BRIGHTNESS TEMPERATURE OF RADIATION FROM THE COUPLING OF OBLIQUE BERNSTEIN MODES IN CORONAL PLASMA

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ABSTRACT. The probability for coalescence of electrostatic Bernstein modes propagating obliquely to an ambient magnetic field and having random phases is evaluated in a kinetic weak turbulence description. Using knowledge of dispersion from linear theory and of the saturation spectrum of the waves in a temperature anisotropy instability from numerical simulation (Gitomer et al., 1972), the brightness temperature T_b of the emitted high-frequency electromagnetic waves is calculated. Rather crude approximations are necessary when evaluating the probability, nevertheless the result is considered to permit a reliable order of magnitude estimate - usually sufficient in astrophysical applications. Characteristic parameter values of the outer corona give $T_b < 10^{12}$K.

ЯРКОСТНАЯ ТЕМПЕРАТУРА ЭЛЕКТРОМАГНИТНОГО ИЗЛУЧЕНИЯ ОПРЕДЕЛЕНА ИЗ ВОЛН БЕРНШТЕЙНА РАСПРОСТРАНЯЮЩИХСЯ ПОД УГЛОМ В КОРОНАЛЬНОЙ ПЛАЗМЕ: В рамках теории слабой турбулентности вычислена вероятность слияния электростатических волн Бернштейна, распространяющихся под углом к внешнему магнитному полю. Используя линейное дисперсионное соотношение и найденный в работе Gitomer и бр.(1972), уровень насыщения неустойчивости, обусловленной анизотропией температуры, мы определили яркостную температуру электромагнитного излучения T . Несмотря на то, что при вычислении вероятности были сделаны весьма сильные упрощения, полученный результат дает возможность оценить T по порядку величины, что обычно бывает достаточно в астрофизических приложениях. Для верхней короны было получено значение $T_b < 10^{12}$ °K.

URČENIE JASOVEJ TEPLOTY ELEKTROMAGNETICKÉHO ŽIARENIA V KORÓNE, ZO ŠIKMÉHO PRECHODU BERNSTEINOVÝCH VĹN CEZ KORONÁLNU PLAZMU: Pomocou teórie slabej turbulencie, boli vypočítané pravdepodobnosti zlučovania vĺn typu Bernsteina. Uvažovaný bol prípad šikmého (k smeru vonkajšieho magnetického poľa) prechodu

Bernsteinových vĺn. Jasová teplota T_b bola počítaná z lineárneho disperzného vzťahu, pričom anizotropia teploty určovala hladinu nasýtenia nestability (Gitomer a i., 1972). Odhliadnuc od toho, že pri výpočte pravdepodobnosti sa vychádzalo zo silného zjednodušenia problému, získaný výsledok umožňuje určiť rád hodnoty jasovej teploty, čo zvyčajne stačí pre astrofyzikálne aplikácie. Pre hornú korónu bola zistená hodnota $T_b < 10^{12}$ °K.

1. INTRODUCTION

It has been argued in the companion paper (Kliem, 1986) that electrostatic electron cyclotron harmonic waves (e.s. ECW) - a generalization of Bernstein waves at oblique propagation angles $0 < |\theta = \arc\tan(k_\perp/k_\parallel) \leq \pi/2$ - might be of importance in certain events of nonthermal radio burst emission from the outer corona with brightness temperatures up to $T_b \sim 10^{10}$K. We shall consider here e.s. ECW that were excited in a compressed plasma with $P = \omega_p^2/\omega_c^2 \sim 1$ and sufficiently anisotropic temperature, $A = T_\perp/T_\parallel \gtrsim 3$ initially, and that grew in a few cyclotron periods to the saturation level E_{sat}, being maintained at this level by further perpendicular compression or decaying slowly on the collisional time scale. Due to fast wave growth the plasma can be characterized by $A \lesssim 3$ most of the time, therefore only the branch below ω_c possessing the lowest instability threshold ($A \approx 3$) need be considered. Linear theory shows that the ranges of unstable wavenumbers and frequencies are, respectively, $0.1\, r_c^{-1} \lesssim k_{1\perp}$, $|k_{1\parallel}| \lesssim 0.5\, r_c^{-1}$, $0.5 \leq \omega^{\sigma}/\omega_c \lesssim 1$, where $r_c = v_{t\perp}/\omega_c$, $v_{t\perp} = (T_\perp/m)^{1/2}$, and $\omega^{\sigma'}(\underline{k}_{1,2}) \equiv \omega^{\sigma'}$ will be used to denote e.s. ECW. The unstable region of the dispersion surface is well aproximated by $\omega^{\sigma'}(\underline{k}_1) = \omega_c + v_g^{\sigma'}\, k_1 = \omega_c(1 - 0.23\,(k_{1\parallel}\, r_c)^{-3/2} \cdot k_{1\perp} r_c)$ (numerical values shift slightly upward for $P > 1$). Numerical simulation (Gitomer et al., 1972) has shown that $E_{sat}(\underline{k}_1)$ scatters rather irregularly inside the unstable domain about the mean value $\bar{E}_{sat}^2/8\pi\, \bar{n} T_\perp = \gamma \sim 6.10^{-4}$. For order of magnitude estimates $E_{sat}(\underline{k}_1) = \bar{E}_{sat}$ may be used. The thermal level of e.s. ECW would be $E^2/8\pi\, \bar{n} T \sim g$ (g - so called plasma parameter $\lesssim 10^{-8}$ in the outer corona).

Coalescence $\sigma' + \sigma'' \rightarrow \sigma$ of two e.s. ECW ($\sigma'' = \sigma'$) into an e.m. mode $\sigma = 0, x$ is considered within the framework of standard weak turbulence descriptions (e.g. Melrose, 1980; Smith, 1970). The nonlinear current density describing the coupling of linear modes in a magnetized plasma is usually calculated in hydrodynamic approximation owing to mathematical difficulties. Very general and complocated formulae have been written down by Sauer (1972) and Melrose and Sy (1972) who used the Vlasov equation for magnetized plasma. Sauer (1972) has also been able to evaluate expressions exactly for perpendicular or parallel propagation of (both) modes σ', σ''. Obliquely propagating modes that are absent in hydrodynamic plasma description are to be treated here. Only electron dynamics must be taken into account.

2. COALESCENCE PROBABILITY

Introduce a pertubation expansion $F = F_0 + F_1 + F_2 + \dots,\ F_{s+1} \ll F_s$, for any quantity $F = f(\underline{x}, \underline{v}, t)$; $\underline{E}(\underline{x}, t)$; $\underline{B}(\underline{x}, t)$ into Vlasov-Maxwell equations and define the Fourier decomposition for a stationary and homogeneous situation.

$$F(\underline{x}, t) = \iint d^3k\, d\omega\ F(\underline{k}, \omega) e^{i(\underline{k}\cdot\underline{x} - \omega t)} \tag{1}$$

We have $E_0 = 0$, $B_1 = B_2 = \dots = 0$. The essence of linear theory is the wave equation $\underline{\underline{D}}(\underline{k}, \omega)\cdot\underline{E}_1(\underline{k}, \omega) = 0$, where $\det(\underline{\underline{D}}) = 0$ gives dispersion, and the relation

$$f_1(\underline{x}, \underline{v}, t) = (e/m) G^{-1}\ \underline{E}_1(\underline{x}, t)\cdot\underline{\nabla}_v f_0 \tag{2}$$

where the "inverse Vlasov operator G^{-1}" means integration along unpertubed particle trajectories (cf. Krall and Trivelpiece, 1973, eq. (8.8.5)). In second order

$$f_2(\underline{x}, \underline{v}, t) = (e/m) G^{-1} \left\{ \underline{E}_2 \cdot \underline{\nabla}_v f_0 + \underline{E}_1 \cdot \underline{\nabla}_v f_1 \right\} \equiv f_{(2)} + f^{(2)} \tag{3}$$

The second term represents the coupling of linear normal modes (σ', σ'') and gives rise to the nonlinear current density $\underline{j}^{(2)}$. The wave equation takes on the form $\underline{\underline{D}}\cdot\underline{E}_2 = -(4\pi i/\omega)\underline{j}^{(2)}(\underline{k}, \omega)$. We write down explicitely

$$f_1(\underline{k}, \underline{v}, \omega) = (e/m) \int_{-\infty}^{t} dt'\ e^{i\ \underline{k}\cdot(\underline{x} - \underline{x}') - \omega(t' - t)}\ \underline{E}_1(\underline{k}, \omega)\cdot \underline{\nabla}_{v'} f_0(\underline{v}') \tag{4}$$

$$f^{(2)}(\underline{k}, \underline{v}, \omega) = (e/m) \int_{-\infty}^{t} dt'\ e^{i\left[\underline{k}\cdot(\underline{x}' - x) - \omega(t' - t)\right]} \iint d^3k_1 d\omega_1 \iint d^3k_2 d\omega_2\ \cdot$$

$$\cdot\ \delta(\underline{k} - \underline{k}_1 - \underline{k}_2)\ \delta(\omega - \omega_1 - \omega_2)\ \underline{E}_1(\underline{k}_1, \omega_1)\cdot \underline{\nabla}_{v'}\ f_1(\underline{k}_2, \underline{v}', \omega_2) \tag{5}$$

The unperturbed trajectories in $\underline{B}_0$ are $\underline{x}'(t')$ with boundary condition $\underline{x}'(t) = \underline{x}$. Introduce the abbreviation $\varkappa_i = (\underline{k}_i, \omega_i)$ and the vector $\underline{U}(\varkappa, \varkappa_1, \varkappa_2)$ by

$$\underline{j}^{(2)}(\varkappa) = -en \int d^3v\ \underline{v} f^{(2)} = \iint d\varkappa_1 d\varkappa_2\ \delta(\varkappa - \varkappa_1 - \varkappa_2)\ E_1(\varkappa_1) E_1(\varkappa_2) \underline{U} \tag{6}$$

The space and time averaged power density, P, of groeth of $\underline{E}_2$ (emission in mode σ), calculated as the work done by $\underline{j}^{(2)}$ against its self consistent field $\underline{E}_2$, may be shown to be

$$P = (2\pi)^{-3} \int d^3k\ P^{\sigma}(\underline{k})$$

$$= \lim_{T_0, V_0 \to \infty} \frac{(2\pi)^6}{T_0 V_0}\ d^3k\ R_E^{\sigma}(\underline{k}) \left| \underline{e}^{\sigma *}(\underline{k})\cdot\underline{j}^{(2)}(\underline{k}, \omega^{\sigma}(\underline{k}))\right|^2 \tag{7}$$

in the approximation of a lossless plasma (Melrose, 1980; Smith, 1970). Here $\underline{e}^{\sigma}(\underline{k})$ is the polarization vector of made σ and $R_E^{\sigma}(\underline{k})$ is the corresponding ratio of electric to total wave energy at $\underline{k}$ (cf. Melrose, 1980, eq. (2.93)). With the random phase approximation, written here as $E_1(\mathcal{H}_1)E_1^*(\mathcal{H}'_1) = |E_1(\mathcal{H}_1)|^2(T_0V_0)^{-1}\delta(\mathcal{H}_1-\mathcal{H}'_1)$, and with $\underline{E}_1(\mathcal{H}_{1,2}) = \underline{E}^{\sigma',\sigma''}_{\underline{k}_{1,2}}\delta(\omega_{1,2}-\omega^{\sigma',\sigma''}(\underline{k}_{1,2}))$ the spectral power density becomes

$$P^{\sigma}(\underline{k}) = \lim_{V_0\to\infty}\frac{(2\pi)^8}{V_0^2}R_E^{\sigma}(\underline{k})\iint d^3k_1d^3k_2\,\delta(\underline{k}-\underline{k}_1-\underline{k}_2)\,\delta(\omega^{\sigma}_{\underline{k}}-\omega^{\sigma'}_{\underline{k}_1}-\omega^{\sigma'}_{\underline{k}_2})\,.$$

$$\cdot\left|\underline{e}^{\sigma *}(\underline{k}).\underline{U}\right|^2\left|E^{\sigma'}_{\underline{k}_1}\right|^2\left|E^{\sigma''}_{\underline{k}_2}\right|^2 \qquad (8)$$

By rewriting this equation in the language of the semi-classical formalism (e.g. Melrose, 1980) the coalescence probability is obtained ($\hbar$ - Planck's constant)

$$w^{\sigma\,\sigma'\,\sigma''}_{\underline{k}\,\underline{k}_1\underline{k}_2} = 16(2\pi)^4\hbar\,\delta(\underline{k}-\underline{k}_1-\underline{k}_2)\,\delta(\omega^{\sigma}_{\underline{k}}-\omega^{\sigma'}_{\underline{k}_1}-\omega^{\sigma''}_{\underline{k}_2})\left|\underline{e}^{\sigma *}(\underline{k}).\underline{U}\right|^2\,.$$

$$\cdot\,(\omega^{\sigma'}_{\underline{k}_1}\omega^{\sigma''}_{\underline{k}_2}/\omega^{\sigma}_{\underline{k}})\,R_E^{\sigma}(\underline{k})\,R_E^{\sigma'}(\underline{k}_1)\,R_E^{\sigma''}(\underline{k}_2) \qquad (9)$$

For $\sigma''=\sigma'$, a factor (1/2) must be added to this expression. We have approximated the dispersion $\omega^{\sigma'}_{\underline{k}_{1,2}}$ and the spectrum $\left|E^{\sigma'}_{\underline{k}_{1,2}}\right|$ in Sec. 1; $\omega^{\sigma}_{\underline{k}}$, $\underline{e}^{\sigma}(\underline{k})$ and $R_E^{\sigma}(\underline{k})$ are known (see Melrose, 1980; Melrose and Sy, 1972) so that we now have to determine $\underline{U}(\underline{k},\underline{k}_1\underline{k}_2)$ by evaluating (4) and (5). Since $A\lesssim 3$ and $\omega^{\sigma'}_r(\underline{k}_1)$ depends only weakly on A (remember the approximation of lossless plasma), we may choose f_0 to be the isotropic Maxwellian (A = 1): $f^M(\underline{v}) = (2\pi v_t^2)^{-3/2}\cdot\exp(-v^2/2v_t^2)$. The integration along the unperturbed trajectories

$x' = (v_\perp/\omega_c)\sin(\omega_c(t'-t)+\varphi) - (v_\perp/\omega_c)\sin\varphi + x,$

$y' = -(v_\perp/\omega_c)\cos(\omega_c(t'-t)+\varphi) + (v_\perp/\omega_c)\cos\varphi + y, z' = v_\parallel(t'-t) + z$

becomes elementary with the usual expansion of $\exp\left[i(k_\perp v_\perp/\omega_c)\sin\varphi\right]$ into a Bessel function series know from linear dispersion theory. Great simplification is achieved by utilizing the freedom to choose $k_y = 0$, $k_x - k_\perp$, here (which will later be $k_{2y} = 0$). (This breaks the usual symmetry of $w^{\sigma\sigma'\sigma''}$, since $k_{1y}\neq 0$, $k_Y\neq 0$ then in general.)

$$f_1(\underline{k},\underline{v},\omega) = i\,\frac{e}{m\,v_t^2}\,\frac{E_1(\underline{k},\omega)}{k}\,f^M(v)\left\{1+\omega\,e^{-i(k_\perp v_\perp/\omega_c)\sin\varphi}\,.\right.$$

$$\left.\cdot\sum_n\frac{Jn\,(k_\perp v_\perp/\omega_c)\,e^{in\varphi}}{k_\parallel v_\parallel-\omega+n\omega_c}\right\} \qquad (10)$$

This function will be integrated over dt' in (5) and then further in (6) over $\underline{v}$ $dv_\parallel\,d\varphi\,v_\perp\,dv_\perp$ where f^M effectively cuts the integration for $v_{\parallel,\perp} > 2v_t$. Consider the integrals in the following order: $dv_\parallel$, $d\varphi$, $dv_\perp$, dt'. The contributions of the resonances give the plasma dispersion function $Z((\omega - n\omega_c)/2^{1/2}k_{\parallel} v_t) \lesssim 1$ for real argument. Most terms vanish under the integration $d\varphi$. As compared to the contribution from unity in (10), ~ 10 terms of order $\lesssim 1$ survive in the remaining sum with different signs. Since later on we will only integrate over f_1, f_2, multiplied by smoothly varying functions, and do not take derivatives or divide by expressions derived from f_1, we may obtain order of magnitude results by (drastically) approximating

$$f_1)\underline{k}, \underline{v}, \omega) \approx i(e/mv_t^2)\ (E_1/k)\ f^M(v) \tag{11}$$

With negligible loss of accuracy we may put $k_{x,y} \to 0$ in the exponential term in (5), and after the now simple integration dt' we may also put $\omega - k_{\parallel} v_{\parallel} \approx \omega$ (because $|\underline{k}| \ll |\underline{k}_{1,2}|$). The result is already

$$f^{(2)}(\underline{k}, \underline{v}, \omega) = -\ (e\bar{n}v_t^2)^{-1}\ f^M(v) \iint d\varkappa_1\ d\varkappa_2\ \delta(\varkappa - \varkappa_1 - \varkappa_2)\ .$$

$$.\ E_1(\varkappa_1) E_1(\varkappa_2) \underline{U}.\underline{v} \tag{12}$$

and we identify

$$\underline{U}(\underline{k}, \underline{k}_1, \underline{k}_2) = \frac{e^3\ \bar{n}/mT}{k_1 k_2 (\omega^2 - \omega_c^2)}\ (k_{1x}\omega - ik_{1y}\omega_c,\ ik_{1x}\omega_c + k_{1y}\omega,$$

$$k_{1\parallel}(\omega^2 - \omega_c^2)/\omega),\ \omega = \omega^{\sigma}(\underline{k}) \tag{13}$$

The function $|\underline{e}^{\sigma *}(\underline{k}).\underline{U}|^2$ simplifies considerably for $k_{\parallel} = 0$ or $k_{\perp} = 0$. We write the result for $\sigma = o$, $k_{\perp} = 0$ in which case the resulting T_b turns out to be higher than in the other 3 complementary cases (within one order of magnitude): $|\underline{e}^{o,x *}(k_{\parallel}).\underline{U}| = 2^{-1/2} k_{1\perp}/(k_1 k_2 (\omega \pm \omega_c))$.

3. BRIGHTNESS TEMPERATURE

We must now integrate (d^3k_1) over the saturation spectrum in (8). The function $\delta(\omega^{\sigma}_{\underline{k}} - \omega^{\sigma'}_{\underline{k}_1} - \omega^{\sigma'}_{\underline{k}-\underline{k}_1})$ is rewritten as

$\approx \delta(\omega^{\sigma}_{\underline{k}} - 2\omega^{\sigma'}_{\underline{k}_1}) = \delta(k_{1\perp} - k_{1\perp 0})/2\,|v_g^{\sigma'}(\underline{k}_1)|$, where $v^{\sigma'}_{g\perp}(\underline{k}_1) \approx$

$\approx -\ 0.23(k_{1\parallel}\ r_c)^{-3/2} v_{t\perp}$ and $\omega^{\sigma'}(k_{1\perp 0}(k_{1\parallel}, k_{\parallel}), k_{1\parallel}) = \omega^{\sigma}(k_{\parallel})/2$, for $k_{\perp} = 0$,

which we choose here for demostration (and maximum T_b). The remaining integral is

$$\int dk_{1\parallel}\ k^3_{1\perp 0}\ /(k_1^2\ k_2^2\ |v^{\sigma'}_{g\perp}|) \approx v_{t\perp}^{-1} \tag{14}$$

to a good approximation. With unstable area $\sim 1/4r_c^2$ in the $k_{1\parallel} - k_{1\perp}$ - plane we have

$$P^{o}(k_{\parallel})=32(2\pi)^3(e^3\bar{n}^2r_c^3T_{\perp}/mT)^2\,\gamma^2 v_{t\perp}^{-1}\omega_c^{-2}Y^2(1+Y)^{-2}R_E^{\sigma}\,(k_{\parallel}), \quad (15)$$

$$Y = \omega_c/\omega^{o}(k_{\parallel}).$$

For parameters such that $f_p = 61$ MHz, $f_c = 56$ MHz, $v_{t\parallel} = c/50$, and $A = 5$ one has $(e^3\bar{n}^2r_c^2\ T_{\perp}/mT)2 \approx 1.5\ 10^{-4}$ erg $cm^{-3}v_{t\perp}^{4}$.

The corresponding brightness temperature is for optical thin radiation

$$T_b^{\sigma}(\underline{k}) = (v_g^{\sigma}(\underline{k}))^{-1}\ P^{\sigma}(\underline{k})1 \quad (16)$$

where $v_g^{\sigma}(\underline{k})$ is the group velocity of mode σ. For the source depth we have from the paper Kliem (1986): $1 < 5.10^5$ cm (for the same parameter values). At a frequency of observation of f = 80 MHz we get finally

$$T^{o}\ (k_{\parallel}) < 10^{12}\ K$$

Evaluating absoption in a similar manner, it is confirmed that the optical depth $\tau^{o}(k_{\parallel}) \sim 1.3\ 10^{-7}\ cm^{-1}.1$ remains small. Details will be presented in a later publication (Kliem, 1986a, in preparation).

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