Can cometary nuclei have originated in collapsing protosolar nebula?

Their distribution before and after the collapse

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Abstract. The idea that comets of the Oort (and also Hills) cloud were already formed in the interstellar molecular cloud, which was the parent cloud of protosolar nebula, is suggested. The dynamics of comets in the free-falling collapse of this nebula is described on the basis of a simplified model. It appears that, if a homogeneous gas nebula collapses in this way, its border falls to the centre freely, whereby the nebula always remains homogeneous, no matter escapes outward, and the motion of a chosen cometary nucleus, located into the nebula, can be described analytically (the decelerating effect of the nebular material is assumed to be insignificant). This description is given. Based on the idea suggested and the above-mentioned simplified model of collapse, which allows only qualitative conclusions to be drawn, one can claim that the resultant conception of the cometary population as a whole does not differ from that regarded as real.

Key words: comets - cosmogony - celestial mechanics - protosolar nebula

1. Introduction

The question of the origin of comets in the Oort cloud still remains unanswered. The primordial theory, assuming the creation of comets in the Uranus-Neptune region (Kuiper, 1951; Safronov, 1972; Greenberg, 1982) has some difficulties. The first is the manner in which the Oort cloud comets gained their angular momentum (Marochnik et al., 1988). A problem is also the ejection of comets from the region last-mentioned into the region of the Oort cloud. The mass of this cloud was estimated at between 0.1 and 1 $M_\oplus$ (Earth mass) at the time the primordial theory was created. However, the best estimate of the mass of the original Oort cloud is now given as 100 to 250 $M_\oplus$ (Weissman, 1990). Therefore, the orbital energy of planet-ejectors (mainly of Neptune (Fernandèz, 1985)) was too low for complete ejection (Neslušan, 1992).

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The weakness in the theory of interstellar origin of comets (McCrea, 1975; Club and Napier, 1982) is the low probability that interstellar comets would be captured by the Solar System.

In this paper, in an effort to answer the question of the cometary origin, it is suggested that the primordial and interstellar theories be combined. We shall assume that comets are generally created in interstellar molecular clouds as McCrea and other authors did. However, at the same time we shall assume that the Oort cloud comets were created in the very parent molecular cloud of the protosolar nebula.

The collapse of the nebula was the consequence of kinetic energy loss of atoms, molecules, and microscopic dust grains by radiation at the time of their deceleration, or thereafter. However, this mechanism of kinetic energy loss was not effective in the case of such large bodies such as cometary nuclei. This is the basic idea of the deliberations that follow.

It has been found that the solution of our problem is too difficult in models of the protosolar nebula collapse such as those of Hayashi (1966), Bodenheimer (1968, 1981), Larson (1969, 1972), and Shu (1977). We shall, therefore, restrict ourselves to a qualitative analysis of the problem by using a very simplified model of protosolar nebula collapse (see Section 2). Consequently, our results should, of course, be only considered as quanlitative.

2. Initial scheme

Drawing on the models of protosolar nebula collapse mentioned in Section 1, it is assumed that the protosolar nebula was spherically symmetric and homogeneous with a mass roughly equal to the present solar mass and with a radius of about 1×10^4 AU before its collapse. It consisted mostly of hydrogen and helium. Heavier chemical elements were rare, their abundances amounting about 1% to 2%.

During the first period, which we are interested in in this paper, the collapse of nebula was of a free-falling character. In more detail, the shock wave spread as a free-falling body from the border of the nebula to its centre. The internal part of the nebula (front of the shock wave) remained roughly homogeneous, and the density in the external part depended on distance r from the nebula's centre as $\propto r^{-2}$.

As in the Introduction, we assumed the existence of cometary nuclei in the protosolar nebula already before its collapse began. Since the nebula was homogeneous, the distribution of nuclei was most probably homogeneous too, the nuclei moved randomly in all directions, and took part in nebular rotation.

In the model used here, the border of the protosolar nebula falls to the nebula's centre freely, whereby the nebula will always remain homogeneous, and no matter will escape outward. The rotation is insignificant during the free-falling collapse (flattening will always be negligible). Gravity will be the only force, which will act on the nuclei during the whole period we are interested in. Once the deceleration effect of the nebular environment becomes significant, the computation will be terminated. (The rate of the deceleration effect in comparison with gravity is estimated in Section 3.) We shall assume that all nuclei still located within the nebula at this time were incorporated in the protoplanetary disc and that they did not become a part of the later cometary population.

In contrast, the nuclei which left the nebula earlier, moved along Keplerian orbits with different semi-major axes. We shall determine the distribution of the orbits.

3. An estimate of deceleration effect

We shall denote the cross-section of the assumed cometary nucleus by σ. Let the nucleus move with velocity v_n within time interval Δt, the velocity being consider constant during this time interval.

If $v_n = 0$, the mean momentum imparted to the nucleus by all particles colliding with it within Δt is zero, since the momenta from opposite directions compensate one another. If $v_n \neq 0$, the particle imparts momentum $(\sigma v_n \Delta t).(\rho v_n)$ to the nucleus within time interval Δt. The momentum is imparted in the direction opposite to that of the motion of the nucleus. Quantity ρ is the density of the nebular environment.

The momentum divided by time interval Δt is in fact the acting deceleration force F_d. Thus,

$$F_d = \frac{(\sigma v_n \Delta t).(\rho v_n)}{\Delta t} = \sigma \rho v_n^2 \qquad (1).$$

We shall assume that the deceleration effect can be neglected, if force F_d is small in comparison with the gravitational force, F_g.

The description of the dynamics of a mass point in a stable, thin, spherically symmetric, homogeneous, material nebula with mass M_o and radius R represents a simple, completely analytically manageable, celestial mechanics problem, which we shall, therefore, not deal with here. It has been found that, the period of mass point motion about the nebular centre is the same for all mass points in the nebula. Hence, velocity v_n is maximal along the longest orbit, i.e. it is circular with its radius almost equal to R. The velocity

$$v_n = \sqrt{\frac{k^2 M_o}{R}} \qquad (2)$$

in this case, where k is the Gauss gravitational constant.

If the mass of the assumed nucleus is denoted m_n, the ratio p_F of the maximum deceleration and gravitational forces is

$$p_F = \frac{F_d}{F_g} = \frac{\sigma \rho R}{m_n} \qquad (3)$$

The density of the nebula ρ depends on radius R as

$$\rho = \frac{3M_o}{4\pi R^3} \tag{4}$$

in our model. If $p_F \leq p_c$, the radius of the nebula R, at which the deceleration effect may still be neglect, is limited by the inequality

$$R \geq \sqrt{\frac{3\sigma M_o}{4\pi p_c m_n}} \tag{5}$$

Quantity p_c is the maximum critical ratio, at which the deceleration effect may still be neglected.

We estimate circular cross-section σ, which can be expressed as πR_n^2, where R_n is the radius of a typical cometary nucleus. We shall regard a nucleus whose mass $m_n \approx 1 \times 10^{13}\ kg$ and density $\rho_n \approx 200\ kg/m^3$ to be a typical cometary nucleus. Since $R_n = [3m_n/(4\pi\rho_n)]^{1/3}$, we can modify (5) to read

$$R \geq \frac{1}{m_n^{1/6}} \left(\frac{3}{4\pi\rho_n}\right)^{1/3} \sqrt{\frac{3M_o}{4p_c}} \tag{6}$$

We shall regard the value $p_c \approx 0.001$ as the appropriate value of the critical ratio. Relation (6) then yields the limit of radius $R \geq 2 \times 10^2\ AU$. (We have put $M_o = 2 \times 10^{30}\ kg$.) By using other admissible values of p_c, m_n, ρ_n, we obtain the lower or higher limiting value of radius R, but the value will always be of the same order. Thus, we shall use the above-mentioned value in our qualitative analysis.

Note that the free-falling collapse of the protosolar nebula terminated, when its density achieved a value of the order $\approx 10^{-10}\ kg/m^3$ (Gaustad, 1963). The radius of the nebula with density $\approx 10^{-10}\ kg/m^3$ is roughly $1 \times 10^2\ AU$ in our model. We can see that the deceleration effect of the nebular material on cometary nuclei is negligible almost during the whole of the free-falling collapse.

4. Motion of a particle in a collapsing homogeneous nebula

Let us consider the simplified model of a collapsing nebula mentioned in Section 2. Let the nebula be homogeneous, spherically symmetric with mass M_o. The rotation of the nebula will be assumed enough to avoid flattening of the nebula during the whole of its free-fall collapse period. (We shall, therefore, not consider it in this section.) The nebula's radius is R_o at time $t = 0$ and its border begins to fall inward, freely, the nebula remaining homogeneous. At time t ($t > 0$) the nebula's radius will be R.

The acceleration $\ddot{R}$ of the nebular border is given by the equation

$$\ddot{R} = -\frac{k^2 M_o}{R^2} \tag{7}$$

and its velocity by

$$\dot{R} = -\sqrt{\frac{2k^2M_o}{R_o}}\sqrt{\frac{R_o}{R} - 1} \tag{8}$$

at time t.

Let the position of the chosen mass point, mass m, be given by vector $\boldsymbol{r} = (x, y, z)$. The first time derivative of this vector is $\dot{\boldsymbol{r}} = (\dot{x}, \dot{y}, \dot{z})$ and the second $\ddot{\boldsymbol{r}} = (\ddot{x}, \ddot{y}, \ddot{z})$. The equations of motion can then be expressed as

$$m\ddot{\boldsymbol{r}} = -\frac{k^2 m M_o}{R^3}\boldsymbol{r} \tag{9}$$

Now, let us replace the time derivatives of vector $\boldsymbol{r}$ by derivatives with respect to R. It holds that $\dot{\boldsymbol{r}} = \boldsymbol{r}'.\dot{R}$ and $\ddot{\boldsymbol{r}} = \boldsymbol{r}".\dot{R}^2 + \boldsymbol{r}'.\ddot{R}$, where $\boldsymbol{r}' = d\boldsymbol{r}/dR$ and $\boldsymbol{r}" = d^2\boldsymbol{r}/dR^2$. Now, using (7) and (8) the equations of motion (9), after simple mathematical handling, can be modified to

$$2R_oR^2\boldsymbol{r}" - 2R^3\boldsymbol{r}" - R_oR\boldsymbol{r}' + R_o\boldsymbol{r} = 0 \tag{10}$$

We shall first seek the solution of this vector differential equation in the form of power expansion

$$\boldsymbol{r} = \sum_{j=0}^{\infty} \boldsymbol{a}_j R^j \tag{11}$$

where each of the vectors $\boldsymbol{a}_j = (a_{xj}, a_{yj}, a_{zj})$ is a constant vector for $j = 0, 1, 2, \ldots$. If we substitute expansion (11) as well as its time derivatives into equations (10), we can easily show that all vectors $\boldsymbol{a}_j$, except $\boldsymbol{a}_1$, are equal to the zero vector. Hence in this case the solution of equations (10) is

$$\boldsymbol{r} = \boldsymbol{a}_1 R \tag{12}$$

Solution (12) describes the motion of such a mass point, which was at rest relative to the nebular material at time $t = 0$. If the position of the point was described by vector $\boldsymbol{r}_o$ at time $t = 0$, then $\boldsymbol{a}_1 = \boldsymbol{r}_o/R_o$.

Therefore, it is necessary to find a more general solution of equations (10) in the form of a more complicated non-integer power expansion

$$\boldsymbol{r} = \sum_{j=0}^{\infty} \boldsymbol{b}_j R^{j+s} \tag{13}$$

where each of the vectors $\boldsymbol{b}_j = (b_{xj}, b_{yj}, b_{zj})$ is again a constant vector for $j = 0, 1, 2, \ldots$. Quantity s is a real constant. If we substitute solution (13) with its time derivatives into equations (10), neither vector $\boldsymbol{b}_o$, nor vectors $\boldsymbol{b}_j$ with $j > 0$ need be zero vectors if

$$2s(s-1) - s + 1 = 0 \tag{14}$$

and provided $s = 1$ or $s = 1/2$. If $s = 1$, then $\boldsymbol{b}_o \neq \mathbf{o}$, generally, but $\boldsymbol{b}_j = \mathbf{o}$ for $j = 1, 2, 3, \ldots$. We have thus in fact obtained solution (12).

If $s = 1/2$, it is possible to prove that

$$\boldsymbol{r} = \boldsymbol{b}_o\sqrt{R}\left\{1 - \frac{R}{2R_o} - \sum_{j=2}^{\infty}\left[\frac{(2j-3)!!}{2^j.j!}\left(\frac{R}{R_o}\right)^j\right]\right\} \equiv \boldsymbol{b}_o\sqrt{R}\sqrt{1 - \frac{R}{R_o}} \tag{15}$$

We shall regard the superposition of solutions (12) and (15) as the definitive solution of equations (10), i.e.

$$\boldsymbol{r} = \boldsymbol{a}_1 R + \boldsymbol{b}_o\sqrt{R}\sqrt{1 - \frac{R}{R_o}} \tag{16}$$

(The drawback of solution (15) itself that the mass point can be positioned at time $t = 0$, when $R = R_o$, only if the origin of the coordinate system $\boldsymbol{r} = \mathbf{o}$.

If the position and velocity of the mass point at time $t = 0$ are given by vectors $\boldsymbol{r}_o = (x_o, y_o, z_o)$ and $\boldsymbol{v}_o = (v_{xo}, v_{yo}, v_{zo})$, respectively, vectors $\boldsymbol{a}_1$, $\boldsymbol{b}_o$ can then be expressed in terms of $\boldsymbol{r}_o$, $\boldsymbol{v}_o$, and relation (16) becomes

$$\boldsymbol{r} = \boldsymbol{r}_o\frac{R}{R_o} + \boldsymbol{v}_o R_o\sqrt{\frac{2R}{k^2 M_o}}\sqrt{1 - \frac{R}{R_o}} \tag{17}$$

We obtain components $\dot{x}$, $\dot{y}$, $\dot{z}$ of velocity vector $\dot{\boldsymbol{r}}$ by differentiating vector (17) with respect to time:

$$\dot{\boldsymbol{r}} = -\boldsymbol{r}_o\frac{1}{R_o}\sqrt{\frac{2k^2M_o}{R}}\sqrt{1 - \frac{R}{R_o}} + \boldsymbol{v}_o\left(2 - \frac{R_o}{R}\right) \tag{18}$$

The mass point cannot move in the internal part of the nebula during the whole of the collapse period. It can leave the nebula. This happens at the moment, when the nebular border achieves a certain radius R_{out}, where $r = R_{out}$ (we have put $|\boldsymbol{r}| = r$). Radius R_{out}, as a function of vectors $\boldsymbol{r}_o$ and $\boldsymbol{v}_o$, can be expressed as the solution of quadratic equation

$$AR_{out}^2 + BR_{out} + C = 0 \tag{19}$$

where

$$A = (R_o^2 - r_o^2)^2 + 2c_v(R_o^2 - r_o^2)v_o^2 + c_v^2 v_o^4 + 4c_v(\boldsymbol{r}_o.\boldsymbol{v}_o)^2 \tag{20}$$

$$B = -2c_v R_o(R_o^2 - r_o^2) - 2c_v^2 R_o v_o^4 - 4c_v R_o(\boldsymbol{r}_o.\boldsymbol{v}_o)^2 \tag{21}$$

$$C = c_v^2 R_o^2 v_o^4 \tag{22}$$

We have put $|\boldsymbol{r}_o| = r_o$, $|\boldsymbol{v}_o| = v_o$, and $c_v = 2R_o^3/(k^2 M_o)$.

5. Distribution of cometary nuclei

We assume a quite uniform distribution of cometary nuclei in the protosolar nebula before the beginning of its free-fall collapse. We also assume their random motion in all possible directions and that the nuclei are taking part in the general slow rotation of the nebula.

The distribution of the absolute values of velocities of the nuclei has proved to be difficult. We shall assume they have a Maxwell distribution, although it is not sure, whether we can apply this distribution to such large bodies as cometary nuclei. The Maxwell distribution can be expressed as

$$d^6n = \frac{\Delta^3 N}{\pi^{3/2} v_p^3}.exp\left(-\frac{v'^2}{v_p^2}\right).v'^2.dv'.d\vartheta.cos\varphi.d\varphi \tag{23}$$

in coordinate system $x' - y' - z'$ rotating with the nebula about the z'−axis. This means that if the number of nuclei in an element of the nebula with volume $\Delta^3 V$ is $\Delta^3 N$, the number of nuclei moving with velocities ranging from v' to $v' + dv'$ in directions described by the angular intervals from ϑ to $\vartheta + d\vartheta$ and from φ to $\varphi + d\varphi$ is equal to d^6n. Constant v_p is the so-called most numbered velocity.

The position of volume element $\Delta^3 V$ can be described in rectangular coordinates x_o, y_o, z_o as well as in spherical coordinates r_o, θ, ϕ in the non-rotating coordinate system (the nebula rotates about the z−axis, the $x - y$ plane is the nebular equatorial plane) at time $t = 0$, where

$$x_o = r_o.cos\theta.cos\phi \tag{24}$$

$$y_o = r_o.sin\theta.cos\phi \tag{25}$$

$$z_o = r_o.sin\phi \tag{26}$$

If the border of the nebula at its equator rotates with velocity v_{PSN} at time $t = 0$, the rectangular components v_{xo}, v_{yo}, v_{zo} of velocity of the assumed nucleus in element $\Delta^3 V$ can then be expressed as

$$v_{xo} = v'_x \tag{27}$$

$$v_{yo} = v'_y + v_{PSN}.cos\phi \tag{28}$$

$$v_{zo} = v'_z \tag{29}$$

in the last-mentioned coordinate system. Quantities v'_x, v'_y, v'_z are rectangular components of velocity of the assumed nucleus in volume element $\Delta^3 V$ in the rotating system:

$$v'_x = v'.cos\vartheta.cos\varphi \tag{30}$$

$$v'_y = v'.sin\vartheta.cos\varphi \tag{31}$$

$$v'_z = v'.sin\varphi \tag{32}$$

The element of volume $\Delta^3 V$ at distance from r_o to $r_o + \Delta r_o$ from the nebular centre in the direction described by angular intervals from θ to $\theta + \Delta\theta$ and from ϕ to $\phi + \Delta\phi$ is

$$\Delta^3 V = r_o^2.\Delta r_o.\Delta\theta.cos\phi.\Delta\phi \qquad (33)$$

and contains

$$\Delta^3 N = \frac{3N}{4\pi R_o^3} r_o^2.\Delta r_o.\Delta\theta.cos\phi.\Delta\phi \qquad (34)$$

nuclei. Quantity N is the total number of nuclei in the whole nebula.

Since we shall be interested in the distribution of orbits at the end of the free-fall collapse, the distribution being independent of direction, we can take into account the axial symmetry of the problem and integrate distribution (23) over all possible values of angle θ (from 0 to 2π *radians*) at once. Then

$$d^5 n = \frac{3N}{2\pi^{3/2} v_p^3 R_o^3} r_o^2.exp\left(-\frac{v'^2}{v_p^2}\right).v'^2.\Delta r_o.cos\phi.\Delta\phi.dv'.d\vartheta.cos\varphi.d\varphi \qquad (35)$$

and we can make the computation in five-dimensional instead of six-dimensional phase space.

The initial dynamics of the chosen cometary nucleus is given by relations (24-29), therefore, we can easily determine radius R_{out} from relation (19) and calculate vectors $\boldsymbol{r}$, $\dot{\boldsymbol{r}}$ (relations (17), (18)) characterizing the position and velocity of the group of nuclei leaving the nebula in the same direction with the same velocity. The group moves along a Keplerian orbit after leaving the nebula, and we can determine the semi-major axis as well as pericentre and apocentre of this orbit on the basis of calculated vectors $\boldsymbol{r}$, $\dot{\boldsymbol{r}}$ in the routine manner. All these Keplerian orbits are those, which were sought.

6. Numerical calculation

The distributions of the semi-major axes as well as of the pericentres and apocentres of the cometary nuclei orbits were calculated on the basis of relation (35). The result is their distribution at the end of the free-fall collapse of the nebula, when its radius $R = 200\ AU$ (using relation (6)).

The integration was made over the whole five-dimensional phase space, where the particular elements of volume were specified by the discrete limiting values of quantities r_o ($0 - 10000\ AU$, step $500\ AU$), ϕ ($-90° - 90°$, step $10°$), v' ($v_p/20 - 5v_p$, step $v_p/20$), ϑ ($0° - 360°$, step $10°$), and φ ($-90° - 90°$, step $10°$) for the chosen pair of free parameters v_p and v_{PSN}. We empirically approximated the upper limit of velocity v', actually equal to infinity, by the value $5v_p$ considering that $exp(-5^2).5^2 \ll 1$.

We assumed the number ν of cometary nuclei in the element of the five-dimensional phase volume to be equal to $\Delta r_o.\Delta\phi.dv'.d\vartheta.d\varphi$. If the total number of nuclei is $N = N_i\nu$, where N_i is the number of nuclei for a given number of

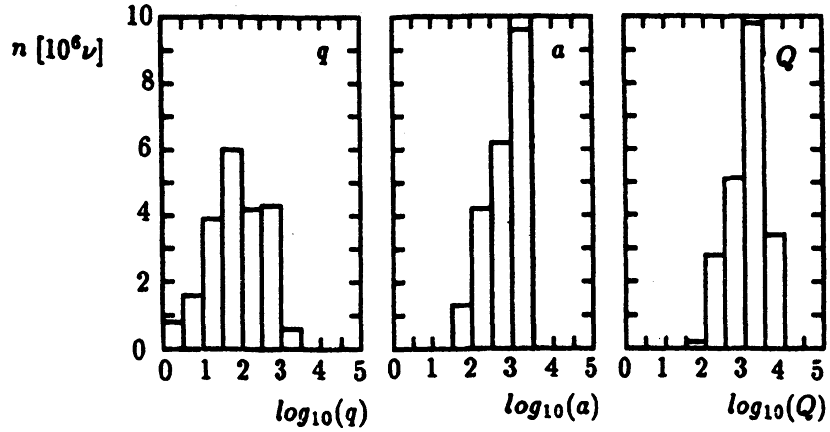

Figure 1. The distributions of pericentres (q), semi-major axes (a), and apocentres (Q) of cometary nuclei orbits at the end of the idealized collapse of the homogeneous nebula for $v_{PSN} = v_I/10$, $v_p = v_I/10$. Quantity n is the number of nuclei with pericentres (semi-major axes, apocentres) of orbits in the given interval on the logarithmic scale. (All these values are given in AU.) The number of nuclei whose orbits have apocentres outside the nebula is $3.11 \times 10^6.\nu$, the number of nuclei captured in the nebula is $9.31 \times 10^5.\nu$, the total number of all nuclei is $N_i\nu = 2.23 \times 10^7.\nu$, and the original radius of the nebula is $R_o = 1 \times 10^4$ AU. (All mentioned quantities are described in the text.)

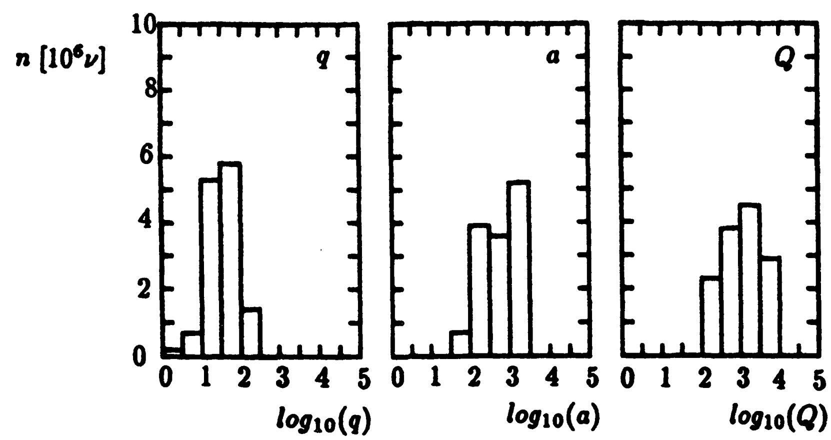

Figure 2. The distributions of pericentres (q), semi-major axes (a), and apocentres (Q) of cometary nuclei orbits at the end of the idealized collapse of the homogeneous nebula for $v_{PSN} = v_I/10$, $v_p = v_I/100$. Quantity n is the number of nuclei with pericentres (semi-major axes, apocentres) of orbits in the given interval on the logarithmic scale. (All these values are given in AU.) The number of nuclei whose orbits have apocentres outside the nebula is $3.11 \times 10^6.\nu$, the number of nuclei captured in the nebula is $8.83 \times 10^6.\nu$, the total number of all nuclei is $N_i\nu = 2.23 \times 10^7.\nu$, and the original radius of the nebula is $R_o = 1 \times 10^4$ AU. (All mentioned quantities are described in the text.)

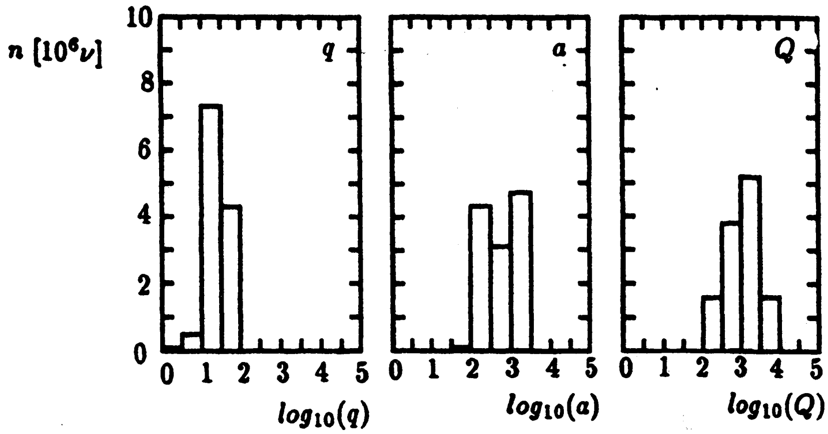

Figure 3. The distributions of pericentres (q), semi-major axes (a), and apocentres (Q) of cometary nuclei orbits at the end of the idealized collapse of the homogeneous nebula for $v_{PSN} = v_I/10$, $v_p = v_I/1000$. Quantity n is the number of nuclei with pericentres (semi-major axes, apocentres) of orbits in the given interval on the logarithmic scale. (All these values are given in AU.) The number of nuclei whose orbits have apocentres outside the nebula is $3.11 \times 10^6.\nu$, the number of nuclei captured in the nebula is $1.02 \times 10^7.\nu$, the total number of all nuclei is $N_i\nu = 2.23 \times 10^7.\nu$, and the original radius of the nebula is $R_o = 1 \times 10^4$ AU. (All mentioned quantities are described in the text.)

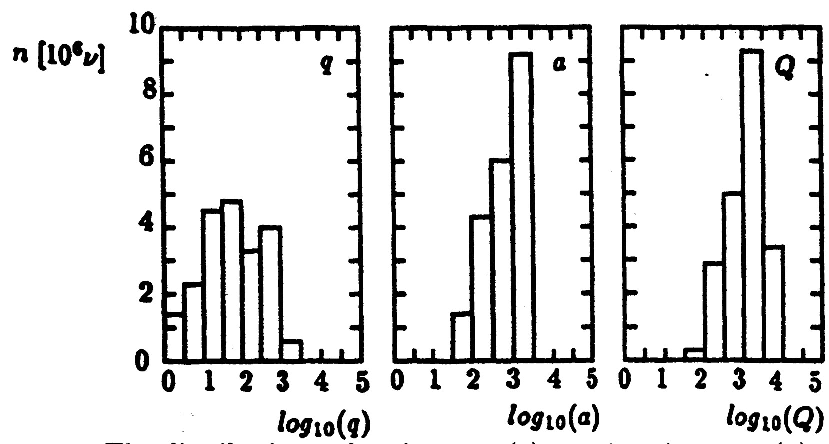

Figure 4. The distributions of pericentres (q), semi-major axes (a), and apocentres (Q) of cometary nuclei orbits at the end of the idealized collapse of the homogeneous nebula for $v_{PSN} = v_I/50$, $v_p = v_I/10$. Quantity n is the number of nuclei with pericentres (semi-major axes, apocentres) of orbits in the given interval on the logarithmic scale. (All these values are given in AU.) The number of nuclei whose orbits have apocentres outside the nebula is $3.11 \times 10^6.\nu$, the number of nuclei captured in the nebula is $1.37 \times 10^6.\nu$, the total number of all nuclei is $N_i\nu = 2.23 \times 10^7.\nu$, and the original radius of the nebula is $R_o = 1 \times 10^4$ AU. (All mentioned quantities are described in the text.)

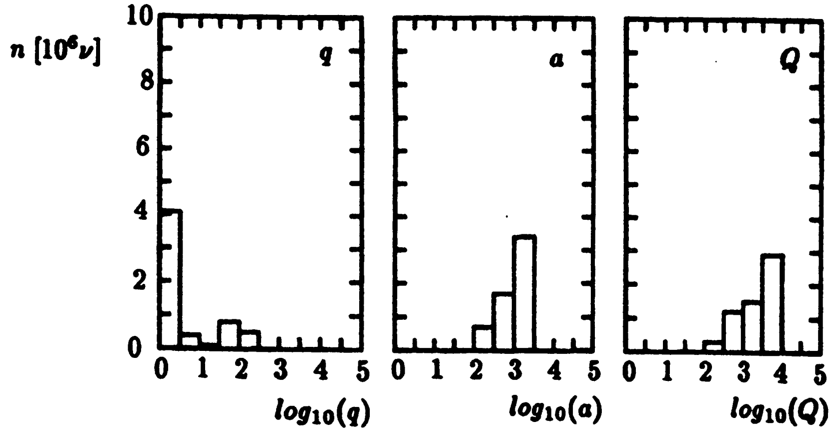

Figure 5. The distributions of pericentres (q), semi-major axes (a), and apocentres (Q) of cometary nuclei orbits at the end of the idealized collapse of the homogeneous nebula for $v_{PSN} = v_I/50$, $v_p = v_I/100$. Quantity n is the number of nuclei with pericentres (semi-major axes, apocentres) of orbits in the given interval on the logarithmic scale. (All these values are given in AU.) The number of nuclei whose orbits have apocentres outside the nebula is $3.11 \times 10^6.\nu$, the number of nuclei captured in the nebula is $1.64 \times 10^7.\nu$, the total number of all nuclei is $N_i\nu = 2.23 \times 10^7.\nu$, and the original radius of the nebula is $R_o = 1 \times 10^4$ AU. (All mentioned quantities are described in the text.)

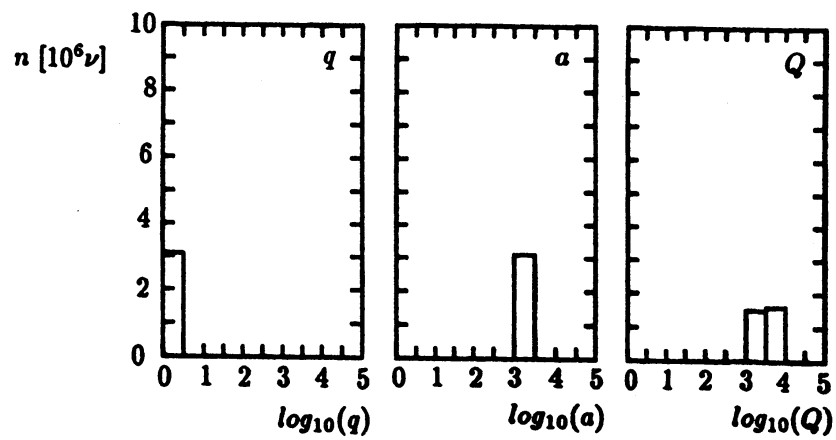

Figure 6. The distributions of pericentres (q), semi-major axes (a), and apocentres (Q) of cometary nuclei orbits at the end of the idealized collapse of the homogeneous nebula for $v_{PSN} = v_I/50$, $v_p = v_I/1000$. Quantity n is the number of nuclei with pericentres (semi-major axes, apocentres) of orbits in the given interval on the logarithmic scale. (All these values are given in AU.) The number of nuclei whose orbits have apocentres outside the nebula is $3.11 \times 10^6.\nu$, the number of nuclei captured in the nebula is $1.92 \times 10^7.\nu$, the total number of all nuclei is $N_i\nu = 2.23 \times 10^7.\nu$, and the original radius of the nebula is $R_o = 1 \times 10^4$ AU. (All mentioned quantities are described in the text.)

elements of the phase volume in a given numerical integration, and if we put the actual total number of nuclei to be equal to N_r, then $\nu = N_r/N_i$. We considered the first solution of quadratic equation (19) as the value of radius R_{out}. (The second solution yielded negative apocentres of the Keplerian orbits for some input values.)

We express velocities v_p and v_{PSN} as multiples of velocity

$$v_I = \sqrt{\frac{k^2 M_o}{R_o}} \tag{36}$$

of the nucleus, moving along a circular orbit located at the border of the nebula at the beginning of its collapse.

All combinations of discrete values $v_I/10$, $v_I/50$, $v_I/100$, $v_I/500$, $v_I/1000$, and $v_I/5000$ were chosen as pairs of v_p and v_{PSN}. Thus, 36 integrations were carried out summarilly. It was found that, the result was practically independent of value v_{PSN}, if $v_{PSN} \leq v_I/50$. Therefore, we present the result for $v_{PSN} = v_I/10$ and $v_{PSN} = v_I/50$, only.

No large differences between the individual results for various v_p if $v_p \leq v_I/1000$. Therefore, we present 2 series of figures for $v_{PSN} = v_I/10$ and $v_{PSN} = v_I/50$ as Figures 1-6, where each series contains 3 figures for $v_p = v_I/10$, $v_p = v_I/100$, and $v_p = v_I/1000$.

The first, second, and third graphs illustrate the distribution of pericentres (q), semi-major axes (a), and apocentres (Q) of orbits, respectively, in every figure.

The first column of each graph, where $0 \leq log_{10}X < 0.5$ (the symbol X represents either q, or a, or Q), demonstrates the number of all such nuclei where $log_{10}X < 0.5$, i.e. the nuclei having orbits with $log_{10}X < 0$ are included, too.

All graphs are related only to the nuclei, which left the nebula even before the end of its free-fall collapse and continued to move along Keplerian orbits. The number of nuclei, which remained in the internal part of nebula at the end of the collapse, is shown in the caption of every figure.

The numbers of nuclei whose original orbits had apocentres at a distance larger than the nebula's original radius R_o are given in the caption of every figure. Free parameter v_p should have an upper limit, the last-mentioned number being small in comparison with the total number of nuclei. Namely, we have no reason to assume the existence of an extensive cloud of comets around the protosolar nebula. The numerical computations indicate that $v_p \leq v_I$.

Free parameter v_{PSN} can also be limited from above. If the ratio of the centrifugal force in the nebula's equatorial plane at distance R_o from its centre and of the gravitational force there does not exceed a critical value f_c (the flattening of the nebula has to be negligible) at time $t = 0$, then $v_{PSN} \leq \sqrt{f_c}.v_I$.

We can see that $log_{10}a < 4$ in Figures 1-6. The main part of the real Oort cloud comets moves along more distant orbits, however. Therefore, we have

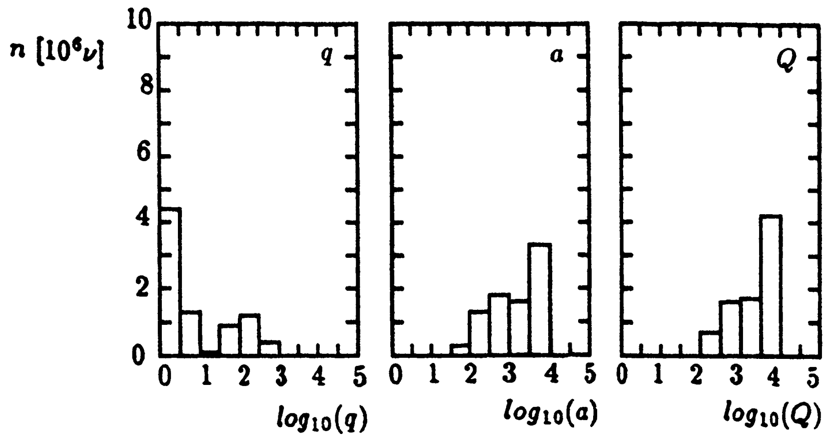

Figure 7. The distributions of pericentres (q), semi-major axes (a), and apocentres (Q) of cometary nuclei orbits at the end of the idealized collapse of the homogeneous nebula for its original radius $R_o = 3 \times 10^4$ AU and low velocities $v_{PSN} = v_I/100$, $v_p = v_I/100$. Quantity n is the number of nuclei with orbital pericentres (semi-major axes, apocentres) in the given interval on the logarithmic scale. (All these values are given in AU.) The number of nuclei whose orbits have apocentres outside the nebula is $3.11 \times 10^6.\nu$, the number of nuclei captured in the nebula is $1.40 \times 10^7.\nu$, and the total number of all nuclei is $N_i\nu = 2.23 \times 10^7.\nu$. (All mentioned values are described in the text.)

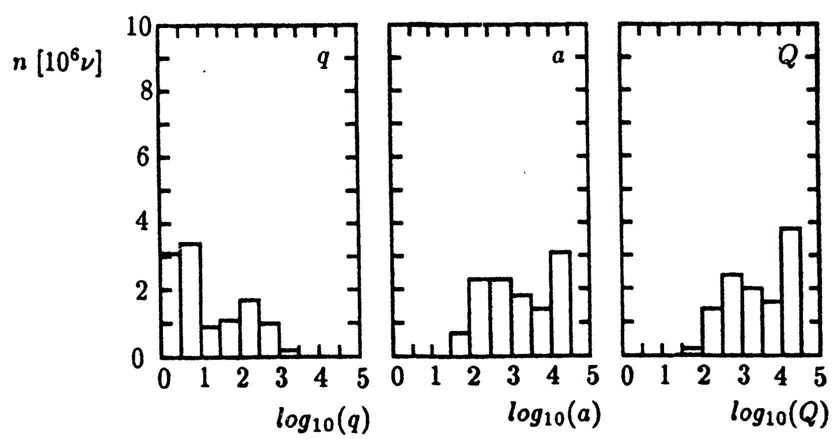

Figure 8. The distributions of pericentres (q), semi-major axes (a), and apocentres (Q) of cometary nuclei orbits at the end of the idealized collapse of the homogeneous nebula for its original radius $R_o = 0.7 \times 10^5$ AU and low velocities $v_{PSN} = v_I/100$, $v_p = v_I/100$. Quantity n is the number of nuclei with orbital pericentres (semi-major axes, apocentres) in the given interval on the logarithmic scale. (All these values are given in AU.) The number of nuclei whose orbits have apocentres outside the nebula is $3.11 \times 10^6.\nu$, the number of nuclei captured in the nebula is $1.08 \times 10^7.\nu$, and the total number of all nuclei is $N_i\nu = 2.23 \times 10^7.\nu$. (All mentioned values are described in the text.)

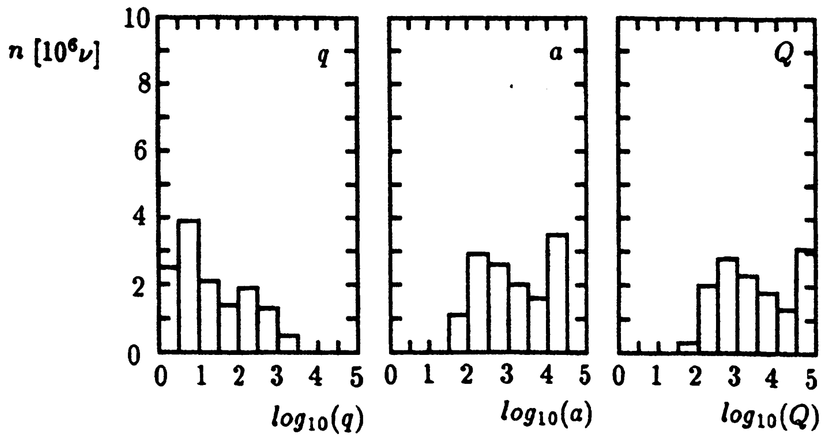

Figure 9. The distributions of pericentres (q), semi-major axes (a), and apocentres (Q) of cometary nuclei orbits at the end of the idealized collapse of the homogeneous nebula for its original radius $R_o = 1.1 \times 10^5$ AU and low velocities $v_{PSN} = v_I/100$, $v_p = v_I/100$. Quantity n is the number of nuclei with orbital pericentres (semi-major axes, apocentres) in the given interval on the logarithmic scale. (All these values are given in AU.) The number of nuclei whose orbits have apocentres outside the nebula is $3.11 \times 10^6.\nu$, the number of nuclei captured in the nebula is $8.71 \times 10^6.\nu$, and the total number of all nuclei is $N_i\nu = 2.23 \times 10^7.\nu$. (All mentioned values are described in the text.)

illustrated the resultant distribution of orbits also for a few values of R_o larger than 1×10^4 AU for fixed values of, v_p, v_{PSN} ($v_p = v_{PSN} = v_I/100$; v_I is expressed for the corresponding value of R_o) in Figures 7-9. The discussion of the larger radius R_o is in Section 7.

We also carried out a less precise integration for all 36 combinations of free parameters v_p and v_{PSN}. In this integration the individual elements of the phase volume were limited by discrete values of quantities r_o (0 – 10000 AU, increment 500 AU), ϕ ($-90°$ – $90°$, increment $15°$), v' ($v_p/4$ – $5v_p$, increment $v_p/4$), ϑ ($0°$ – $360°$, increment $15°$), and φ ($-90°$ – $90°$, increment $15°$). We did not find any differences between the more and less precise results exceeding 1%. Hence, we can say that the inaccuracy of the numerical integration did not exceed 1%.

7. Conclusion

As we mentioned in Section 1, our analysis had a qualitative character only. It is possible to state that the simplified model of protosolar nebula collapse, which was used, yielded almost all qualitative results, which were necessary for outlined hypothesis of formation of cometary cloud to be right.

The calculated distance of the Oort cloud smaller than real one, was the only disagreement. It will also probably be necessary to include in the considerations phenomena existing before the formation of the protosolar nebula with radii roughly 1×10^4 AU and with densities $\approx 10^{-16}$ kg/m^3.

We shall have to assume a uniform distribution of the cometary nuclei moving in randomly oriented directions the velocities, whose absolute values display a Maxwellian distribution, already from the moment, when the nebula separated from its parent interstellar molecular cloud. The density of such clouds is about $\approx 10^{-19}$ kg/m^3 (Spitzer-jr., 1985), hence, the nebular radius, if homogeneity and spherical symmetry of the nebula is assumed, would be $\approx 1.1 \times 10^5$ AU.

In the Figures we have illustrated the resultant distribution of nuclei also on the basis of assuming that the nebular radius was larger than $\approx 1 \times 10^4$ AU at the beginning of the collapse. The distance of the Oort cloud be qualitatively in agreement with the actual in this case, although the problem, whether the nebula began to collapse by free-falling already from the distance of its border $\approx 1.1 \times 10^5$ AU, or whether it collapsed gradually down to radius $\approx 1 \times 10^4$ AU at first, which was then followed by the free fall, remains unsolved.

If we increase the original nebular radius, it is interesting, that the distribution of orbits has two maxima, as can be seen in Figures 8 and 9, where this radius is already of the order of $\approx 10^5$ AU. The first, less numerous maximum at the larger distance can be related to the outer Oort cloud, and the second, more numerous at the lesser distance can be related to the internal Hills cloud.

The results of the calculations indicate that a rather significant group of nuclei (mainly, if low values of v_p and v_{PSN} are considered) was captured in the collapsed central part of the nebula, which gradually converted to the protosun and protoplanetary disc. There had to exist an intermediate group of nuclei between the groups of nuclei located outside the nebula and the captured nuclei.

The nuclei of this intermediate group were partly decelerated by the material of the collapsed nebula and carried by it in the direction of nebular rotation. However, these effects were not strong enough in the case of the intermediate group of nuclei and, therefore, these finally remained outside the nebula, which gradually collapsed further, but they moved along significantly shorter orbits.

In the case of the above-mentioned effects, it is possible to conclude that the intermediate group contained the more nuclei, the less their orbits were inclined relatively to the equatorial plane of the nebula. We can thus also explain the existence of the Kuiper belt of comets beyond Neptune, if we assume the presence of cometary nuclei already at the beginning of the existence of the protosolar nebula (and also at the end of its free-falling collapse, of course).

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References

Bodenheimer, P.: 1968, *Astrophys. J.* **153**, 483
Bodenheimer, P.: 1981, in *Fundamental Problems in the Theory of Stellar Evolution, IAU Symp. 93*, eds.: D. Sugimoto , D.A. Lamb and D.N. Schramm, Reidel, Dordrecht, 5
Clube, S.V.M. and Napier, W.M.: 1982, *Quart. J. Roy. Astron. Soc.* **23**, 45
Fernandèz, J.A.: 1985, in *Dynamics of Comets: Their Origin and Evolution*, eds.: A. Carusi and G.B. Valsecchi, Reidel, Dordrecht, 45
Gaustad, J.E.: 1963, *Astrophys. J.* **138**, 1050
Hayashi, C.: 1966, *Ann. Rev. Astron. Astrophys.* **4**, 171
Kuiper, G.P.: 1951, in *Astrophysics*, ed.: J.A. Hynek, McGraw Hill Book Co., New York, 357
Larson, R.B.: 1969, *Mon. Not. R. Astron. Soc.* **145**, 271
Larson, R.B.: 1972, *Mon. Not. R. Astron. Soc.* **157**, 121
Marochnik, L.S., Mukhin, L.M. and Sagdeev, R.Z.: 1988, *Science* **242**, 547
McCrea, W.H.: 1975, *Observatory* **95**, 239
Neslušan, L.: 1992, *Contrib. Astron. Obs. Skalnaté Pleso* **22**, 209
Safronov, V.S.: 1972, in *The Motion, Evolution of Orbits, and Origin of Comets*, eds.: C.A. Chebotarev and E.I. Kazimirchak-Polonskaya, Reidel, Dordrecht, 329
Shu, F.H.: 1977, *Astrophys. J.* **214**, 488
Spitzer-jr., L.: 1985, *Physica Scripta* **T11**, 5
Weissman, P.R.: 1990, *Nature* **344**, 825