Photoelectric photometry of the eclipsing contact binaries: EF Dra, GW Cep and CW Cas

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Abstract. New photoelectric BV observations of the eclipsing contact systems EF Dra, GW Cep and CW Cas obtained in autumn 2000 are presented and analyzed. A photometric solution of CW Cas indicates the poor thermal contact of the components. Photometric elements of EF Dra determined from our light curves combined with published spectroscopic elements yield the absolute parameters of the components: m_1 = 1.81±0.10 $M_\odot$, m_2 = 0.29±0.03 $M_\odot$, R_1 = 1.701±0.026 $R_\odot$ and R_2 = 0.778±0.012 $R_\odot$. The detected third light in EF Dra is used to estimate the mass of the third body $m_3 \approx 0.75$ $M_\odot$ as well as its K1-2 spectral type. The observed period changes of the eclipsing pair indicate either a 1360 days or more than 12 years orbital period of the third body. The observed period decreases in GW Cep and CW Cas are probably influenced by the presence of their third bodies.

Key words: contact binaries – photometry – orbital period

1. Introduction

The number of eclipsing binaries displaying the W UMa type light curve (EW) is quickly increasing. While the 4th edition of the General Catalogue of the variable stars listed 561 objects as EW variables, the 5th edition lists as many as 715 such variables. The group of contact type binaries (EW light curve) is extended mainly by the OGLE experiment (Rucinski, 1997a,1997b), ROTSE CCD survey (Akerlof et al., 2000) and EINSTEIN X-ray survey (Gioia et al., 1989). The data available for W UMa variables are augmented by good quality radial velocity observations at the David Dunlap observatory (Lu & Rucinski, 1999, Lu et al., 2000). In spite of this, the number of systems with reliable light and radial velocity solutions based on the Roche geometry is lower than 40 (Maceroni & van't Veer, 1996). The presence of the third light in the photometric light-curve solutions, apparent orbital period changes due to the light-time effect and the

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presence of the additional spectral lines using the high-dispersion spectroscopy have shown that many contact binaries are members of multiple systems.

The present paper is the first in a series analyzing the photoelectric observations obtained at the Stará Lesná and Skalnaté Pleso Observatories in the frame of the survey of neglected and faint contact systems. The aim of this survey is the determination of the principal photometric and absolute parameters of the studied systems and analysis of their orbital periods and light-curve changes.

2. New photoelectric observations

The present BV photoelectric observations of EF Dra were carried out in six nights in September and November, 2000 at the Stará Lesná (SL) observatory of the Astronomical Institute of the Slovak Academy of Sciences. BD+69°965 was used as the comparison star. The BV photoelectric photometry of GW Cep was obtained during five nights from September to December, 2000 at the SL observatory. BD+39°965 was used as the comparison star. The BV photoelectric photometry of CW Cas was performed in six nights from September to December, 2000 at the SL and Skalnaté Pleso (SP) observatories of the same Institute. GSC 4020-1645 served as the comparison star. The journal of photoelectric observations is given in Table 1.

At both observatories a single-channel pulse-counting photoelectric photometer installed at the Cassegrain focus of the 0.6m reflector was used. Observations at SP were carried out through standard B and V filters using photomultiplier HAMAMATSU R4457P. Standard B and V filters and photomultiplier EMI 9789QB were used at SL. The integration time of one measurement was 10 seconds. The standard B,V magnitudes were obtained from instrumental b,v magnitudes using the following transformations

$$\begin{aligned} \mathrm{V} &= \mathrm{v} - k_V(\mathrm{b}-\mathrm{v}) \\ \mathrm{B}-\mathrm{V} &= k_{BV}(\mathrm{b}-\mathrm{v}) \end{aligned} \tag{1}$$

The transformation coefficients k_V, k_{BV} were determined by the observations of the standard stars in the Pleiades, Praesepe and IC 4665 open clusters. These observations are performed routinely on both telescopes every two or three months. The average seasonal extinction coefficients were used for the differential extinction correction. The individual B and V observations of EF Dra, GW Cep and CW Cas are plotted in Figs. 1 and 2. The maximum I (phase 0.25) of CW Cas was covered during bad observational conditions (nights September 27 and November 29) only.

The LCs of all three systems were fairly symmetric during the observing runs. Since most orbital phases were covered only once, night-to-night variations cannot be ruled out. The secondary minimum of EF Dra covered on September 1,10 and 12 shows variable depth.

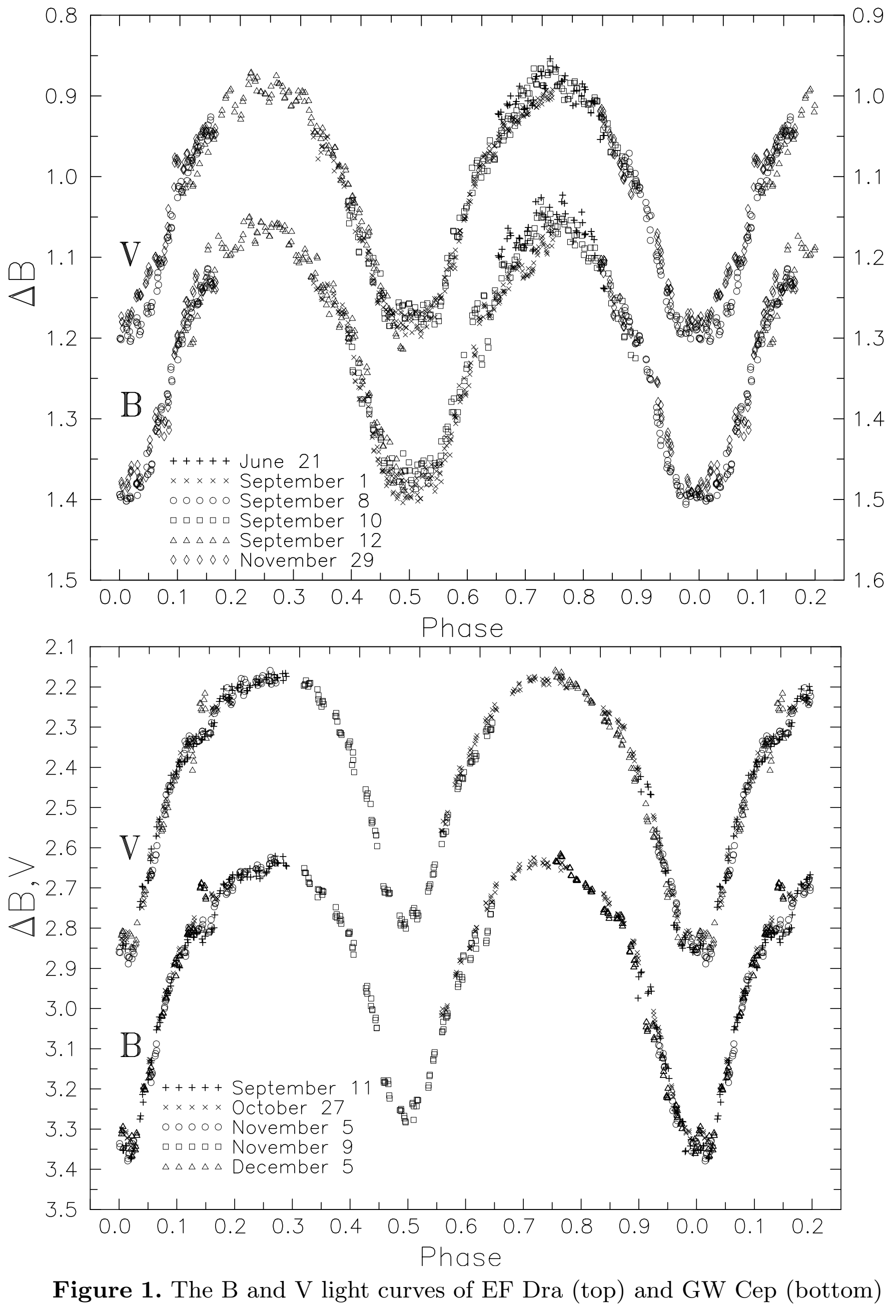

Figure 1. The B and V light curves of EF Dra (top) and GW Cep (bottom)

Table 1. Journal of photoelectric observations of EF Dra, GW Cep and CW Cas obtained at the SL and SP Observatories. Date, intervals of phases (according to ephemerides (4),(6) and (8)), observatory, number of observations in one filter (N), estimated standard deviation of an individual observation in V/B filter (σ) and observer/program

Date	HJD_{mean} 2 400 000+	Phases	Obs.	N	σ	Program
EF Dra						
Jun 21, 2000	51717.444	0.653 – 0.836	SL	48	0.012	pa-pr
Sep 1, 2000	51789.447	0.333 – 0.747	SL	235	0.006	pr
Sep 8, 2000	51796.418	0.871 – 1.159	SL	158	0.007	pa-va
Sep 10, 2000	51798.387	0.394 – 0.893	SL	179	0.011	pr-va-pa
Sep 12, 2000	51800.377	0.109 – 0.489	SL	147	0.011	pa-va
Nov 29, 2000	51878.264	0.829 – 1.616	SL	138	0.007	va
GW Cep						
Sep 11, 2000	51799.514	0.898 – 1.289	SL	104	0.009	pa-va
Oct 27, 2000	51845.355	0.557 – 1.195	SL	172	0.007	pa-pr
Nov 5, 2000	51854.353	0.934 – 1.271	SL	104	0.013	va
Nov 9, 2000	51858.303	0.320 – 0.646	SL	108	0.012	va-pr
Dec 5, 2000	51884.598	0.756 – 1.150	SL	128	0.011	va
CW Cas						
Sep 18, 2000	51806.482	0.549 – 1.432	SP	208	0.026	pr-sc
Sep 27, 2000	51815.531	0.112 – 0.619	SP	54	0.025	cho-ur
Oct 28, 2000	51846.318	0.722 – 1.101	SL	153	0.017	pr-pa
Nov 29, 2000	51787.377	0.112 – 0.821	SP	161	0.013	pr-ur
Dec 01, 2000	51880.381	0.619 – 0.902	SL	124	0.011	pr
Dec 02, 2000	51881.278	0.377 – 0.600	SL	86	0.010	pr

Program and observers: cho - Chochol, pa - Parimucha, pr - Pribulla, sc - Schalling, ur - Urban, va - Vaňko

Our observations led to the determination of 4 new minima times of EF Dra, 5 minima times of GW Cep and 4 minima times of CW Cas (Table 2). The minima times were determined separately for the BV passbands using the parabola fit, Kwee and Van Woerden (K&W) method, sliding integration method, the tracing paper and the "center of mass" method, which were described in detail by Ghedini (1982). The computer code was kindly provided by R. Komžík (2000). To eliminate the influence of the LC asymmetries on the determination of minima times, only the observations around minima in the ± 0.10 phase intervals were used. No systematic shifts between minima times in the B and V passbands were found indicating low photospheric activity. The mean error of the minima times was 0.00015 days for CW Cas and GW Cep and 0.00025 days for EF Dra.

Table 2. Photoelectric and photographic (typed in italics) primary (I) and secondary (II) minima of EF Dra, GW Cep and CW Cas

JD_{hel} 2 400 000+	type	Ref.	JD_{hel} 2 400 000+	type	Ref.	JD_{hel} 2 400 000+	type	Ref.
EF Dra								
47701.8201	II	1	47820.3335	I	2	49763.6371	I	8
47701.8201	II	2	48467.3956	I	3	49763.6379	I	8
47715.8125	II	1	48475.4498	I	4	50301.5180	II	9
47716.8710	I	1	48488.3835	II	4	50301.5194	II	9
47727.8971	I	1	48491.5633	I	4	50571.4095	I	10
47730.8650	I	1	49132.4789	II	5	51789.4252	II	11
47734.8902	II	1	49132.4793	II	5	51789.4274	II	11
47752.4872	I	2	49416.5707	II	6	51796.4217	I	11
47759.4876	II	2	49465.5490	I	7	51796.4222	I	11
47760.3305	II	2	49465.5500	I	7	51798.3304	II	11
47763.5069	I	2	49580.4595	I	7	51798.3312	II	11
47792.3481	I	2	49580.4606	I	7			
GW Cep								
44200.4820	I	12	48909.2942	I	5	51799.4832	I	11
47689.447	I	13	48909.2959	I	5	51799.4846	I	11
47787.330	I	13	49592.5452	I	7	51854.3953	I	11
48504.3788	I	4	50283.4470	I	15	51845.3954	I	11
48504.3839	I	4	51854.3238	I	11	51858.3080	II	11
48544.8710	I	14	51854.3241	I	11	51858.3082	II	11
CW Cas								
32467.061	I	16	36544.3740	I	17	48176.3894	II	20
33567.945	II	16	36552.3450	I	17	48176.5449	I	20
33929.989	I	16	36552.5047	II	17	48176.5493	I	20
33946.108	II	16	36573.3901	I	17	49594.3761	II	7
35745.4570	II	17	36574.3469	I	17	50048.2793	I	21
35745.6156	I	17	36578.3339	II	17	50798.5611	I	22
35747.3711	II	17	37259.4270	II	17	50813.0660	II	22
35747.5280	I	17	37319.3742	II	17	51072.6290	II	22
35755.4995	I	17	37332.2879	I	17	51806.4840	I	11
35757.4146	I	17	37343.2880	II	17	51806.4845	I	11
35757.5736	II	17	41634.4147	I	19	51806.4855	I	11
35812.2610	I	17	41649.4034	I	19	51815.5722	II	11
35814.4920	I	17	41632.3450	II	19	51815.5728	II	11
35835.220	I	18	41650.3595	I	19	51815.5730	II	11
35835.382	II	18	41650.5189	II	19	51846.3416	I	11
36135.5870	I	17	41658.3308	I	19	51846.3428	I	11
36136.5450	I	17	41658.4906	II	19	51881.2574	II	11
36137.3418	II	17	48176.3880	II	20	51881.2588	II	11

References: (1) - Robb & Scarfe (1989), (2) - Plewa et. al (1991), (3) - Wünder & Agerer (1992), (4) - BAV-M 60 (1992), (5) - BAV-M 62 (1993), (6) - BAV-M 68 (1994), (7) - Agerer & Hübscher (1995), (8) - Agerer & Hübscher (1996), (9) - Agerer & Hübscher (1997), (10) - Agerer & Hübscher (1999), (11) - present paper, (12) - Hoffmann (1982), (13) - BAV-M 56 (1989), (14) - Landolt (1992), (15) - BBSAG 112 (1996), (16) - Koho (1952), (17) - Broglia (1964), (18) - Zonn & Semeniuk (1959), (19) - Burchi & De Santis (1975), (20) - BAV-M 59 (1991), (21) - BBSAG 111 (1996), (22) - AAVSO 5 (1999)

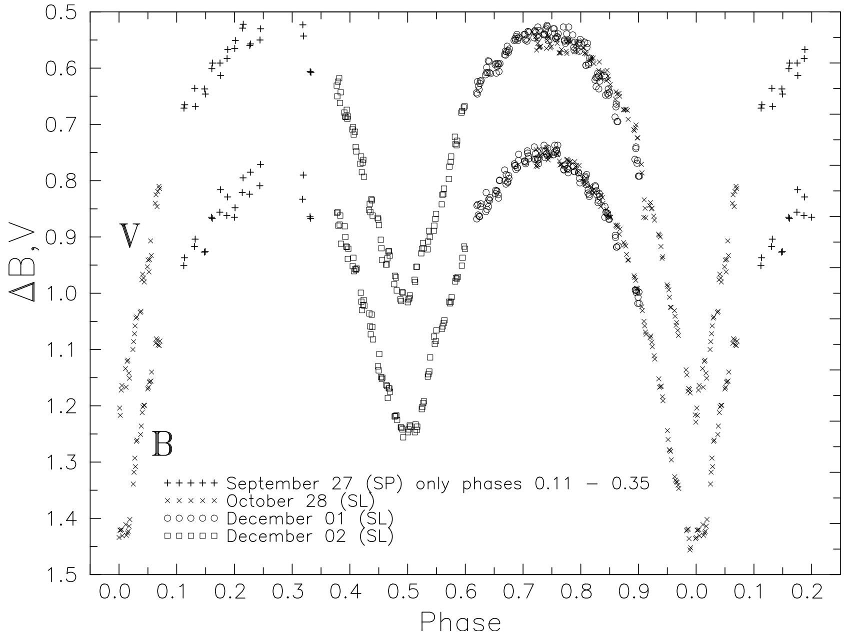

Figure 2. The B and V light curves of CW Cas. The observations obtained on September 18, 2000 are not displayed due to their lower quality.

3. Interpretation

For the study of O-C diagrams of our three systems we used all available photographic, photoelectric and CCD minima times. The photoelectric and CCD minima times were assigned three times larger weights than the photographic ones. As the minima times determined from visual estimates exhibit very large scatter, they were neglected in further analysis of EF Dra and CW Cas, but they were taken into account (with the same weight as the photographic minima times) in the case of GW Cep, where only a few photoelectric and photographic minima times were available. Due to the fact that we did not find any systematic shifts between the photoelectric minima times in different passbands, we used all minima together.

For the determination of the photometric elements, the synthetic LCs and the differential corrections code developed by Wilson & Devinney (1971) (hereafter W&D) and improved by Wilson (1992) were used. Since the number of observations for individual stars is rather large and night-to-night LC variations were not recorded, we computed about 150 normal points for each passband and system. The normal points were determined by running averages of all observations calculated with ephemerides (4), (6) and (8) appropriate for recent minima times. The B and V LCs were solved simultaneously. For each LC the values of σ were evaluated as described by Wilson (1979). The initial parameters of EF Dra were taken from Plewa et al. (1991), for GW Cep and CW Cas from Maceroni & van't Veer (1996). Mode 3 of the W&D code appropriate for the contact binaries was employed assuming synchronous rotation and zero eccentricity. For the computation of monochromatic luminosities the approximate atmospheric models option of the W&D program was used. Coefficients of the gravity darkening $g_1 = g_2 = 0.32$ (Lucy, 1967) and coefficients $A_1 = A_2 = 0.5$ (e.g., Rucinski, 1969) were fixed as appropriate for the convective envelopes ($T_{eff} < 7500$ K). The limb darkening coefficients were interpolated from Table 1 of Al-Naimiy (1978). The temperatures of the primary components were fixed according to their spectral types. The photometric elements were determined by the subset solutions of the non-correlated parameters.

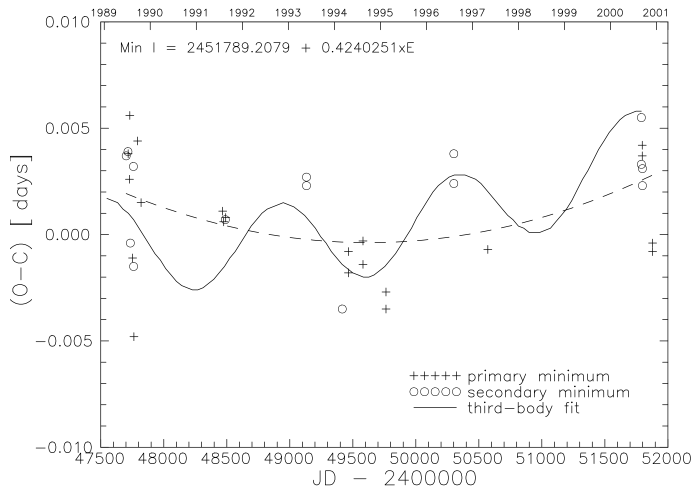

Figure 3. The (O-C) diagram of EF Dra, the mean linear ephemeris is given at the top of the figure. The third-body fit was obtained assuming a circular orbit

3.1. EF Dra

EF Dra ($V_{max} \approx 10.9$ mag, $P = 0.424026$ days, sp. type F9V) was discovered as an X-ray source (1E1806.1+6944) in the Einstein Observatory Extended Medium Sensitivity Survey (Gioia et al., 1987). Fleming et al. (1989) obtained three spectra of EF Dra and suggested that the object is probably a W UMa type binary with spectral type F9V. The authors determined the distance to the system $d = 153$ pc. Robb & Scarfe (1989) confirmed from the VRI photometry that EF Dra is a W UMa system and derived its period $P = 0.42400$ days. The amplitude of its light curve (hereafter LC) is $\Delta V \approx 0.32$ mag. Plewa et al. (1991) classified the system as an A-subtype W UMa binary and noted that the system is totally eclipsing, implying the constraints on the mass ratio $0.1 < q < 0.15$ and inclination of the system $i > 70°$.

Lu (1993) analyzed 43 high and low dispersion spectra of the system obtained at the David Dunlap Observatory. The broadening functions clearly showed the presence of the third component at the radial velocity (hereafter RV) -38 km.s^{-1}, close to the systemic velocity V_γ = -42 km.s^{-1}, suggesting that the third component is physically related to the eclipsing pair. A detailed analysis of the spectra was published by Lu & Rucinski (1999). Circular-orbit fit to the 16 RVs of both components determined from the high-dispersion spectra yielded K_1 = 48.9±4.3 km.s^{-1}, K_2 = 306.1±5.4 km.s^{-1}, q = 0.160±0.014, V_γ = -42.2±3.5 km.s^{-1}, $m_1 \sin^3 i$ = 1.699±0.091 M$_\odot$ and $m_2 \sin^3 i$ = 0.271±0.032 M$_\odot$. The authors explained the discrepancy of the photometric and spectroscopic mass ratios as being due to the third light neglected in previous photometric analysis.

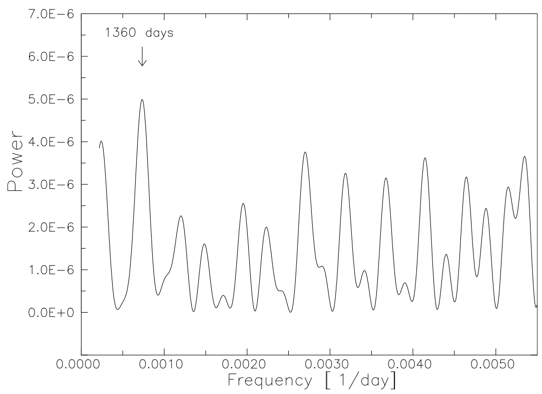

Figure 4. Power spectrum of the (O-C) residuals from the parabolic fit for EF Dra

The observed long-term orbital period increase of EF Dra, seen in Fig. 3, can be interpreted either as a part of a light-time orbit caused by the presence of another body in the system with an orbital period of at least 12 years or by the mass transfer from the less to more massive component. The observed deviations of the (O-C) residuals from the parabolic ephemeris:

$$\begin{array}{lllll} \text{Min I} & = 2\,451\,789.2135 & + 0.4240270 & \times E & + 1.41 \;\; 10^{-11} \times E^2. \\ & \quad\quad \pm 9 & \quad\quad \pm 4 & & \quad \pm 45 \end{array} \quad (2)$$

can be caused by a light-time effect with period P = 1360±150 days, found by the Fourier period analysis of the (O-C) residuals in the range 0.5 - 12 years (see Fig. 4). Unfortunately, the small amount of data and its large scatter prevented us from determining its orbital elements reliably. The mass function of the third body can be expressed as follows:

$$f(m_3) = \frac{(m_3 \sin i_3)^3}{(m_1 + m_2 + m_3)^2} = \frac{4\pi^2 c^3}{G} \frac{A^3_{light}}{P^2_3}, \quad (3)$$

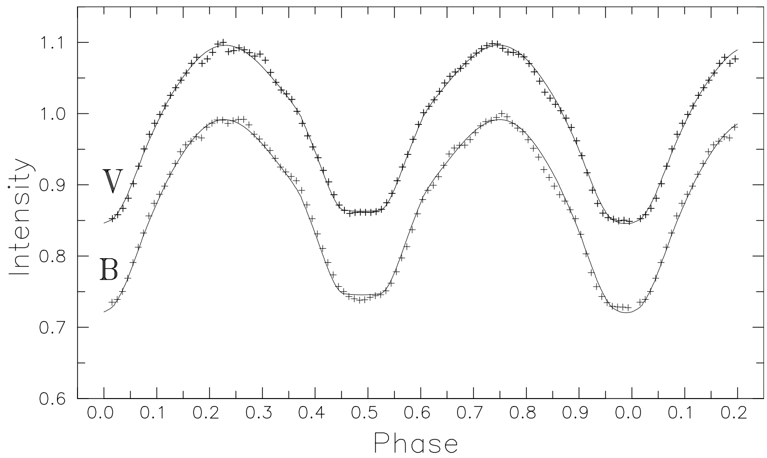

Figure 5. The B and V normal points of EF Dra and their fits corresponding to the optimal photometric elements given in Table 3

where c is the speed of light, A_{light} is the semi-amplitude of the observed light-time effect, P_3 is the orbital period, m_3 the mass and i_3 the orbital inclination of the third body. The values of $A_{light} = 0.002$ days and $P_3 = 1360$ days found from the best fit for circular orbit provide the mass function $f(m_3) = 2.99\ 10^{-3}$ M$_\odot$. In the case of coplanar orbits we get the mass of the third body $m_3 = 0.26$ M$_\odot$. To get an accordance with the mass $m_3 = 0.75$ M$_\odot$ estimated from the photometrically detected third light (see below), the orbital inclination should be $i_3 = 21°$.

The recent photoelectric and CCD minima times (taken since 1995) were used to determine the linear ephemeris of EF Dra suitable for phasing of the new data as well as for future forecasts of the minima times:

$$\begin{array}{lll} \text{Min I } = 2\,451\,789.2132 & + 0.4240262 & \times E. \\ \qquad\qquad\qquad \pm 7 & \qquad \pm 2 & \end{array} \tag{4}$$

The determination and reliability of the photometric elements of EF Dra was simplified by the spectroscopic mass ratio $q = 0.160 \pm 0.014$ (Lu & Rucinski, 1999) fixed in our analysis and the fact that the system is totally eclipsing - the secondary eclipse is clearly the eclipse of the smaller and cooler secondary component (the A-type LC). The situation is, however, complicated by the fact that EF Dra is a triple system. The broadening function in Fig. 1 of Lu (1993) indicates that the light contribution of the third component is at least 0.1 of the total luminosity of the system. Therefore, we have not fixed the zero third light. The resulting third light is much larger in the V passband indicating the late spectral type of the third component (≈K1-2). If it is the main-sequence object, its mass should be about 0.75 M$_\odot$. The long-term orbital period of the third body (P_3 ≈3.7 years or P_3 >12 years) is also supported by the constancy of its RV on broadening functions of Lu (1993).

The resulting photometric elements are given in Table 3 and corresponding fits are shown in Fig. 5.

3.2. GW Cep

GW Cep ($V_{max} \approx 11.4$ mag, $P = 0.318829$ day, sp. type G2) was discovered as a variable by Geyer et al. (1955). Photoelectric LCs of the system were published by Meinunger & Wenzel (1965) and Hoffmann (1982). They recognized it as a W UMa, W-subtype eclipsing variable with total eclipses. Kaluzny (1984) re-analyzed Hoffmann's photoelectric LC of GW Cep, determined photometric elements and found an inclination angle i = 83.9±0.7° and mass ratio q = 0.370±6. These data were used by Maceroni & van't Veer (1996) to calculate the absolute parameters of the system: m_1=1.06M$_\odot$, m_2=0.39M$_\odot$, R_1=1.05R$_\odot$ R_2=0.67R$_\odot$, supposing that both components of GW Cep are located on the ZAMS. Spectroscopic observations of GW Cep are not available.

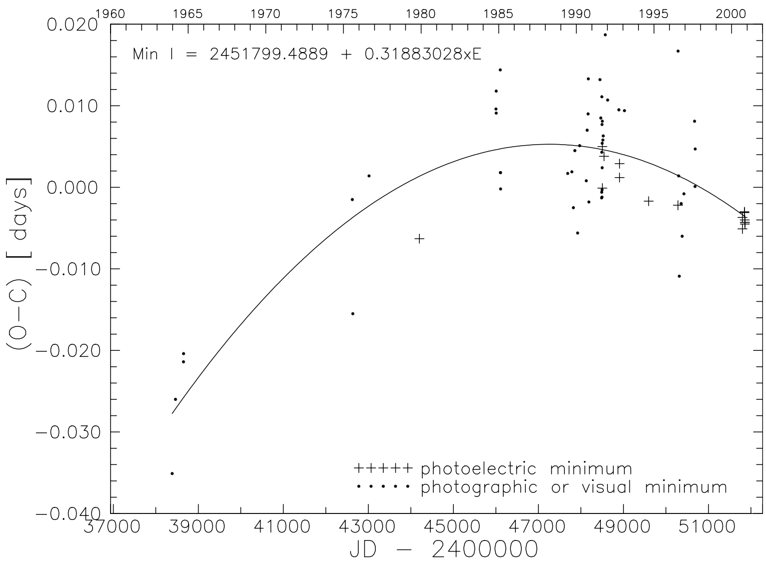

Figure 6. The (O-C) diagram of GW Cep from mean linear ephemeris given at the top of the figure

Due to the fact that only 10 photoelectric minima times of GW Cep were available, we have also used all available photographic and visual minima times in our analysis. The (O-C) diagram from the mean linear ephemeris (see Fig. 6) clearly shows that the orbital period of the system is decreasing. A weighted parabolic fit to all data yields the following quadratic ephemeris:

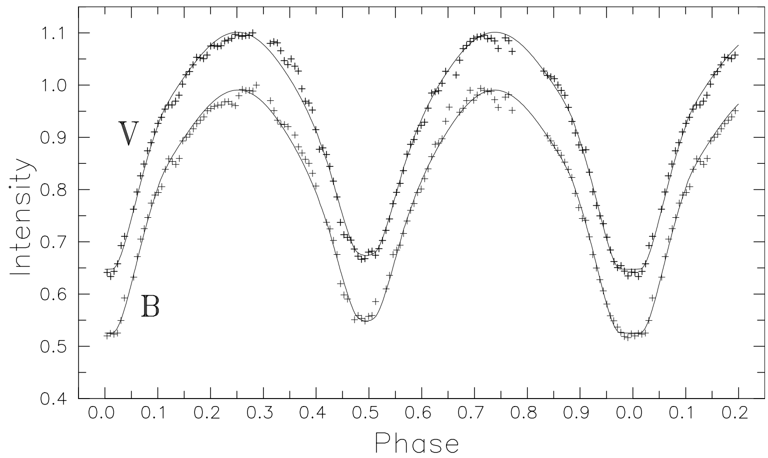

Figure 7. The B and V normal points of GW Cep and their fits corresponding to the optimal photometric elements given in Table 3

$$\begin{aligned} \text{Min I} &= 2\,451\,799.4846 + 0.31882908 \times E - 4.3\ 10^{-11} \times E^2. \\ &\qquad \pm 10 \qquad\qquad \pm 17 \qquad\qquad \pm 5 \end{aligned} \tag{5}$$

The observed period decrease can be interpreted by the mass transfer from the more to less massive component. If we use the masses of the components $m_1 + m_2 = 1.45$ M$_\odot$ determined by Maceroni & van't Veer (1996), we need a mass transfer rate $\Delta m/\Delta t = (6.6 \pm 0.8)\ 10^{-8}$ M$_\odot$ year^{-1} to explain the observed period decrease $\Delta P/P = (3.09 \pm 0.36)\ 10^{-7}$ year^{-1}. It is interesting to note that the weighted sum of squared residuals from the 3rd order polynomial is 7% lower than for the 2nd order polynomial. This indicates the existence of a further effect affecting the observed period change, most probably a light-time effect caused by the presence of a third body. The weighted linear fit to recent minima times (since 1990) provides the following ephemeris:

$$\begin{aligned} \text{Min I} &= 2\,451\,799.4844 + 0.31882946 \times E. \\ &\qquad \pm 10 \qquad\qquad \pm 13 \end{aligned} \tag{6}$$

GW Cep is a W-subtype contact system. Hence the primary minimum is the eclipse of the smaller and brighter secondary component. Furthermore we assume the more massive and cooler component to be the primary one. The spectroscopic mass ratio for this system is unknown. Due to the fact that the photometric mass ratio and orbital inclination correlate with the third light, we have fixed zero third light. Kaluzny (1984) determined by the LC solution $q = m_2/m_1 = 0.371 \pm 0.008$. Therefore we have performed several separate solutions for fixed mass ratios from 0.35 to 0.40. The χ^2 varied in this interval only slightly with minimum at 0.38. The zero third light was fixed in all solutions.

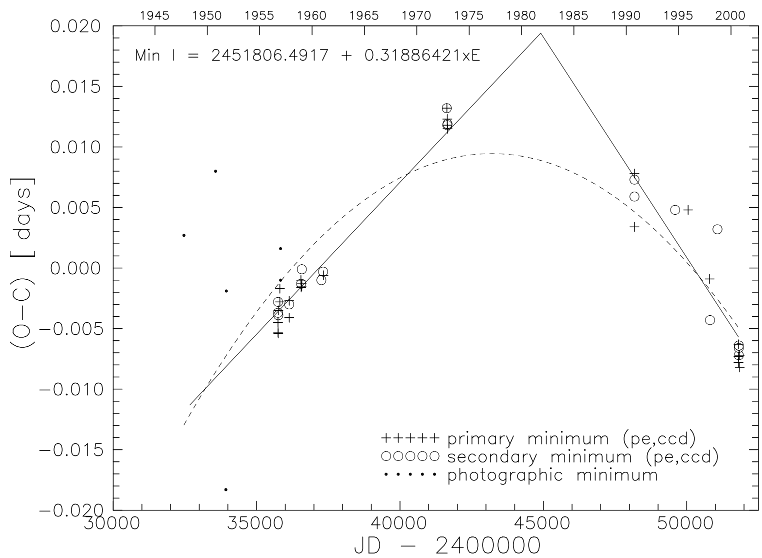

Figure 8. The (O-C) diagram of CW Cas, the mean linear ephemeris is given at the top of the figure

The resulting photometric elements are given in Table 3 and corresponding fits are shown in Fig. 7.

3.3. CW Cas

CW Cas ($V_{max} \approx 11.0$ mag, $P = 0.318863$ day, sp. type G8) is a solar type contact binary (Hilditch & Hill, 1975). Its classification and determination of photometric elements is negatively influenced by the fact that the system is far from being totally eclipsing. The photometric solution of the V LC, obtained by Burchi & De Santis (1975) and performed by Burchi et al. (1977) using the Wilson & Devinney (1971) code, indicated an A subtype i.e., the primary minimum is the transit as proposed by Broglia (1964). Barone et al. (1988) re-analyzed this LC using the improved W&D code and found two possible solutions interpreting the primary minimum as a transit or occultation. On the basis of χ^2 and physical characteristics the authors preferred the occultation solution (W-subtype LC) and found the following optimal geometric elements: i=73.4±0.2°, $q = 0.543 \pm 0.024$ (the cooler and more massive component as the primary one) and fill-out $F = 0.11$. These data were used by Maceroni & van't Veer (1996) to calculate the absolute parameters of the system: $m_1 = 0.99$ M$_\odot$, $m_2 = 0.54$ M$_\odot$, $R_1 = 0.99$ R$_\odot$, $R_2 = 0.75$ R$_\odot$, supposing that both components of CW Cas are located on the ZAMS. Spectroscopic observations of CW Cas are not available.

The (O-C) diagram of CW Cas from the mean linear ephemeris indicates either a continuous or sudden orbital period decrease (Fig. 8).

In the first case, the weighted parabolic fit to all photographic, CCD and photoelectric minima times yields the following ephemeris:

$$\text{Min I} = \underset{\pm 11}{2\,451\,806.4862} + \underset{\pm 13}{0.31886293} \times E - \underset{\pm 24}{2.44}\ 10^{-11} \times E^2. \tag{7}$$

A continuous period decrease could be caused by the mass transfer from the more to less massive component. If we use the masses of the components $m_1 + m_2 = 1.53$ M$_\odot$, determined by Maceroni & van't Veer (1996), we need a mass transfer rate $\Delta m/\Delta t$ = (6.6±0.7) 10^{-8} M$_\odot$year^{-1} to explain the observed period decrease $\Delta P/P$ = (1.75±0.17) 10^{-7} year^{-1}. Unfortunately, the parabolic fit does not explain data well. The weighted sum of squared residuals from the 3rd order polynomial is 17% lower than for the 2nd order polynomial. The deviations from the parabolic fit could be caused by a light-time effect.

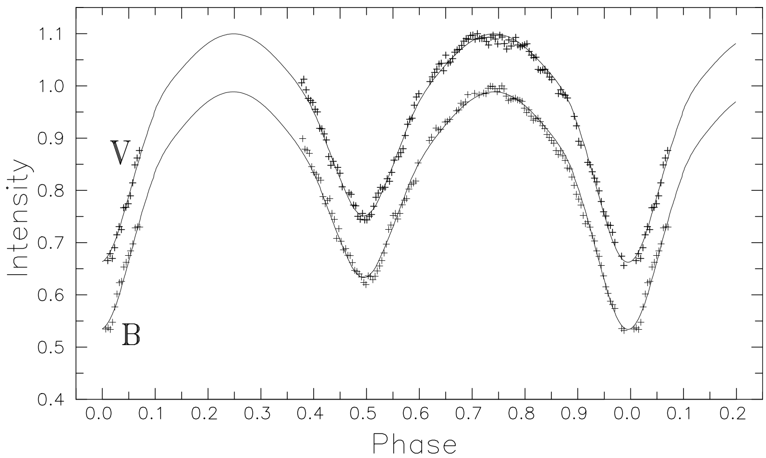

Figure 9. The B and V normal points of CW Cas and their fits corresponding to the optimal photometric elements given in Table 3

In the second case, the sudden period decrease $\Delta P/P$ = -(6.1±0.6) 10^{-6}, which equals 0.169 seconds, occurred sometimes between 1979 and 1984. The exact time of the sudden period change cannot be determined reliably. A sudden mass transfer Δm = (2.3±0.2) 10^{-6} M$_\odot$ from the more to less massive component is necessary to explain the observed period decrease. The separate fits to the data prior and after 1975 give the weighted sum of the squared residuals 31% lower as in the parabolic case.

The linear fit to the minima times (since 1975) provides the following ephemeris:

$$\text{Min I} = \underset{\pm 11}{2\,451\,806.4857} + \underset{\pm 17}{0.31886304} \times E \tag{8}$$

The classification of CW Cas is complicated by the fact that the system is far from being totally eclipsing and the spectroscopic observations are not available. We have accepted W-subtype LC classification, indicating that the primary minimum is an occultation, in agreement with Barone et al. (1988). We adopted their photometric elements as initial parameters for our study. Although the phases 0.1 - 0.4 of the LC are not covered by data of sufficient quality, both minima and maximum at phase 0.75 are well covered. The resulting photometric elements are given in Table 3 and corresponding fits are shown in Fig. 9.

Table 3. Photometric elements and their standard errors (σ) - i - inclination; $q = m_2/m_1$ - mass ratio; Ω - surface potential; r_1, r_2 - volume mean fractional radii; T_1, T_2 - polar temperatures. $\sum w(O-C)^2$ is the weighted sum of squares of residuals for all light curves. Parameters not adjusted in the solution are denoted by a superscript "a"

	EF Dra		GW Cep		CW Cas	
Parameter		σ		σ		σ
i [°]	78.13	0.33	83.96	0.34	74.59	0.11
q	0.16^a	–	0.38^a	–	0.533^a	
Ω	2.0831	0.0023	2.5823	0.0032	2.9317	0.0031
Fill-out	0.455	0.023	0.235	0.014	0.022	0.010
r_1	0.5596	0.0009	0.4797	0.0009	0.4364	0.0007
r_2	0.2560	0.0012	0.3127	0.0010	0.3261	0.0013
T_1 [K]	6000^a	–	5800^a	–	5086^a	–
T_2 [K]	6054	10	6115	9	5510	8
$L_{1B}/(L_{1B}+L_{2B})$	0.8244	0.0012	0.6449	0.0006	0.5338	0.0009
$L_{1V}/(L_{1V}+L_{2V})$	0.8252	0.0011	0.6530	0.0005	0.5480	0.0008
l_{3B}	0.1331	0.0057	0.0^a	–	0.0^a	–
l_{3V}	0.1557	0.0055	0.0^a	–	0.0^a	–
$\sum(O-C)^2$	0.00073	–	0.00534	–	0.00273	–

4. Discussion and conclusion

The scarcity of the minima times makes the interpretation of the (O-C) diagrams ambiguous. The most probable interpretation of the observed period changes is the mass transfer between the components combined with the light-time effect caused by the third body. The presence of the third body in EF Dra was conclusively proved. The analysis of the (O-C) diagrams also indicates its possible presence in GW Cep and CW Cas.

The photometric fits explain the normal points quite well, although there is some additional light at the shoulders of the secondary minimum (GW Cep and CW Cas). The discrepancy can be reduced by decreasing the coefficient of gravity darkening. There are indications that the gravity darkening coefficient is smaller both in the case of convective and radiative envelopes (see e.g., Rucinski, 1993). For zero gravity darkening we get much better accord of the fits and observed LCs. While the fill out factors of EF Dra (0.455) and GW Cep (0.235) are large, the fill-out factor of CW Cas is only $F = 0.022 \pm 0.010$. This value corresponds to the temperature differences of the components 424±9 K. Therefore, the system CW Cas is in rather poor thermal contact. This can be assesed using the period-colour relation (e.g., Wang, 1994):

$$(\mathrm{B}-\mathrm{V})_0 = 0.062 - 1.31 \log P(\mathrm{days}), \tag{9}$$

where $(\mathrm{B\text{-}V})_0$ is the intrinsic colour index of the system. While the observed colours EF Dra and GW Cep roughly correspond to the above relation, CW Cas is much redder for its period indicating poor contact of the components (see also Fig. 10).

Although the absolute parameters of GW Cep and CW Cas were not determined reliably, some clues on the evolutionary status of their components can be inferred from the mean densities of the components. It can be easily shown (see e.g., Mochnacki, 1985) that mean densities of the components can be determined without knowledge of the absolute parameters:

$$\overline{\rho_1} = \frac{4\pi^2}{GP^2}\frac{1}{1+q}\frac{1}{V_1(F,q)}, \tag{10}$$

$$\overline{\rho_2} = \frac{4\pi^2}{GP^2}\frac{q}{1+q}\frac{1}{V_2(F,q)}, \tag{11}$$

where $V_{1,2}(F,q)$ are the volumes of the components expressed in the semi-major axis as a unit. The resulting mean densities of all three systems together with ZAMS densities (Lang, 1992) are shown in Fig. 10. The primary components of all three systems have lower mean densities for their spectral type due to the energy tranfer to the secondary components as expected for contact systems. The densities of both components of EF Dra are much lower than expected for their spectral type F9 (Fleming et al., 1987). This could be partly caused by the fact that A-type W UMa systems are evolutionary closer to TAMS. The more probable explanation is an earlier spectral type of the primary ≈ F0. The determination of its spectral type is, however, negatively influenced by the fact that the spectrum of the eclipsing pair is heavily blended with the lines of the third component. The determination from the observed colour indices is complicated also by non-negligible interstellar reddening ($d \approx 150$pc).

Unfortunately, the studied systems were not included into the Hipparcos astrometric mission. Spectroscopic elements of CW Cas and GW Cep are not available. Hence their absolute parameters cannot be determined reliably. The published spectroscopic elements of EF Dra combined with photometric elements including its orbital inclination $i = 78.1°$ yield the absolute parameters of the components: $m_1 = 1.81 \pm 0.10$ $\mathrm{M}_\odot$, $m_2 = 0.29 \pm 0.03$ $\mathrm{M}_\odot$, $R_1 = 1.701 \pm 0.026$ $\mathrm{R}_\odot$ and $R_2 = 0.778 \pm 0.012$ $\mathrm{R}_\odot$.

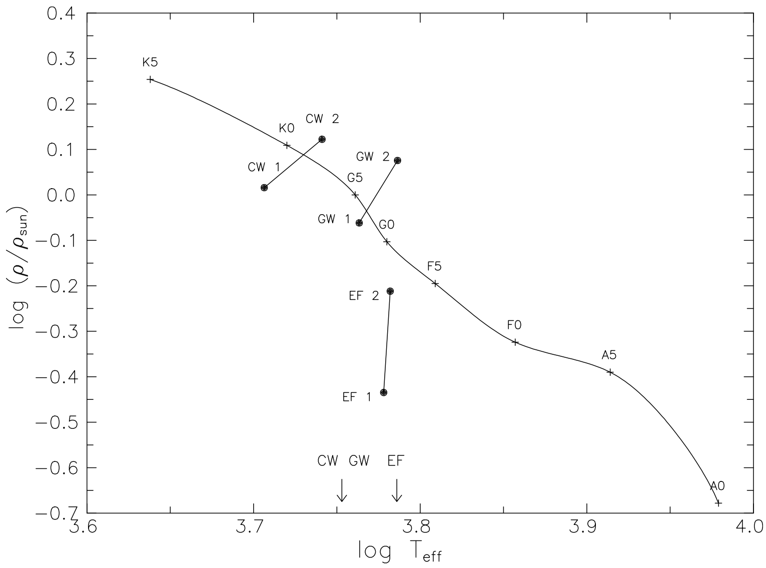

Figure 10. The ZAMS mean densities (Lang, 1992) and the mean densities of the primary(1) and the secondary(2) components of EF Dra, GW Cep and CW Cas. The logarithms of the effective temperatures expected from the period-colour relation (9) are indicated by arrows

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References

Agerer, F., Hübscher, J.: 1995, *Inf. Bull. Variable Stars* , 4222

Agerer, F., Hübscher, J.: 1996, *Inf. Bull. Variable Stars* , 4383

Agerer, F., Hübscher, J.: 1997, *Inf. Bull. Variable Stars* , 4472

Agerer, F., Hübscher, J.: 1999, *Inf. Bull. Variable Stars* , 4711

Akerlof, C., Ambrose, S., Balsano, R., Bloch, J., Casperson, D., Fletcher, S., Gisler, G., Hills, J., Kehoe, R., Lee, B., Marshall, S., Mc Kay, C., Pawl, A., Schaefer, J., Szymanski, J., Wren, J.: 2000, *Astron. J.* **119**, 1901

Al-Naimiy H.M.: 1978, *Astrophys. Space Sci.* **53**, 181

Barone, F., Maceroni, C., Milano, L., Russo G.: 1988, *Astron. Astrophys., Suppl. Ser.* **197**, 347

Broglia, P.: 1964, *Mem. Soc. Ital.* **35**, 23

Burchi, R., De Santis, R,: 1975, *Inf. Bull. Variable Stars* , 962

Burchi, R., Milano, L., Russo, G.: 1977, *Astrophys. Space Sci.* **47**, 35

Fleming, T.A., Gioia, I.M., Maccacaro, T.: 1989, *Astron. J.* **98**, 692

Geyer, E., Kippenhahn, R., Strohmeier, W.: 1955, *Kleine Veröff. Remeis Sternw. Bamberg* , No. 9

Ghedini, S.: 1982, *Software for Photometric Astronomy* , Willmann-Bell Publ. Comp., Richmond
Gioia, I.M., Maccacaro, T., Wolter, A.: 1987, in *Symp. 124, Observational Cosmology*, eds.: A. Hewitt, G. Burbidge and L.Z. Fang, D. Reidel, Dodrecht, 593
Hilditch, R.W., Hill, G.: 1975, *Mem. Roy. Astron. Soc* **79**, 101
Hoffmann, M.: 1982, *Astrophys. Space Sci.* **83**, 195
Kaluzny, J.: 1984, *Acta Astron.* **34**, 217
Koho, S.: 1952, *Tokyo Bull. 2nd. Ser.* **49**, 383
Komžík, R.: 2000, *private comunication* ,
Landolt, A.U.: 1992, *Publ. Astron. Soc. Pac.* **104**, 336
Lang, K.R.: 1992, *Astrophysical Data Planets and Stars* , Springer Verlag
Lu, W.: 1993, *Inf. Bull. Variable Stars* , 3947
Lu, W., Rucinski, S.M.: 1999, *Astron. J.* **118**, 515
Lu, W., Rucinski, S.M., Mochnacki, S.W.: 2000, *Astron. J.* **120**, 1133
Lucy, L.B.: 1967, *Z. Astrophys* **65**, 89
Maceroni, C., van't Veer, F.: 1996, *Astron. Astrophys., Suppl. Ser.* **311**, 523
Meinunger, L., Wenzel, W.: 1965, *Mitt. Ver. Sterne* **2**, 164
Mochnacki, S.W.: 1985, in *Interacting Binaries*, eds.: P.P. Eggleton and J.E. Pringle, D. Reidel Publishing Company, 51,
Plewa, T., Haber, G., Wlodarcik, K.J., Krzesinski, J.: 1991, *Acta Astron.* **41**, 291
Robb, R.M., Scarfe, C.D.: 1989, *Inf. Bull. Variable Stars* , 3370
Rucinski, S.M.: 1969, *Acta Astron.* **19**, 245
Rucinski, S.M.: 1997a, *Astron. J.* **113**, 407
Rucinski, S.M.: 1997b, *Astron. J.* **113**, 1112
Rucinski, S.M.: 1993, in *The Realm of the Interacting Binaries*, eds.: J. Sahade, Y. Kondo and G. McCuskey, Kluwer Acad. Publishers, Dordrecht, 111
Wang, J.M.: 1994, *Astrophys. J.* **434**, 277
Wilson, R.E., Devinney, E.J.: 1971, *Astrophys. J.* **166**, 605
Wilson, R.E.: 1979, *Astrophys. J.* **234**, 1054
Wilson, R.E.: 1992, *private communication* ,
Wunder, E., Agerer, F.: 1992, *Inf. Bull. Variable Stars* , 3811
Zonn, W., Semeniuk, I.: 1959, *Acta Astron.* **9**, 141