Neptune would produce many fragments that could take many different paths away from the explosion. A few fragments could be "captured" into an orbit around the planet. The collisions could lead to a change of the components of orbital velocity and change initial orbits to orbits with different perihelion and aphelion distances, different eccentricities and inclinations. These phenomena are very important for the long evolution of the structure and space distribution of the Kuiper belt bodies and their migration to other regions of the solar system.

Our numerical study is in a good agreement with the theory of Orlov (1950). It is shown that bodies of the Kuiper belt, especially its Classical objects, revolve around the Sun on stable periodic orbits. Their eccentricities, inclinations, and semi-major axes have periodic perturbations only. But additional forces can change their orbits such that the bodies can migrate to the inner or outer part of the solar system.

A possible mechanism for the change of periodical orbits of primordial objects from the Kuiper belt is their mutual collisions or their collisions with other small bodies of the solar system moving on high eccentric orbits.

### 4. Acknowledgements

This work was supported by the Slovak Academy of Sciences Grant VEGA 2/7040/27.

### References

Collisions as a possible case of TNOs migrations


Meech, K.J., Belton, M.J.S.: 1990, Astron. J. 100, 1323


Orlov, A.A.: 1950, Trudy GAISH (in Russian) XV, 71


