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Cremonian space–time(s) as an emergent phenomenon

Metod Saniga

Astronomical Institute, Slovak Academy of Sciences, 05960 Tatranská Lomnica, Slovak Republic

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Abstract

It is shown that the notion of fundamental elements can be extended to *any*, i.e. not necessarily homaloidal, web of rational surfaces in a three-dimensional projective space. A Cremonian space–time can then be viewed as an *emergent* phenomenon when the condition of ''homaloidity'' of the corresponding web is satisfied. The point is illustrated by a couple of particular types of ''almost-homaloidal'' webs of quadratic surfaces. In the first case, the quadrics have a line and two distinct points in common and the corresponding pseudo-Cremonian manifold is endowed with just two spatial dimensions. In the second case, the quadrics share six distinct points, no three of them collinear, that lie in quadruples in three different planes, and the corresponding pseudo-Cremonian configuration features three time dimensions. In both the cases, the limiting process of the emergence of generic Cremonian space–times is explicitly demonstrated.

Although the theory of Cremonian space–time, first introduced in [1], is relatively new, it has already proven to be remarkably fertile and attracted attention of both the lay public [2] and specialists [3] alike. The concept not only offers us a feasible explanation for why our Universe is, at the macroscopic scale, endowed with three spatial and one time dimensions [1,4–7], but also indicates unsuspected intricacies of a coupling between the two [6]. Moreover, it gives us important qualitative hints as for a possible underlying algebraic geometrical structure of a large variety of non-ordinary forms of psychological time (and space) [6,8]. These fascinating properties alone are enough to realize that the theory deserves further serious exploration.

One of the most natural and fruitful ways of getting a deeper insight into a(ny) theory is to relax one (or several) assumptions that the theory is based on and see what structural and conceptual changes such a step involves. In order to pursue this strategy in our case one has to recall the underlying geometrical principle behind our concept of Cremonian space–time: the existence of a *homaloidal* web of (quadric) surfaces in a three-dimensional projective space, P_3 [1,4,5]. An aggregate of surfaces (of any order, not necessarily quadratic) in P_3, is homaloidal [1,9]: if (a) it is linear and of freedom three (i.e. contains a triple infinity of surfaces), (b) all its surfaces are rational, and (c) any three distinct members of the set have only one free (variable) intersection. The generalized theory outlined in what follows is based on abandoning the last assumption, i.e. on allowing any three distinct surfaces of the web to have two (or more) points in common.

Of the couple of different types of homaloidal web of quadrics we have so far dealt with we shall first consider that whose corresponding Cremonian space–time was found to mimic best the basic observed macroscopic properties of the Universe, viz. a web whose base configuration comprises a (straight-)line and three distinct, non-collinear points, none incident with the line in question [1,6]. So, in this case, any web of quadrics that features a base line and where the number of isolated base points falls short of three may serve our purpose.

E-mail address: msaniga@auriga.ta3.sk (M. Saniga).

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We shall, naturally, focus on the case where there are just two base points as this case represents, obviously, the smallest possible deviation from the "homaloidity" condition. In order to facilitate our reasoning, we shall choose a system of homogeneous coordinates $\breve{z}_i$, $i = 1,2,3,4$, in which the equation of base line, $\widehat{\mathscr{L}}$, reads

$$\widehat{\mathscr{L}} : \breve{z}_1 = 0 = \breve{z}_2 \tag{1}$$

and the two isolated base points, $\widehat{B}_1$ and $\widehat{B}_2$, coincide, respectively, with the vertices V_1 and V_2 of the coordinate tetrahedron, i.e.

$$\widehat{B}_1 : \varrho\breve{z}_i = (1,0,0,0), \tag{2}$$

$$\widehat{B}_2 : \varrho\breve{z}_i = (0,1,0,0), \tag{3}$$

where ϱ is, in what follows, a non-zero proportionality factor. Now, employing the equation of a generic quadric, $\mathscr{D}$, of P_3,

$$\begin{aligned}\mathscr{D} &\equiv \sum_{i,j=1}^{4} d_{ij}\breve{z}_i\breve{z}_j \\ &= d_{11}\breve{z}_1^2 + d_{22}\breve{z}_2^2 + d_{33}\breve{z}_3^2 + d_{44}\breve{z}_4^2 + 2d_{12}\breve{z}_1\breve{z}_2 + 2d_{13}\breve{z}_1\breve{z}_3 + 2d_{14}\breve{z}_1\breve{z}_4 + 2d_{23}\breve{z}_2\breve{z}_3 + 2d_{24}\breve{z}_2\breve{z}_4 + 2d_{34}\breve{z}_3\breve{z}_4 \\ &= 0,\end{aligned} \tag{4}$$

we find that the system of quadrics that contain $\widehat{\mathscr{L}}$ and pass through $\widehat{B}_1$ and $\widehat{B}_2$ is given by

$$d_{12}\breve{z}_1\breve{z}_2 + d_{13}\breve{z}_1\breve{z}_3 + d_{14}\breve{z}_1\breve{z}_4 + d_{23}\breve{z}_2\breve{z}_3 + d_{24}\breve{z}_2\breve{z}_4 = 0, \tag{5}$$

where each d's may acquire any real value. This aggregate is, however, not a web for it effectively depends on four, instead of three, parameters. In order to get a web from it, a linear constraint has to be imposed on the parameters d's. As $d_{12} \neq 0$—otherwise the aggregate would contain another base line (the $\breve{z}_3 = 0 = \breve{z}_4$ one)—this constraint can be written, without any loss of generality, in the form

$$d_{12} = \kappa_1 d_{13} + \kappa_2 d_{23} + \kappa_3 d_{14} + \kappa_4 d_{24}, \tag{6}$$

where κ_i, $i = 1,2,3,4$, are regarded as fixed constants. After substituting the last equation into Eq. (5), we get the web desired

$$\mathscr{W}^{\clubsuit}(\vartheta) = \sum_{i=1}^{4}\vartheta_i\mathscr{D}_i^{\clubsuit} \equiv \vartheta_1\breve{z}_1(\breve{z}_3 + \kappa_1\breve{z}_2) + \vartheta_2\breve{z}_2(\breve{z}_3 + \kappa_2\breve{z}_1) + \vartheta_3\breve{z}_1(\breve{z}_4 + \kappa_3\breve{z}_2) + \vartheta_4\breve{z}_2(\breve{z}_4 + \kappa_4\breve{z}_1) = 0, \tag{7}$$

where we simplified the notation by putting $d_{13} \equiv \vartheta_1$, $d_{23} \equiv \vartheta_2$, $d_{14} \equiv \vartheta_3$, and $d_{24} \equiv \vartheta_4$.

At this point we recall the definition of a Cremonian space–time as a configuration composed of the totality of *fundamental* elements associated with a homaloidal web [1,4–7]. A fundamental element of a given homaloidal web is any algebraic geometrical object (a curve, or a surface) whose only intersections with a member of the web are the base elements of the latter [9]. In the case of quadrics, the fundamental elements are of two distinct kinds, namely lines and conics, and form pencils, i.e. linear, singly-parametrical aggregates/systems. A pencil of lines is taken to generate/represent a macroscopic dimension of space, while that of conics—time. From its definition it readily follows that the concept of a fundamental element, and so that of a Cremonian space–time, is not tied solely to homaloidal webs, but it can be extended perfectly to *any* web whatsoever! A crucial property of fundamental elements is that they are located on the Jacobian surface of the web [9], i.e. on the surface formed by the totality of vertices of the cones contained in the web; or, what amounts to the same, the totality of singular points of degenerate/composite quadrics in the web.

Our forthcoming task is thus to find the form of the Jacobian surface, $\mathscr{J}$, for the web given by Eq. (7). From the above-given properties it follows that given a generic web of quadrics,

$$\mathscr{W}(\vartheta) = \sum_{i=1}^{4}\vartheta_i\mathscr{D}_i = 0, \tag{8}$$

its $\mathscr{J}$ is the locus of points satisfying the following Eq. (9)

$$\mathscr{J} = \det\begin{pmatrix} \partial\mathscr{D}_1/\partial\breve{z}_1 & \partial\mathscr{D}_2/\partial\breve{z}_1 & \partial\mathscr{D}_3/\partial\breve{z}_1 & \partial\mathscr{D}_4/\partial\breve{z}_1 \\ \partial\mathscr{D}_1/\partial\breve{z}_2 & \partial\mathscr{D}_2/\partial\breve{z}_2 & \partial\mathscr{D}_3/\partial\breve{z}_2 & \partial\mathscr{D}_4/\partial\breve{z}_2 \\ \partial\mathscr{D}_1/\partial\breve{z}_3 & \partial\mathscr{D}_2/\partial\breve{z}_3 & \partial\mathscr{D}_3/\partial\breve{z}_3 & \partial\mathscr{D}_4/\partial\breve{z}_3 \\ \partial\mathscr{D}_1/\partial\breve{z}_4 & \partial\mathscr{D}_2/\partial\breve{z}_4 & \partial\mathscr{D}_3/\partial\breve{z}_4 & \partial\mathscr{D}_4/\partial\breve{z}_4 \end{pmatrix} = 0, \tag{9}$$

which when combined with Eq. (7) yields

$$\mathscr{J}^{\clubsuit} = 2\breve{z}_1\breve{z}_2\overline{\mathscr{D}}^{\clubsuit} = 0, \tag{10}$$

where

$$\overline{\mathscr{D}}^{\clubsuit} \equiv \breve{z}_4(\kappa_1\breve{z}_2 - \kappa_2\breve{z}_1) - \breve{z}_3(\kappa_3\breve{z}_2 - \kappa_4\breve{z}_1). \tag{11}$$

It is a quite straightforward task for the reader to verify that the fundamental elements of web (7) are, in a general case, located only in the planes $\breve{z}_2$ and $\breve{z}_1$, in both the cases these elements are lines forming pencils centered, respectively, at $\widehat{B}_1$, viz.

$$\widetilde{\mathscr{L}}_1(\vartheta) : \breve{z}_2 = 0 = \vartheta_1\breve{z}_3 + \vartheta_3\breve{z}_4, \tag{12}$$

and $\widehat{B}_2$, viz.

$$\widetilde{\mathscr{L}}_2(\vartheta) : \breve{z}_1 = 0 = \vartheta_2\breve{z}_3 + \vartheta_4\breve{z}_4; \tag{13}$$

for a line meets a quadric in two (not necessarily distinct and/or real) points and these are in our cases furnished, respectively, by $\widehat{B}_1$ and $\widehat{B}_2$ and a point of $\widehat{\mathscr{L}}$. The quadric $\overline{\mathscr{D}}^{\clubsuit} = 0$ contains, in general, only one further fundamental element apart from a couple of lines shared with $\widetilde{\mathscr{L}}_1(\vartheta)$ and $\widetilde{\mathscr{L}}_2(\vartheta)$ namely the line joining the points $\widehat{B}_1$ and $\widehat{B}_2$. Yet, this quadric is the most interesting piece of $\mathscr{J}^{\clubsuit}$ because when it becomes *composite* (singular), the web $\mathscr{W}^{\clubsuit}$ becomes *homaloidal* and the quadric itself exhibits two different, singly-infinite aggregates of fundamentals.

In order to see that explicitly, one recalls [1,9] that a quadric $\mathscr{D}$, Eq. (4), is composite iff

$$\det\begin{pmatrix} d_{11} & d_{12} & d_{13} & d_{14} \\ d_{21} & d_{22} & d_{23} & d_{24} \\ d_{31} & d_{32} & d_{33} & d_{34} \\ d_{41} & d_{42} & d_{43} & d_{44} \end{pmatrix} = 0 \tag{14}$$

and, for Eq. (11), this equation reduces to

$$\kappa_1\kappa_4 - \kappa_2\kappa_3 = 0. \tag{15}$$

To find the form of this composite quadric, $\overline{\mathscr{D}}^{\clubsuit}_{\odot}$, we may assume, without any substantial loss of generality, that κ_1 is non-zero, rewrite Eq. (15) as

$$\kappa_4 = \frac{\kappa_2\kappa_3}{\kappa_1} \tag{16}$$

and insert the latter into Eq. (11) to arrive at

$$\overline{\mathscr{D}}^{\clubsuit}_{\odot} = \frac{1}{\kappa_1}(\kappa_1\breve{z}_2 - \kappa_2\breve{z}_1)(\kappa_1\breve{z}_4 - \kappa_3\breve{z}_3); \tag{17}$$

that is, $\overline{\mathscr{D}}^{\clubsuit}_{\odot}$ consists of a pair of *planes*, $\kappa_1\breve{z}_2 - \kappa_2\breve{z}_1 = 0$ and $\kappa_1\breve{z}_4 - \kappa_3\breve{z}_3 = 0$. As Eq. (7) acquires under Eq. (16) the form

$$\mathscr{W}^{\clubsuit}_{\odot}(\vartheta) = \vartheta_1\breve{z}_1(\breve{z}_3 + \kappa_1\breve{z}_2) + \vartheta_2\breve{z}_2(\breve{z}_3 + \kappa_2\breve{z}_1) + \vartheta_3\breve{z}_1(\breve{z}_4 + \kappa_3\breve{z}_2) + \vartheta_4\breve{z}_2\left(\breve{z}_4 + \frac{\kappa_2\kappa_3}{\kappa_1}\breve{z}_1\right) = 0, \tag{18}$$

it is easy to spot that this "constrained" web contains, indeed, a *third* base point, viz.

$$\widehat{B}^{\odot}_3 : \varrho\breve{z}_i = (\kappa_1, \kappa_2, -\kappa_1\kappa_2, -\kappa_2\kappa_3), \tag{19}$$

and is thus homaloidal [1,6,9], featuring, in addition to $\widetilde{\mathscr{L}}_1(\vartheta)$ and $\widetilde{\mathscr{L}}_2(\vartheta)$, one more pencil of fundamental *lines*, viz.

$$\widetilde{\mathscr{L}}^{\odot}_3(\vartheta) : \kappa_1\breve{z}_2 - \kappa_2\breve{z}_1 = 0 = \left(\frac{\kappa_1}{\kappa_2}\vartheta_1 + \vartheta_2\right)(\breve{z}_3 + \kappa_1\breve{z}_2) + \left(\frac{\kappa_1}{\kappa_2}\vartheta_3 + \vartheta_4\right)(\breve{z}_4 + \kappa_3\breve{z}_2), \tag{20}$$

and a pencil of fundamental *conics*, viz.

$$\widetilde{\mathscr{Q}}^{\odot}(\vartheta):\kappa_1\breve{z}_4-\kappa_3\breve{z}_3=0=\left(\vartheta_1+\frac{\kappa_3}{\kappa_1}\vartheta_3\right)\breve{z}_1(\breve{z}_3+\kappa_1\breve{z}_2)+\left(\vartheta_2+\frac{\kappa_3}{\kappa_1}\vartheta_4\right)\breve{z}_2(\breve{z}_3+\kappa_2\breve{z}_1), \tag{21}$$

the two aggregates being located, as expected, in the two sheets of $\overline{\mathscr{D}}^{\clubsuit}_{\odot}$. Our findings can be rephrased as follows. The "pseudo-", or "proto-" Cremonian configuration associated with a generic $\mathscr{W}^{\clubsuit}$ consists of two space dimensions ($\widetilde{\mathscr{L}}_1(\vartheta)$ and $\widetilde{\mathscr{L}}_2(\vartheta)$) and transforms into a fully-developed "classical" Cremonian space–time [1,3,6], endowed with an additional space dimension ($\widetilde{\mathscr{L}}^{\odot}_3(\vartheta)$) and time ($\widetilde{\mathscr{Q}}^{\odot}(\vartheta)$), whenever the web becomes homaloidal. This phenomenon can be given another nice geometrical picture. We may regard κ_i's as homogenous coordinates of a variable point in an abstract three-dimensional projective space. Eq. (15) (or, equivalently, Eq. (16)) then defines a quadric surface in this space and the "homaloidity" condition simply answers to the fact that the point happens to fall on this quadric.

In order to get a deeper insight into the nature of this "emergence phenomenon", we shall consider a web of quadrics through the following six distinct points

$$\begin{aligned}&\widehat{B}_1:\varrho\breve{z}_i=(1,0,0,0),\quad \widehat{B}_4:\varrho\breve{z}_i=(\kappa,0,0,1),\\&\widehat{B}_2:\varrho\breve{z}_i=(0,1,0,0),\quad \widehat{B}_5:\varrho\breve{z}_i=(0,\kappa,0,1),\\&\widehat{B}_3:\varrho\breve{z}_i=(0,0,1,0),\quad \widehat{B}_6:\varrho\breve{z}_i=(0,0,\kappa,1),\end{aligned} \tag{22}$$

where κ is a variable real parameter. These points are, obviously, all real, lying in quadruples in three different planes and no three on the same line. The web they define is of the form

$$\mathscr{W}^{\spadesuit}(\vartheta)=\vartheta_1\breve{z}_2\breve{z}_3+\vartheta_2\breve{z}_1\breve{z}_3+\vartheta_3\breve{z}_1\breve{z}_2+\vartheta_4\breve{z}_4(\breve{z}_1+\breve{z}_2+\breve{z}_3-\kappa\breve{z}_4)=0 \tag{23}$$

and its Jacobian reads

$$\mathscr{J}^{\spadesuit}=2\breve{z}_1\breve{z}_2\breve{z}_3(\breve{z}_1+\breve{z}_2+\breve{z}_3-2\kappa\breve{z}_4)=0. \tag{24}$$

Although the Jacobian features four distinct planes, in a general case, $\kappa\neq 0$, only three of them carry sets of fundamental elements. These are the planes $\breve{z}_1=0$, $\breve{z}_2=0$ and $\breve{z}_3=0$, and the corresponding fundamental elements are *conics* in the following pencils:

$$\widetilde{\mathscr{Q}}_1(\vartheta):\breve{z}_1=0=\vartheta_1\breve{z}_2\breve{z}_3+\vartheta_4\breve{z}_4(\breve{z}_2+\breve{z}_3-\kappa\breve{z}_4) \tag{25}$$

$$\widetilde{\mathscr{Q}}_2(\vartheta):\breve{z}_2=0=\vartheta_2\breve{z}_1\breve{z}_3+\vartheta_4\breve{z}_4(\breve{z}_1+\breve{z}_3-\kappa\breve{z}_4) \tag{26}$$

and

$$\widetilde{\mathscr{Q}}_3(\vartheta):\breve{z}_3=0=\vartheta_3\breve{z}_1\breve{z}_2+\vartheta_4\breve{z}_4(\breve{z}_1+\breve{z}_4-\kappa\breve{z}_4) \tag{27}$$

respectively. The conics (of a triply-infinite system),

$$\widetilde{\mathscr{Q}}^{\star}(\vartheta):\breve{z}_1+\breve{z}_2+\breve{z}_3-2\kappa\breve{z}_4=0=\vartheta_1\breve{z}_2\breve{z}_3+\vartheta_2\breve{z}_1\breve{z}_3+\vartheta_3\breve{z}_1\breve{z}_2+\kappa\vartheta_4\breve{z}_4^2, \tag{28}$$

cut out from $\mathscr{W}^{\spadesuit}$ by the remaining Jacobian plane, $\breve{z}_1+\breve{z}_2+\breve{z}_3-2\kappa\breve{z}_4=0$, cannot be fundamental elements as they do not contain any of the base points (see Eq. (22)). $\mathscr{W}^{\spadesuit}$ becomes homaloidal for $\kappa\to 0$,

$$\mathscr{W}^{\spadesuit}_{\odot}(\vartheta)\equiv\mathscr{W}^{\spadesuit}_{\kappa\to 0}(\vartheta)=\vartheta_1\breve{z}_2\breve{z}_3+\vartheta_2\breve{z}_1\breve{z}_3+\vartheta_3\breve{z}_1\breve{z}_2+\vartheta_4\breve{z}_4(\breve{z}_1+\breve{z}_2+\breve{z}_3)=0, \tag{29}$$

in which case, in addition to the three pencils of fundamental conics

$$\widetilde{\mathscr{Q}}^{\odot}_{\alpha}(\vartheta)\equiv\widetilde{\mathscr{Q}}^{\kappa\to 0}_{\alpha}(\vartheta):\breve{z}_\alpha=0=\vartheta_\alpha\breve{z}_\beta\breve{z}_\gamma+\vartheta_4\breve{z}_4(\breve{z}_\beta+\breve{z}_\gamma),\quad \alpha\neq\beta\neq\gamma,\quad \alpha,\beta,\gamma=1,2,3, \tag{30}$$

we also have a pencil of fundamental *lines*, namely

$$\widetilde{\mathscr{L}}^{\odot}(\vartheta)\equiv\widetilde{\mathscr{Q}}^{\star}_{\kappa\to 0}(\vartheta):\breve{z}_1+\breve{z}_2+\breve{z}_3=0=\vartheta_1\breve{z}_2\breve{z}_3+\vartheta_2\breve{z}_1\breve{z}_3+\vartheta_3\breve{z}_1\breve{z}_2; \tag{31}$$

these lines share point V_4 ($\varrho\breve{z}_i=(0,0,0,1)$), the common merger of three of the base points, viz. $\widehat{B}_4$, $\widehat{B}_5$ and $\widehat{B}_6$, and the point at which all the quadrics of $\mathscr{W}^{\spadesuit}_{\odot}$ touch the plane $\breve{z}_1+\breve{z}_2+\breve{z}_3=0$ [5,9]. Although this emerging Cremonian space–time features three time dimensions ($\widetilde{\mathscr{Q}}^{\odot}_{\alpha}(\vartheta)$) and a single spatial one ($\widetilde{\mathscr{L}}^{\odot}(\vartheta)$), and is thus an inverse to that offered to our senses, the case itself serves as an important illustration of the intricacy of the coupling between the *extrinsic* structure of time (i.e. the *type* of a pencil of conics) and the *number* of spatial coordinates. For the three time dimensions of our pseudo-Cremonian manifold, $\widetilde{\mathscr{Q}}_\alpha(\vartheta)$, represent each a generic pencil of conics, i.e. a pencil endowed with four distinct base points, while those of the limiting Cremonian sibling, $(\widetilde{\mathscr{Q}}^{\odot}_\alpha(\vartheta))$, are each a pencil with three distinct base points only, namely $\widetilde{B}_\beta$, $\widetilde{B}_\gamma$ and V_4, the last one being of multiplicity two (see Figs. 1(a) and (b) of Ref. [6]). One thus sees that the ‘‘birth’’ of a space dimension, $\widetilde{\mathscr{L}}^{\odot}(\vartheta)$, entails serious structural changes in all the three time coordinates. This feature dovetails nicely with what we found for strictly homaloidal transitions [6], characterized, however, by a drop in the number of spatial dimensions.

From this reasoning it is obvious that a Cremonian space–time is a rather exceptional structure, whose emergence is of a fairly complex nature. Following the strategy employed above, it should represent no difficulty for the interested reader to examine other potential transitions, in particular those where pseudo-Cremonian configurations feature only a finite (or even zero) number of elements. Insights might also be obtained from an analysis of (pseudo-)Cremonian space–times associated with webs of cubic and/or higher order surfaces. All this implies a wealth of additional possibilities to those outlined in [7] regarding ''Cremonian'' scenarios of how our Universe might have come into being. Here, we are confronted with a fascinating possibility that the Universe may have spent a substantial fraction of its lifetime in some pseudo-Cremonian regime and acquired its current generic ‘‘quadro-cubic’’ Cremonian form [1,6,7] only ‘‘relatively recently’’. This intriguing scenario will be examined in more detail in a separate paper.

As a final note, it is worth stressing that all the foregoing pieces of reasoning have been based on an implicit assumption that the ground field of the background projective setting of our model is identical with that of the real numbers. Yet, all the basic emergence properties can easily be shown to be valid for an *arbitrary* ground field, including, for example, the well-known non-Archimedean field of p-adic numbers, or finite (Galois) fields. In these cases, however, our (pseudo-)Cremonian space–time loses not only its ordering (the former case), but even its continuity and differentiability (the latter one). And these are obviously aspects where our theory bears again a striking formal resemblance, as pointed out on several occasions earlier [1,4,5], to the so-called Cantorian ($\varepsilon^{(\infty)}$) space–time. This notable and intriguing concept was first introduced by El Naschie more than a decade ago [10], and has been subsequently elaborated in numerous papers by the author himself (see Ref. [11] for a recent review) as well as by many others (see, e.g., Refs. [12–14] for most interesting generalizations/applications). The last-mentioned paper, [14], deserves a particular attention as it represents a bold attempt to generalize this Cantorian approach employing algebras/number systems having *zero* divisors, the latter being the issue of renewed interest in pure mathematics [15–17].

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