Contrib. Astron. Obs. Skalnaté Pleso **34**, 105 – 118, (2004)

The dynamical stability of extra-solar planets in binary systems

N.A. Solovaya and E.M. Pittich

Astronomical Institute of the Slovak Academy of Sciences, Interplanetary Matter Division, Dúbravská cesta 9, 845 04 Bratislava, The Slovak Republic

Received: January 21, 2004; Accepted: April 21, 2004

Abstract. The question about dynamical stability of extra-solar planets is considered in the frame of the general three-body problem, i.e. a planet in the binary system revolves around one of the components. The distance between the star's components is much longer than between the orbiting star and the planet. In the differential equations with regard to the eccentricity and the argument of the perigee we used the Hamiltonian without the short-periodic terms, excluded by von Zeipel's method.

The possible conditions of the dynamical stability of extra-solar planets are presented by their orbital parameters – the mutual inclination of orbits and the argument of the perigee of the planet.

The theory has been applied to the systems Gliese 86, γ Cephei, and 61 Cygni. The results were verified by the numerical integration.

Key words: three-body problem – extra-solar planets – dynamical stability – Gliese 86 – γ Cephei – 61 Cygni

1. Introduction

With discoveries of the extra-solar planets a series of celestial mechanics questions arose. It is interesting to know how the planets and planetary systems form and evolve. The principal question is the dynamical stability of planetary orbits as far as they have high eccentricites and inclinations. Extra-solar planets were discovered at single stars and in binary stellar systems. The present study deals with the investigation of the dynamical stability of the planetary orbits in binary systems, wherein the ratio of the semi-major axes of orbits of a planet and of the distant star is less than or equal 0.1.

The dynamical stability is understood as the conservation of the configuration of the system over an astronomically long time interval – the eccentricity of the planetary orbit remains less than 1, the mutual inclination of the orbits changes in small intervals, and there are no close approaches among bodies which can lead to the destruction of the system.

We studied the motion of extra-solar planets with masses from $1\,\mathrm{m_J}$ up to $50\,\mathrm{m_J}$ ($\mathrm{m_J}$ is the mass of Jupiter). The problem was considered in the frame of the general three-body problem, using the analytical theory (Orlov and Solovaya, 1988). The planet in a binary system revolves around one of the components. The motion is considered in the Jacobian coordinate system and the invariant plane is taken as the reference plane. We used the canonical Delaunay elements L_j, G_j, and g_j ($j = 1$ for the planet's orbit, $j = 2$ for the star's orbit). They can be expressed through the Keplerian elements as

$$L_j = \beta_j \sqrt{a_j}, \qquad G_j = L_j \sqrt{1 - e_j^2}, \qquad g_j = \omega_j \,, \tag{1}$$

where

$$\beta_1 = k \frac{m_0 m_1}{\sqrt{m_0 + m_1}}, \qquad \beta_2 = k \frac{(m_0 + m_1)\, m_2}{\sqrt{m_0 + m_1 + m_2}} \,. \tag{2}$$

In the previous expressions the notation has the usual meaning; m_0, m_2 – the masses of the stars, m_1 – the mass of the planet, k – the Gaussian constant, a_j – the semi-major axis, e_j – the eccentricity, and ω_j – the argument of the perigee.

The eccentricity of the star orbit can take any value from zero to one. We used the Hamiltonian of the system without the short-periodic terms. The short-periodic perturbations in the motion of the both components with the period of the revolution on the orbits are very small (Solovaya, 1972). Their values are entirely insensible to the contemporary precision of the definition of the elements or are on the boundary of that precision. Expanded in terms of the Legendre polynomials and truncated after the second-order terms the Hamiltonian has the form

$$\begin{aligned} F \;=\; & \frac{\gamma_1}{2L_1^2} + \frac{\gamma_2}{2L_2^2} - \gamma_3 \frac{L_1^4}{L_2^3 G_2^3} \left[\left(1 - 3\, q^2\right) \left(5 - 3\, \eta^2\right) - \right. \\ & \left. -15 \left(1 - q^2\right) \left(1 - \eta^2\right) \cos 2\, g_1 \right] , \end{aligned} \tag{3}$$

where the coefficients γ_1, γ_2, and γ_3 depend on mass as follows

$$\gamma_1 = \frac{\beta_1^4}{\mu_1}, \qquad \gamma_2 = \frac{\beta_2^4}{\mu_2}, \qquad \gamma_3 = k^2 \mu_1 \mu_2 \frac{\beta_2^6}{\beta_1^4}, \tag{4}$$

and

$$\begin{aligned} \mu_1 \;&=\; \frac{m_0 m_1}{m_0 + m_1}, \qquad\qquad \mu_2 = \frac{(m_0 + m_1)\, m_2}{m_0 + m_1 + m_2}, \\ q \;&=\; \frac{c^2 - G_1^2 - G_2^2}{2 G_1 G_2}, \qquad \eta = \sqrt{1 - e_1^2} \,. \end{aligned} \tag{5}$$

c is the constant of the angular momentum, g_1 is the argument of the perigee of the planet orbit in the invariable plane, and q is cosine of the mutual inclination of the orbits.

In a general case the motion is defined by the masses of components and by the six pairs of the initial values of Keplerian elements. For extra-solar planets in Schneider's Extra-solar Planets Catalog (Schneider, 2004) it is not possible to obtain the complete set of elements for orbits. However, we can perform qualitative investigation of the differential equations with incomplete data. There are presented several examples of the discovered extra-solar planets for which orbital stability was investigated.

2. The circular orbits

The canonical system of the equations of motion, corresponding to the Hamiltonian (3), divides into following mutually combined equations with regard to the eccentricity and the argument of perigee of the planet:

$$\frac{\mathrm{d}G_1}{\mathrm{d}t} = -\frac{15}{8}\gamma_3 \frac{L_1^4}{L_2^3 G_2^3}\left(1-q^2\right)\left(1-\eta^2\right)\sin 2\,g_1\,, \tag{6}$$

$$\begin{aligned}\frac{\mathrm{d}g_1}{\mathrm{d}t} &= \frac{3}{8}\gamma_3 \frac{L_1^3}{L_2^3 G_2^3}\frac{1}{\eta}\left\{-\eta^2 + 5\,q^2 + \frac{1}{G_2}\,\eta\,q\,\left(5-3\,\eta^2\right)\right.\\ &\left. +5\left[\left(\eta^2-q^2\right) - \frac{1}{G_2}\,\eta\,q\,\left(1-\eta^2\right)\right]\cos 2\,g_1\right\}.\end{aligned} \tag{7}$$

These equations have the equilibrium solutions. The right part of Eq. (6) converts to zero in one of the following cases:

$$q = \pm 1\,, \qquad \eta = 1\,, \qquad \sin 2\,g_1 = 0\,.$$

The first case, for which $q = \pm 1$, belongs to the planar case. We do not study it.

Consider the case of circular orbits when $\eta = 1$. For this purpose we introduce new variables:

$$\lambda_1 = e_1\,\cos g_1\,, \qquad \lambda_2 = e_1\,\sin g_1\,.$$

Then

$$\eta = \sqrt{1-\lambda_1^2-\lambda_2^2}\,, \qquad \overline{q} = \frac{\overline{c}^2 - \overline{G}_2^2 - 1}{2\,\overline{G}_2}\,, \qquad \overline{G}_2 = \frac{G_2}{L_1}\,, \qquad \overline{c} = \frac{c}{L_1}\,.$$

The differential equations of motion with the new variables will be

$$\begin{aligned}\frac{\mathrm{d}\lambda_1}{\mathrm{d}t} &= -\frac{\eta}{L_1}\frac{\partial F}{\partial \lambda_2}\\ &= \frac{N}{\eta}\left[\left(3-5\,\overline{q}^2\right)\left(1-\lambda_1^2\right) - 3\,\lambda_2^2 - \frac{\eta\,\overline{q}}{\overline{G}_2}\left(1-\lambda_1^2+4\,\lambda_2^2\right)\right]\lambda_2\,,\end{aligned} \tag{8}$$

$$\begin{aligned}\frac{\mathrm{d}\lambda_2}{\mathrm{d}t} &= \frac{\eta}{L_1}\frac{\partial F}{\partial \lambda_1} \\ &= \frac{N}{\eta}\left[2\left(1-\lambda_1^2\right)-\left(2-5\,\overline{q}^2\right)\lambda_2^2+\frac{\eta\,\overline{q}}{\overline{G}_2}\left(1-\lambda_1^2+4\,\lambda_2^2\right)\right]\lambda_1\,, \end{aligned} \tag{9}$$

where $N = \gamma_3\,\dfrac{L_1^4}{L_2^3\,G_2^3}$ is a positive constant.

Eqs. (8) and (9) have the equilibrium solution $\lambda_1 = \lambda_2 = 0$. Let us investigate the stability of the linearized system of these equations.

When keeping in right-hand part of Eqs. (8) and (9) only first order terms of λ_1 and λ_2, then

$$\frac{\mathrm{d}\lambda_1}{\mathrm{d}t} = N\left(3-5\,\overline{q}^2-\frac{\overline{q}}{\overline{G}_2}\right)\lambda_2\,, \tag{10}$$

$$\frac{\mathrm{d}\lambda_2}{\mathrm{d}t} = N\left(2+\frac{\overline{q}}{\overline{G}_2}\right)\lambda_1\,, \tag{11}$$

where $\eta = 1$ when $\lambda_1 = \lambda_2 = 0$. The corresponding characteristic equation is:

$$\chi^2 = -N^2\left(5\,\overline{q}^2+\frac{\overline{q}}{\overline{G}_2}-3\right)\left(2+\frac{\overline{q}}{\overline{G}_2}\right)\,. \tag{12}$$

The stability properties for $t \geq t_0$ of the linearized system may be the following (Lyapunov, 1950):

i) If the right-hand part of equation (12) is negative, we have two pure imaginary roots. Its linearized solution is simply stable.

ii) If the right-hand part of equation (12) is positive, its solution is unstable. This is the case if

$$5\,\overline{q}^2+\frac{\overline{q}}{\overline{G}_2}-3<0\,,$$

since always $\overline{G}_2 > 1$. It may be when

$$q_1 = \frac{-1-\sqrt{60\,\overline{G}_2^2+1}}{10\,\overline{G}_2} < \overline{q} < \frac{-1+\sqrt{60\,\overline{G}_2^2+1}}{10\,\overline{G}_2} = q_2\,.$$

So, the possible conditions of the stability of the circular orbits of extrasolar planets are characterized by their orbital parameters, i.e. by the angle of mutual inclination I and the parameter $\overline{G}_2$, which is a function of the ratio of the semi-major axes of orbits of the planet and the distant star, the eccentricity of the orbit of the distant star, and masses of all components of the system.

If the mass of the planet changes in the range from 1 to 50 $\mathrm{m_J}$ and the ratio of semi-major axes of the orbits of the planet and the distant star lies in the

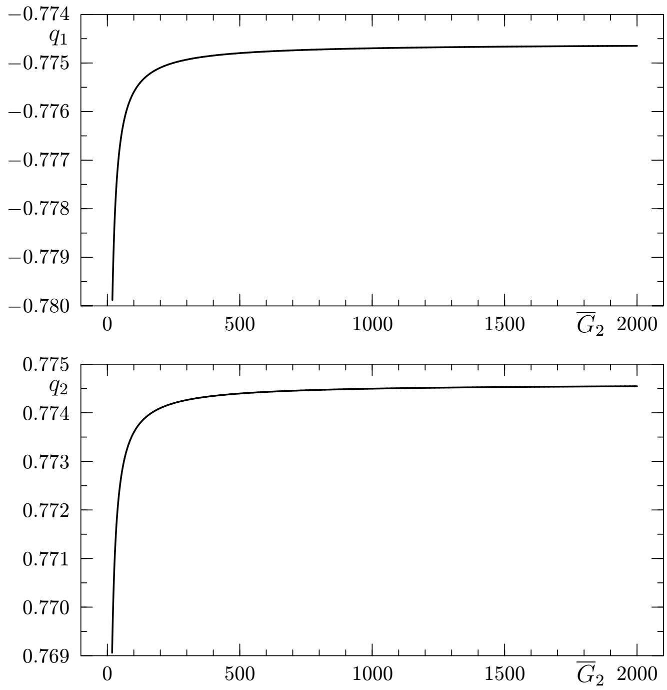

Figure 1. The changes of the cosine of the mutual inclination for the parameter $\overline{G}_2$ in the interval 20–2000, for q_1 (top) and q_2 (bottom).

range of 0.01 to 0.10, then the parameter $\overline{G}_2$ will change within the limits of 20 to 2000. The boundaries of the stability of motion of a planet are plotted in Fig. 1.

The condition for the stability of motion of a planet is that the angle of the mutual inclination must be $141° < I < 39°$.

As an example we took the planet *Gliese* 86 from Schneider's Catalog (Schneider, 2004). It revolves around one of the star components of the long-periodic spectroscopical binary system. The masses of the components are $m_0 = 0.79\,\mathrm{m}_\odot$, $m_1 = 4.9\,\mathrm{m_J}$, and $m_2 = 0.4\,\mathrm{m}_\odot$. The orbital elements of the planet (index 1) and the distant star (index 2) are following:

$$
\begin{aligned}
m_1 \sin i_1 &= 4.01\,\mathrm{m_J}, \\
e_1 &= 0.041, && e_2 = 0.200, \\
a_1 &= 0.117 \text{ AU}, && a_2 = 19 \text{ AU}, \\
& && i_2 = 164.0°, \\
\Omega_1 &= 266°, \\
T_1 &= JD(2\,450\,000) : 1146.54\,.
\end{aligned}
$$

A considerably different mass of the planet, $m_1 = 15.5\,\mathrm{m_J}$, we found in Han's paper (Han et. al, 2001). We produced the investigation of the stability of motion of the planet for these two masses.

From the known value of $m_1\, sin i_1$ we obtained for the accepted masses two values of the inclination of the planetary orbit. If the mass of the planet $m_1 = 4.9\,\mathrm{m_J}$, then the angle of the inclination of the planetary orbit $i_1 = 54.921°$ and if $m_1 = 15.5\,\mathrm{m_J}$, then $i_1 = 14.994°$.

So as the node of the orbit of the distant star is unknown, we supposed its value $\Omega_2 = 0°$ and $\Omega_2 = 180°$. From theory we obtained the following results.
For $m_1 = 4.9\,\mathrm{m_J}$:

i) If $\Omega_2 = 0°$, then the angle of mutual inclination of orbits $I = 124.62°$.

ii) If $\Omega_2 = 180°$, then $I = 124.67°$.

In both cases the angle of the mutual inclination is located out of the limits of stability and small deviations of initial elements may become large in the future.
For $m_1 = 15.5\,\mathrm{m_J}$:

i) If $\Omega_2 = 0°$, then the angle of mutual inclination of orbits $I = 158.66°$.

ii) If $\Omega_2 = 180°$, then $I = 159.02°$.

Such orbits are stable within the whole time interval.

In general, the planet in this binary system has the stable orbit when $I \geq 141°$. This takes place when the mass of the planet $m_1 \geq 10\mathrm{m_J}$.

3. The near circular orbits

Consider the motions close to a circular motion. The eccentricity may have the meaning $e_1 = \sqrt{1-\xi}$. In general case, the relation between ξ and t is defined by the following equation (Orlov and Solovaya, 1988):

$$\frac{1}{12}\,\overline{G}_2^2 \int_{\xi_1}^{\xi} \frac{1}{\sqrt{\Delta}}\,\mathrm{d}\xi = \frac{B_3}{A_1} + \frac{1}{16}\,\gamma\,\frac{m^2}{\sqrt{(1-e_2^2)^3}}\, n_1\,(t-t_0)\;, \tag{13}$$

where

$$n_1 = \frac{k}{a_1}\sqrt{\frac{m_0+m_1}{a_1}}\,, \qquad n_2 = \frac{k}{a_2}\sqrt{\frac{m_0+m_1+m_2}{a_2}}\,,$$

$$m = \frac{n_2}{n_1}\,, \qquad \gamma = \frac{m_2}{m_0+m_1+m_2}\,.$$

Δ is the polynomial of the fifth order. It can be separated into two polynomials of the second and the third order, which have the form:

$$f_2(\xi) = \xi^2 - 2\left(\overline{c}^2 + 3\,\overline{G}_2^2\right)\xi + \left(\overline{c}^2 - \overline{G}_2^2\right)^2 + \frac{2}{3}\,(10+A_3)\,\overline{G}_2^2\,, \tag{14}$$

$$\begin{aligned} f_3(\xi) \;=\; & \xi^3 - \left(2\,\overline{c}^2 + \overline{G}_2^2 + \frac{5}{4}\right)\xi^2 + \\ & + \left[\frac{5}{2}\left(\overline{c}^2 + \overline{G}_2^2\right) + \left(\overline{c}^2 - \overline{G}_2^2\right) - \frac{1}{6}\,\overline{G}_2^2\,(10 + A_3)\right] - \\ & - \frac{5}{4}\left(\overline{c}_2 - \overline{G}_2^2\right), \end{aligned} \tag{15}$$

where

$$A_3 = 2 - 6\,\eta_0^2\,q_0^2 - 6\,\left(1 - \eta_0^2\right)\left[2 - 5\,\left(1 - q_0^2\right)\sin^2 g_{1_0}\right]. \tag{16}$$

For qualitative investigation of motion it is necessary to know the roots of the equations $f_2(\xi) = 0$ and $f_3(\xi) = 0$. The subscript or superscript 0 denotes initial values of all parameters.

The solution of this system of equations has a meaning in the region where $f_2(\xi)\,f_3(\xi) > 0$. All roots are real and positive, but only two of them, ξ_1 and ξ_2, are always less than 1.

The meaning of the variable ξ, we are interested in, must be defined by interval

$$\xi_1 \le \xi \le \xi_2\,.$$

So $\xi = 1 - e_1^2$.

In the initial moment for near circular orbits the value of

$$q_0 = \cos\left(i_{1_0} + i_{2_0}\right)$$

may be arbitrary and η_0 is defined from (5) as

$$\eta_0 = 1 - \varepsilon\,,$$

where ε is a small positive quantity.

We will find the values of the three smallest roots by restricting to the first order of ε. Then the smallest root of the equation of the second order (Eq. 14), which we denote as α_1, is

$$\alpha_1 = 1 + \frac{Q}{4\,\overline{G}_2\,\left(2\,\overline{G}_2 + q_0\right)}\,\varepsilon \tag{17}$$

and the two smallest roots of the equation of the third order (Eq. 15), denoted as α_2 and α_3, are:

$$\alpha_2 = 1 + \frac{Q}{4\,B}\,\varepsilon\,, \tag{18}$$

$$\begin{aligned} \alpha_3 \;=\; & a + \varepsilon\left[(a-1)\left(2\,a - 5\,\overline{G}_2\,q_0 + \frac{5}{2}\right)\left(1 + \overline{G}_2\,q_0\right) - \frac{1}{4}Q\right] \times \\ & \times \frac{A + \sqrt{A^2 - B}}{2\,B\,\sqrt{A^2 - B}}, \end{aligned} \tag{19}$$

where

$$
\begin{aligned}
A &= \frac{1}{8} + \frac{3}{2}\,\overline{G}_2^2 + 2\,\overline{G}_2\, q_0\,, \\
B &= 5\,\overline{G}_2^2\, q_0^2 + \overline{G}_2\, q_0 - 3\,\overline{G}_2^2\,, \\
Q &= -4\,\left[2\,\overline{G}_2^2 + \overline{G}_2\, q_0 - 5\,\overline{G}_2^2\,\left(1 - q_0^2\right)\,\sin^2 g_{1_0}\right]\,, \\
a &= 1 + A - \sqrt{A^2 - B}\,.
\end{aligned}
\tag{20}
$$

The coefficient Q is always less than zero. The coefficient B maybe positive or negative in dependence on the mutual inclination of the orbits.

Consider the case when the mutual inclination of the orbits is such that $B > 0$. Then $\alpha_1 < 1$, $\alpha_2 < 1$, $\alpha_3 > 1$, and $\alpha_1 < \alpha_2$. Consequently,

$$
\alpha_2 \le \eta^2 \le \alpha_1\,. \tag{21}
$$

When $\varepsilon \to 0$, $\alpha_1 \to 1$, and $\alpha_2 \to 1$, then $\eta^2 \to 1$.

We may take ε so small that always $\left|\eta^2 - 1\right| < \delta$ for $t > t_0$. It means that if $\alpha_2 \le \eta^2 < \alpha_1$ the circular motion is stable with respect to e_1 (Chetaev, 1965).

In the case when the mutual inclination of the orbits is such that $B < 0$ the root α_3 is the smallest root and the lower limit of $\eta^2 = \alpha_3$. When $\varepsilon \to 0$ then $\eta^2 = a < 1$ and it is possible to pick up such small $\delta > 0$ that in some moment $\left|\eta^2 - 1\right| > \delta$, for an arbitrary small initial ε. In this case the circular motion is unstable.

As an example of the motion near to circular consider the star system γ Cephei. It is a binary star system with period $P = 70$ years. The following data are again taken from Schneider's Catalog (Schneider, 2004): The masses of components are $m_0 = 1.59\,\mathrm{m}_\odot$, $m_1 = 1.76\,\mathrm{m_J}$ and $m_2 = 0.58\,\mathrm{m}_\odot$. The orbital elements of the planet (index 1) and the distant star (index 2) are following:

$$
\begin{aligned}
m_1 \sin i_1 &= 1.59\,\mathrm{m_J}, & & \\
e_1 &= 0.2, & e_2 &= 0.439, \\
a_1 &= 2.1 \text{ AU}, & a_2 &= 22 \text{ AU}, \\
& & i_2 &= 51.3^\circ, \\
\omega_1 &= 95^\circ, & \omega_2 &= 162.1^\circ, \\
\Omega_1 &= 75.6^\circ, & & \\
T_1 &= JD(2\,450\,000) : 53\,156.8, & T_2 &= JD(2\,450\,000) : 48\,506.
\end{aligned}
$$

From the known value of $m_1\, sini_1$ we obtained value of the inclination of the planetary orbit $i_1 = 64.61^\circ$.

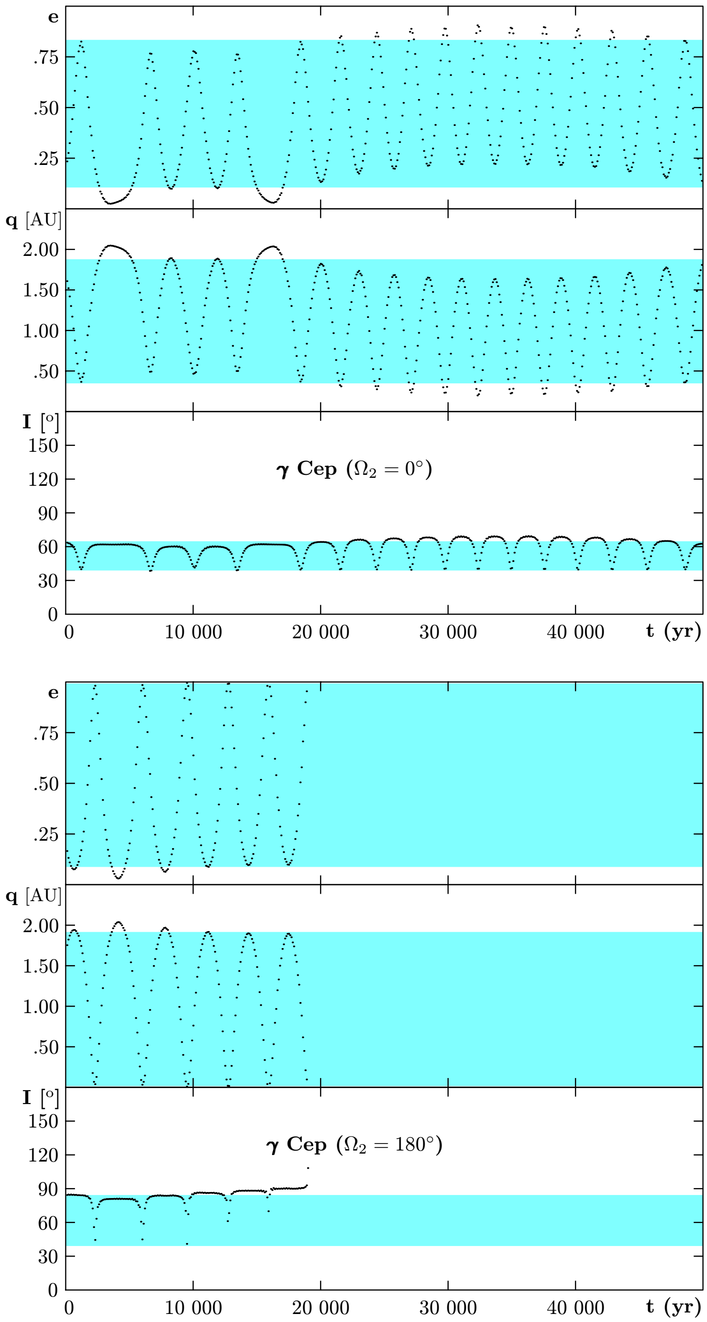

Figure 2. Orbital evolution of the extrasolar planet with mass 1.76 m_J (0.00168 $m_\odot$) in the binary star system γ Cep. Up for $\Omega_2 = 0°$ and down for $\Omega_2 = 180°$.

Because the longitude of the ascending node of the orbit of the distant star is unknown we supposed its value $\Omega_2 = 0°$, $\Omega_2 = 90°$, and $\Omega_2 = 180°$. From theory we obtained following results.

i) If $\Omega_2 = 0°$ the angle of mutual inclination of orbits $I = 64.082°$. Within the period of 3576 years the angle of the mutual inclination can change within the interval $38.865° < I < 64.487°$ and the eccentricity within the range $0.105 < e_1 < 0.835$. At the maximum value $e_{1_{max}} = 0.835$ the minimum perigee distance $r_{T_1} = 0.347$ AU.

ii) If $\Omega_2 = 90°$ the angle of mutual inclination of orbits $I = 18.098°$. Within the period of 2011 years the angle of the mutual inclination can change within the range $17.560° < I < 18.560°$ and the eccentricity within the interval $0.192 < e_1 < 0.219$.

iii) If $\Omega_2 = 180°$ the angle of mutual inclination of orbits $I = 84.628°$. Within the period of 3431 years the angle of the mutual inclination can vary as $39.052° < I < 84.718°$ and the eccentricity as $0.087 < e_1 < 0.993$. At the maximum value $e_{1_{max}} = 0.993$ the minimum perigee distance $r_{T_1} = 0.015$ AU.

In the first and third cases $B < 0$ the small deviations of initial elements may become large in the future. In the second case $B > 0$ the motion of the planet is stable.

The comparison of these results with the results obtained by the numerical integration (see Fig. 2) for the case of γ Cephei showed that the used analytical method gives good results. The boundaries of dark zones were computed from the theory. The curves are results of the numerical integration.

The results indicate that the motion of such orbital system can be stable only in the case when the mutual inclination of the orbits is within a defined interval. When the angle of the mutual inclination approaches a certain value, the upper limit of the eccentricity of the planet orbit increases. In the case of the unstable circular or near circular orbits it means that any small initial deviations of the elements of the planet orbit may become substantial in the its future motion.

4. Orbits with high eccentricities

Consider the third case, when the orbit of a planet has the eccentricity $e_1 > 0$ and $\sin 2\, g_1 = 0$. In the case when $g_1 = 0$, the expression in the curly braces of Eq. (7) is equal to

$$2\,\eta^2 \left(2 + \frac{1}{\overline{G}_2}\,\eta\, q\right) . \tag{22}$$

This expression converts to zero for $\eta = 0$. The case when the initial value of $e_1 = 1$ we do not consider.

In the case when $g_1 = \pi/2$, the expression in the curly braces of Eq. (7) is equal to

$$2 \left[5\, q^2 - 3\, \eta^2 + \frac{1}{\overline{G}_2}\, \eta\, q \, \left(5 - 4\, \eta^2\right) \right] . \tag{23}$$

This expression converts to zero for

$$q = \frac{\eta \left[4\, \eta^2 - 5 \pm \sqrt{60\, \overline{G}_2^2 + \left(5 - 4\, \eta^2\right)^2} \right]}{10\, \overline{G}_2} \,,$$

which we denoted as q_{01} for the minus sign before the root term, and q_{02} for the plus sign before the root term.

In the case $g_1 = \pi/2$

$$A_3 = A_{3max} = 20 - 18\, \eta_0^2 - 30\, q_0^2 + 24\, \eta_0^2\, q_0^2 \,. \tag{24}$$

Then we can rewrite the equations of the second and the third orders as

$$\begin{aligned} f_2\,(\xi) \;&=\; \left(\xi - \eta_0^2\right) \left[\left(\xi - \eta_0^2 - 4\, \overline{G}_2\, \eta_0\, q_0 + 8\, \overline{G}_2^2\right) + \right. \\ &\left. \quad + \, 4\, \overline{G}_2^2 \left(1 - 5\, q_0^2\right) \left(1 - \eta_0^2\right) + 16\, \overline{G}_2^2 \right] , \end{aligned} \tag{25}$$

$$\begin{aligned} f_3\,(\xi) \;&=\; \left(\xi - \eta_0^2\right) \left[\left(\xi - \frac{5}{4}\right) \left(\xi - \eta_0^2\right) - \right. \\ &\left. \quad - \, 3\, \overline{G}_2^2\, \xi + \overline{G}_2\, \eta_0\, q_0 \, \left(5 - 4\, \xi\right) + 5\, \overline{G}_2^2\, q_0^2 \right] . \end{aligned} \tag{26}$$

If expression (23) is negative then $q_{01} < q_0 < q_{02}$ and $\xi = \eta_0^2$ is the least root of the equation of the second order (Eq. 25), which is less than 1. In this case $\xi_{1\,min} = \sqrt{1 - \eta_0^2}$. The value of $\xi_{1\,max}$ can increase. In this case the motion of the planet is unstable.

If expression (23) is positive, then $q_0 < q_{01}$ or $q_0 > q_{02}$ and ξ is the least root of the equation of the third order (Eq. 26), which is less than 1. In this case $e_{1_{max}} = \sqrt{1 - \eta_0^2}$. The maximum value of the eccentricity of the planet's orbit cannot exceed the initial value of the eccentricity. The motion of the planet is stable.

As an example we used the double star system 61 Cyg. The following elements of both orbits, the planet and the star, were taken from the Sixth Catalog of Orbits of Visual Binary Stars (Hartkopf and Mason, 2003). The masses of

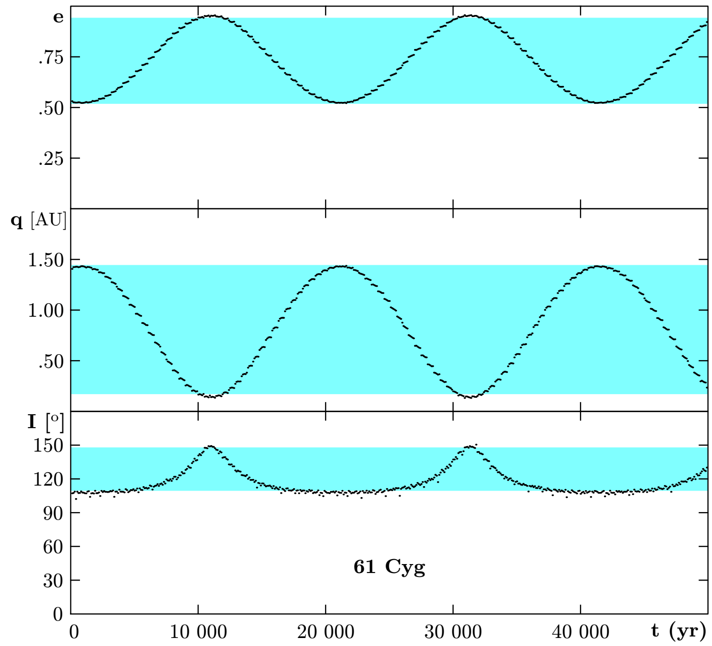

Figure 3. Orbital evolution of the extra-solar planet with mass 16.5 $\mathrm{m_J}$ (0.01 $\mathrm{m_\odot}$) in the binary star system 61 Cyg.

the components are: $m_0 = 1.11\,\mathrm{m_\odot}$, $m_1 = 10.5\,\mathrm{m_J}$, $m_2 = 1\,\mathrm{m_\odot}$. The orbital elements of the planet (index 1) and the distant star (index 2) are following:

$$\begin{aligned} e_1 &= 0.53, & e_2 &= 0.48, \\ a_1 &= 3 \text{ AU}, & a_2 &= 80 \text{ AU}, \\ i_1 &= 134^\circ, & i_2 &= 54^\circ, \\ \omega_1 &= 295^\circ, & \omega_2 &= 146^\circ, \\ \Omega_1 &= 94^\circ, & \Omega_2 &= 176^\circ, \\ T_1 &= 1953.2, & T_2 &= 1697. \end{aligned}$$

From the theory we obtained $I = 109.52^\circ$ and $q_{01} < q_0$. Within the period of 22 412 years the angle of the mutual inclination can change as $109.36^\circ < I < 148.11^\circ$ and the eccentricity as $0.518 < e_1 < 0.943$. At the maximum value $e_{1_{max}} = 0.943$ the minimum perigee distance $r_{T_1} = 0.170$ AU. In this system the encounters of the planet and the star is possible. The tidal phenomena can occur.

From the numerical integration of the equation of the motion we obtained the results which are in good agreement with the results obtained from the theory (see Fig. 3). The dark zones and curves obtained are similar to those in Fig. 2. The boundary of dark zones were computed from the theory. The curves are results of the numerical integration.

5. Conclusion

From this study we can draw the following conclusion. Theoretically, discovered extra-solar planets in binary star systems move on stable or unstable orbits around one of the components. They can revolve around the main star many thousand years even on unstable orbits. The inclination and eccentricity of the most observed extra-solar planets are large. Their orbits are different from the orbits of the planets in the solar system.

From our theory it is possible to calculate orbital parameters of an extra-solar planet – the angle of mutual inclination between the planet and the distant star orbits and the parameter $\overline{G}_2$, which is a function of the ratio of the semi-major axes of orbits of the planet and the distant star, the eccentricity of the orbit of the distant star, and masses of all components. The parameters can be used for the prediction of the stability of the motion of the extra-solar planet over an astronomically long time interval. We showed this in the case of the binary system Gliese 86. At one set of parameters the planet revolves around the main star on the stable orbit. On the second set of the parameters the planet's orbit is unstable and can be destructed after some time by tidal forces of the star.

From observations of the extra-solar planet, which cover a short timescale only, we cannot check the stability or unstability of its motion. For the solution of the question of the stability we must use the analytical theory or numerical integration.

The theory was verified by numerical integration on two examples. In the case of orbits near to circular on the system γ Cephei, and in the case of high eccentricity orbits on the system 61 Cyg. These binary system belong to the best observed binary systems in which extra-solar planets were discovered. In both cases we obtained very good agreement between the results obtained by the theory and the numerical integration within the interval of 50 000 years, covering several revolutions of the distant star.

For celestial mechanics the discoveries of extra-solar planets in the star systems raise many new interesting questions connected with the stability of their motion and their dynamical evolution.

Acknowledgements. This work was supported by the Slovak Academy of Sciences Grant VEGA No. 2/4002/04. The computations were done on the ORIGIN 2000 computer of the Computer center of the Slovak Academy of Sciences.

References

Chetaev, N.G.: 1965, *Ustojchivost' dvizheniya*, Fizmatgiz, Moscow

Han, I., Black, D.C., Gatewood, G.: 2001, *Astrophys. J.* **548**, L57

Hartkopf, W.I., Mason, B.D.: 2003, *Sixth Catalog of Orbits of Visual Binary Stars* , http://ad.usno.navy.mil/wds/orb6.html

Lyapunov, A.M.: 1950, *Obshchaya zadacha ob ustojchivosti dvizheniya*, Gostechizdat, Moscow

Orlov, A.A., Solovaya, N.A.: 1974, *Trudy GAISH* **XLV**, 137

Orlov, A.A., Solovaya, N.A.: 1988, in *The Few Body Problem*, ed.: M.J. Valtonen, Kluwer Acad. Publish., Dordrecht, 243

Schneider, J.: 2004, *Extra-solar Planets Catalogue* , http://www.obs.pm.fr/encycl/catalog.html

Solovaya, N.A.: 1972, *Trudy GAISH* **XLIII**, 38

Solovaya, N.A.: 1998, in *Fourth International Workshop on Positional Astronomy and Celestial Mechanics*, eds.: A.L. García , L.I. Yagudin , M.J. Martínez Usó , A.C. Barbero , E.I. Yagudina and J.A. Moraño, Observatorio Astronomico de la Universitat de Valencia, Valencia, 121