On Cremonian dimensions qualitatively different from time and space

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Abstract

We examine a particular kind of six-dimensional Cremonian universe featuring one dimension of space, three dimensions of time and other two dimensions that can*not* be ranked as either time or space. One of these two, generated by a one-parametric aggregate of (straight-)lines lying on a quadratic cone, is more similar to the spatial dimension. The other, represented by a singly-parametrical set of singular space quartic curves situated on a proper ruled quadric surface, bears more resemblance to time. Yet, the two dimensions differ profoundly from both time and space because, although being macroscopic, they are *not* accessible to (detectable by) *every* Cremonian observer. This toy-model thus demonstrates that there might exist extra-dimensions that need not necessarily be compactified to remain unobservable.
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There are a number of features of the macroscopic physical world that still remain substantially beyond grasp of theoretical physics. Among them, the non-trivial structure of time and the observed dimensionality of the universe obviously represent a case in question. As we found [1,2] and have repeatedly stressed [3–5], the two properties seem to be intimately intertwined and ask, therefore, for a conceptually new approach to be properly understood. A (very promising) piece of such a formalism is undoubtedly the concept/theory of Cremonian space-times [6–14].

The Cremonian picture of space-time is indeed remarkable in several aspects. The first, and perhaps most notable, fact is that without employing any concept of metric (measure), this approach fundamentally distinguishes the time dimension(s) from spatial ones and, in its most trivial form, it straightforwardly leads to their observed number (4) and respective ratio (1 + 3) as well [6,7,9,10]. Second, it demonstrates that these dimensions are not primordial, but emerge from more fundamental algebraic geometrical structures [13]. Third, it indicates that the universe with the inverse signature might evolutionary be intimately connected with our universe [12]. And last, but not least, when the observer (subject) is concerned, it qualitatively reproduces our ordinary perception of time as well as a whole variety of altered/non-ordinary forms of mental space-times [10,15]; moreover, every observer in this basic Cremonian universe is found to face an intricate 2 + 1 break-up among the space dimensions themselves [14].

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In this paper, we introduce and examine a particular kind of a more complex, six dimensional Cremonian universe whose spatio-temporal sector is still four dimensional, yet featuring three dimensions of time and just a single spatial coordinate. The character of other two dimensions is neither that of space nor time; in addition, these dimensions are only conditionally observable/accessible. This Cremonian universe sits in the 3-dimensional projective space over the fields of the real numbers $\Re$ and is generated by the configuration of fundamental elements of a homaloidal web of cubic (i.e., third-order) surfaces that share a proper conic, $\widehat{\mathscr{Q}}$, a (straight-)line, $\widehat{\mathscr{L}}$, incident with the conic and not lying in its plane, and three different non-collinear points, $\widehat{B}_i$ ($i = 1, 2, 3$), none of them incident with either $\widehat{\mathscr{L}}$ or the plane of $\widehat{\mathscr{Q}}$. The cubics of the web have also two double points, D_1 and D_2, in common; both the points lie on $\widehat{\mathscr{L}}$, the former being the intersection of $\widehat{\mathscr{L}}$ and $\widehat{\mathscr{Q}}$ [16,17]. Selecting an allowable system of homogeneous coordinates $\check{z}_\alpha$ ($\alpha = 1, 2, 3, 4$) in such a way that

$$\widehat{\mathscr{L}} : \check{z}_1 = 0 = \check{z}_2, \tag{1}$$

$$\widehat{\mathscr{Q}} : \check{z}_4 = 0 = -2\check{z}_1\check{z}_2 + \check{z}_1\check{z}_3 + \check{z}_2\check{z}_3 \equiv \overline{\mathscr{C}}, \tag{2}$$

$$\widehat{B}_1 : \varrho\check{z}_\alpha = (0, a, b, c), \quad a, c \neq 0, \tag{3}$$

$$\widehat{B}_2 : \varrho\check{z}_\alpha = (f, 0, g, h), \quad f, h \neq 0, \tag{4}$$

$$\widehat{B}_3 : \varrho\check{z}_\alpha = (k, k, l, m), \quad k, l, m \neq 0, \tag{5}$$

$$D_1 : \varrho\check{z}_\alpha = (0, 0, 1, 0), \tag{6}$$

$$D_2 : \varrho\check{z}_\alpha = (0, 0, 0, 1), \tag{7}$$

where ϱ is a non-zero proportionality factor, and assuming, without any substantial loss of generality, that

$$\frac{a}{c} = \frac{f}{h} = 2\frac{k(l-k)}{lm} \equiv -\Theta, \tag{8}$$

the web in question is given by

$$\mathscr{W}(\eta_\alpha) : \eta_1\check{z}_1\check{z}_4\left(\frac{k(g\check{z}_1 - f\check{z}_3)}{fl - gk} + \check{z}_2\right) + \eta_2\check{z}_2\check{z}_4\left(\check{z}_1 + \frac{k(b\check{z}_2 - a\check{z}_3)}{al - bk}\right) + \eta_3\check{z}_1\overline{\mathscr{D}} + \eta_4\check{z}_2\overline{\mathscr{D}} = 0, \tag{9}$$

with

$$\overline{\mathscr{D}} \equiv -2\check{z}_1\check{z}_2 + \check{z}_1\check{z}_3 + \check{z}_2\check{z}_3 + \Theta\check{z}_3\check{z}_4 = \overline{\mathscr{C}} + \Theta\check{z}_3\check{z}_4, \tag{10}$$

and $\eta_\alpha \in \Re$. This web generates the following Cremona transformation [16]

$$\varrho\check{z}_1' = \check{z}_1\check{z}_4\left(\frac{k(g\check{z}_1 - f\check{z}_3)}{fl - gk} + \check{z}_2\right), \tag{11}$$

$$\varrho\check{z}_2' = \check{z}_2\check{z}_4\left(\check{z}_1 + \frac{k(b\check{z}_2 - a\check{z}_3)}{al - bk}\right), \tag{12}$$

$$\varrho\check{z}_3' = \check{z}_1\overline{\mathscr{D}}, \tag{13}$$

$$\varrho\check{z}_4' = \check{z}_2\overline{\mathscr{D}}, \tag{14}$$

where $\check{z}_\alpha'$ are the homogeneous coordinates of an allowable system in the second ("primed") projective space.

Our forthcoming task is to find the *fundamental* elements of $\mathscr{W}(\eta_\alpha)$. To begin with, we recall [6,16] that the fundamental element of a Cremona transformation between two 3-dimensional projective spaces is a curve, or a surface, in one space whose corresponding image in the other space is a single *point*;[1] the loci of fundamental elements of the same kind being mapped, in general, into *curves*, i.e., *one*-dimensional geometrical objects, of the second space. Employing Eqs. (11)–(14), it is quite a straightforward task to spot that in our present case the loci of such elements are the plane of

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[1] Equivalently, a fundamental element associated with a given homaloidal web of surfaces is, in general, any curve/surface whose only intersections with a generic member of the web are the base (i.e., common to all the members) elements of the latter [16].

the conic $\widehat{\mathcal{Q}}$, the three planes $\widehat{B_i\mathcal{L}}$, the quadric $\overline{\mathcal{D}}=0$ and the quadratic cone $\overline{\mathcal{C}}=0$, for their images in the second space are indeed curves, namely lines (for the planes and the quadric) and/or a twisted cubic (for the cone) [17]. To be more explicit, the plane of $\widehat{\mathcal{Q}}$ host a pencil (i.e., linear, single parametrical set) of fundamental *lines* ($\vartheta_{1,2} \in \mathfrak{R}$)

$$\widetilde{\mathcal{L}}(\vartheta_{1,2}) : \vartheta_1 \check{z}_1 + \vartheta_2 \check{z}_2 = 0 = \check{z}_4, \tag{15}$$

whose point of concurrence is the point D_1; a line from this pencil has for its primed counterpart a point of the line $^s\widehat{\mathcal{L}}' : \check{z}_1' = 0 = \check{z}_2'$, the latter being single (hence the superscript "s") on the surfaces of the inverse homaloidal web. The three planes $\widehat{B_i\mathcal{L}}$ contain each a pencil of fundamental *conics* whose four base (i.e., shared by all the members) points are D_1, D_2, $\widehat{B}_i$ and K_i – the last one being the point, not on $\widehat{\mathcal{L}}$, in which the plane in question cuts the conic $\widehat{\mathcal{Q}}$; in particular,

$$\widetilde{\mathcal{Q}}_{i=1}(\vartheta_{1,2}) : \check{z}_1 = 0 = \vartheta_1 \check{z}_4(b\check{z}_2 - a\check{z}_3) + \vartheta_2 \check{z}_3(\check{z}_2 + \Theta\check{z}_4), \tag{16}$$

$$\widetilde{\mathcal{Q}}_{i=2}(\vartheta_{1,2}) : \check{z}_2 = 0 = \vartheta_1 \check{z}_4(g\check{z}_1 - f\check{z}_3) + \vartheta_2 \check{z}_3(\check{z}_1 + \Theta\check{z}_4), \tag{17}$$

$$\widetilde{\mathcal{Q}}_{i=3}(\vartheta_{1,2}) : \check{z}_1 - \check{z}_2 = 0 = \vartheta_1 \check{z}_4(l\check{z}_2 - k\check{z}_3) + \vartheta_2(-2\check{z}_2^2 + 2\check{z}_2\check{z}_3 + \Theta\check{z}_3\check{z}_4). \tag{18}$$

It is readily verified that a conic of $\widetilde{\mathcal{Q}}_i(\vartheta_{1,2})$ corresponds to a point of the line $^d\widehat{\mathcal{L}}_i'$, where $^d\widehat{\mathcal{L}}_{i=1}' : \check{z}_1' = 0 = \check{z}_3'$, $^d\widehat{\mathcal{L}}_{i=2}' : \check{z}_2' = 0 = \check{z}_4'$, and $^d\widehat{\mathcal{L}}_{i=3}' : \check{z}_3' - \check{z}_4' = 0 = (l - gk/f)\check{z}_1' - (l - bk/a)\check{z}_2'$, respectively; all these lines are double ("d") on a generic homaloid of the inverse web. The intrinsic structure and mutual coupling between these four pencils are depicted in Fig. 1. If one compares this configuration with the one introduced and studied in detail in [6], which is associated with a homaloidal web of quadric surfaces and which reproduces what is macroscopically observed, one finds that the two configurations are prefect inverses of each other; it was, among other things, also this feature that motivated us to examine thoroughly this particular kind of Cremonian universe.

Now we turn to quadratic loci of fundamental elements. The quadric $\overline{\mathcal{D}} = 0$, which is proper (i.e., non-composite) and ruled (i.e., containing infinity of lines), accommodates a single parametrical aggregate of fundamental *quartics*, i.e., curves of order four,

$$\widetilde{\mathcal{F}}(\vartheta_{1,2}) : \vartheta_1 \check{z}_1 \left(\frac{k(g\check{z}_1 - f\check{z}_3)}{fl - gk} + \check{z}_2 \right) + \vartheta_2 \check{z}_2 \left(\check{z}_1 + \frac{k(b\check{z}_2 - a\check{z}_3)}{al - bk} \right) = 0 = \overline{\mathcal{D}}; \tag{19}$$

these quartics share the five points $\widehat{B}_i$ ($i = 1, 2, 3$), D_1 and D_2, and, in the primed space, they correspond to the points of the line $^q\widehat{\mathcal{L}}' : \check{z}_3' = 0 = \check{z}_4'$, the latter being of multiplicity four ("q") on the inverse homaloids. All the proper quartics in the set are singular, D_2 being their common double point, and, as it is also obvious from Fig. 2, they are genuine space curves. There are just three composite quartics within this aggregate, each comprising a pair of conics, namely ($\vartheta \equiv \vartheta_2/\vartheta_1$)
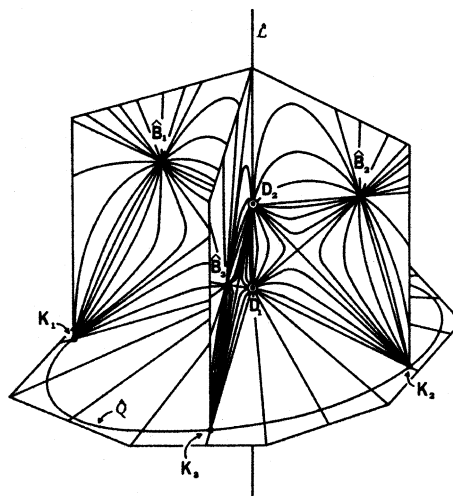

Fig. 1. A schematic sketch of the structure of the configuration of the four "fundamental" pencils defined by Eqs. (15)–(18). In each pencil, out of an infinite number of its members only several are drawn. This configuration represents the space-time "sector" of the corresponding Cremonian manifold, with three time dimensions ($\widetilde{\mathcal{Q}}_i$) and a single space one ($\widetilde{\mathcal{L}}$).
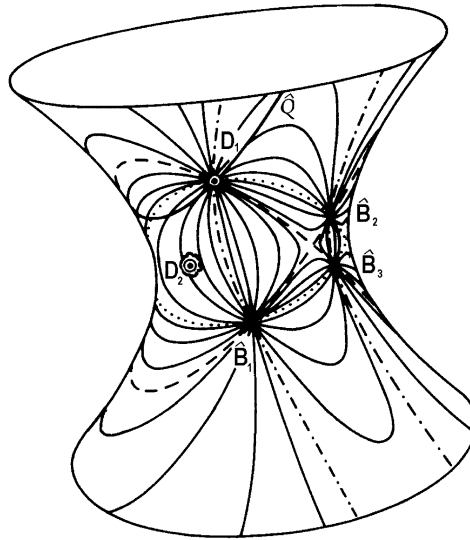
Fig. 2. A schematic sketch of the structure of the singly-infinite set of quartics defined by Eq. (19) for the generic case where the constraint imposed by Eq. (8) is relaxed. As in the previous figure, only a few quartics are illustrated, including the composites $\widetilde{\overline{\mathscr{F}}}_0^{\circ}$ (dot-dashed), $\widetilde{\overline{\mathscr{F}}}_\infty^{\circ}$ (dotted) and $\widetilde{\overline{\mathscr{F}}}_\wp^{\circ}$ (dashed).

$$\widetilde{\overline{\mathscr{F}}}(\vartheta = 0) \equiv \widetilde{\overline{\mathscr{F}}}_0^{\circ} : \breve{z}_1 = 0 = \overline{\mathscr{D}} \cup \frac{k(g\breve{z}_1 - f\breve{z}_3)}{fl - gk} + \breve{z}_2 = 0 = \overline{\mathscr{D}}, \tag{20}$$

$$\widetilde{\overline{\mathscr{F}}}(\vartheta = \infty) \equiv \widetilde{\overline{\mathscr{F}}}_\infty^{\circ} : \breve{z}_2 = 0 = \overline{\mathscr{D}} \cup \breve{z}_1 + \frac{k(b\breve{z}_2 - a\breve{z}_3)}{al - bk} = 0 = \overline{\mathscr{D}}, \tag{21}$$

$$\widetilde{\overline{\mathscr{F}}}(\vartheta = \wp) \equiv \widetilde{\overline{\mathscr{F}}}_\wp^{\circ} : \breve{z}_1 - \breve{z}_2 = 0 = \overline{\mathscr{D}} \cup ag\breve{z}_1 + bf\breve{z}_2 - af\breve{z}_3 = 0 = \overline{\mathscr{D}}, \tag{22}$$

where $\wp \equiv -f(al - bk)/a(fl - gk)$. Fig. 2 illustrates the shape of this aggregate for a generic case where each composite quartic comprises a pair of *proper* conics. (This property does not hold in our constrained case (see Eq. (8)), where one of the conics of both $\widetilde{\overline{\mathscr{F}}}_0^{\circ}$ and $\widetilde{\overline{\mathscr{F}}}_\infty^{\circ}$ is *composite* (a line pair).)

In the case of the quadric cone, $\overline{\mathscr{C}} = 0$, the fundamental elements are again *lines*, forming the following singly-infinite family

$$\widetilde{\mathscr{L}}_{\overline{\mathscr{C}}}(\vartheta) : \breve{z}_1 - \vartheta\breve{z}_2 = 0 = (\vartheta + 1)\breve{z}_3 - 2\vartheta\breve{z}_2, \tag{23}$$

with the parameter ϑ running through all the real numbers and infinity as well; for substituting the last equation into Eqs. (11)–(14) yields ($\varsigma \neq 0$)

$$\varsigma\breve{z}_1' = \vartheta\left(\vartheta(\vartheta + 1)\frac{gk}{fl - gk} + (\vartheta + 1) - 2\vartheta\frac{fk}{fl - gk}\right), \tag{24}$$

$$\varsigma\breve{z}_2' = \vartheta(\vartheta + 1) + (\vartheta + 1)\frac{bk}{al - bk} - 2\vartheta\frac{ak}{al - bk}, \tag{25}$$

$$\varsigma\breve{z}_3' = 2\Theta\vartheta^2, \tag{26}$$

$$\varsigma\breve{z}_4' = 2\Theta\vartheta, \tag{27}$$

which say that a generic line of $\widetilde{\mathscr{L}}_{\overline{\mathscr{C}}}(\vartheta)$ corresponds indeed to a single point of the second projective space. The structure of this aggregate can be discerned from Fig. 3.

At this point we invoke our fundamental "Cremonian" postulate [6–10] which says that each single parametrical sets of fundamental elements generates/represents a unique dimension of a Cremonian universe, with *pencil*-borne aggregates having a special status of generating space (lines) and time (conics). Hence, the Cremonian universe under discussion is six-dimensional, with three dimensions of time ($\widetilde{\mathscr{Q}}_i$), one dimension of space ($\widetilde{\mathscr{L}}$) and two
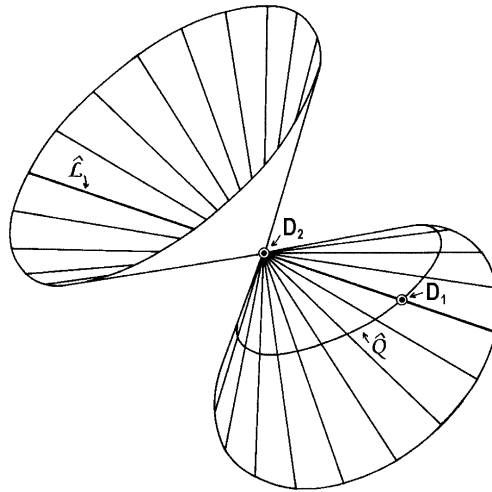
Fig. 3. A schematic sketch of the structure of the singly-infinite set of lines "sweeping" the quadric cone $\overline{\mathscr{C}} = 0$. As in the previous two figures, only a finite number of lines are drawn.

additional dimensions ($\widetilde{\mathscr{F}}$ and $\widetilde{\mathscr{L}_{\overline{\mathscr{C}}}}$) of a *different* nature. It is these two extra-dimensions which are of our next concern.

It is obvious that the $\widetilde{\mathscr{F}}$-dimension bears more resemblance to time, for its constituents are, like conics, of non-linear character, whereas the $\widetilde{\mathscr{L}_{\overline{\mathscr{C}}}}$-one, whose elements are lines, is more similar to the spatial dimension. Yet, there exists a profound difference between the four pencil-dimensions and these two "extra-"dimensions. This difference stems from the fact that the former are all *planar* configurations, whereas the latter are both located on *quadratic* surfaces, and acquires its most pronounced form when a particular Cremonian observer is concerned. For a Cremonian observer is represented by a line [10,14,15], and whilst *any* line in a three-dimensional projective space is incident with *any* plane (see, e.g., Refs. [18,19]), this is no longer the case for a pair comprising a line and a non-composite quadric; given any non-composite quadric (whether proper, or a cone), there exist an infinite number of lines incident with it, but also an infinity of lines which have no intersection with this quadric. If we take this incidence relation as a synonym of the observer's awareness of the particular dimension, then we see that the four "planar" dimensions (i.e., time and space) will be observed by (accessible to) every observer, while the two "quadratic" ones not! In other words, for each of these two non-planar dimensions, there exist two distinct groups of the observers; one comprising observers who perceive this dimension, the other those who do not. This finding thus amounts to saying that these two extra-dimensions are observable only *conditionally*.

It is of crucial importance to realize here that the conditional observability of these extra-dimensions has *nothing* to do with their length (compactification), as no concept of the measure/metric has so far been introduced into our model. Instead, it is of a purely algebraic geometrical origin, based solely on the incidence relations between relevant geometrical objects and intimately linked with the fact that the ground field of the background projective space, taken to be that of the real numbers, is *not* algebraically closed; for the *geometrical* problem of finding the common points of a line and a quadric in a 3-dimensional projective space defined over a given ground field reduces to the *algebraic* one of solving/factoring a quadratic equation in the given field, which is not always possible unless the field is algebraically closed (see, e.g., Ref. [20]). This toy-model thus demonstrates that there might exist extra-dimensions that need not necessarily be compactified/curled-up to remain unobservable. And this is a truly serious implication, especially for cosmology and high energy particle physics.

As a concluding note, it is worth pointing out that such "conditionally-emergent" extra-dimensions of the universe may also be inherent in El Naschie's concept of Cantorian space-time (see, e.g., Refs. [21,22]), for this model has recently been recognized to be, in principle, extendible over *different* algebras/number systems, even those having zero divisors [23]. This property thus dovetails nicely with a number of other intriguing features the two theories/concepts have already been found to share [6,7,13,14].

Acknowledgement

References

[1] Saniga M. Arrow of time and dimensionality of space. In: Sato K, Suginohara T, Sugiyama N, editors. Cosmological constant and the evolution of the universe. Tokyo: Universal Academy Press Inc.; 1996. p. 283–4.
[2] Saniga M. On the transmutation and anihilation of pencil-generated spacetime dimensions. Astrophys Space Sci 1996;244:283–90.
[3] Saniga M. Pencils of conics: a means towards a deeper understanding of the arrow of time? Chaos, Solitons & Fractals 1998;9:1071–86.
[4] Saniga M. Geometry of psycho(patho)logical space-times: a clue to resolving the enigma of time? Noetic J 1999;2:265–74.
[5] Saniga M. Algebraic geometry: a tool for resolving the enigma of time? In: Buccheri R, Di Gesù V, Saniga M, editors. Studies on the structure of time: from physics to psycho(patho)logy. New York: Kluwer Academic/Plenum Publishers; 2000. p. 137–66.
[6] Saniga M. Cremona transformations and the conundrum of dimensionality and signature of macro-spacetime. Chaos, Solitons & Fractals 2001;12:2127–42.
[7] Saniga M. On 'spatially anisotropic' pencil-space-times associated with a quadro-cubic Cremona transformation. Chaos, Solitons & Fractals 2002;13:807–14.
[8] Saniga M. Quadro-quartic Cremona transformations and four-dimensional pencil-space-times with the reverse signature. Chaos, Solitons & Fractals 2002;13:797–805.
[9] Saniga M. Homaloidal webs, space Cremona transformations and the dimensionality and signature of macro-spacetime. In: Amoroso RL, Hunter G, Kafatos M, Vigier J-P, editors. Gravitation and cosmology: from the Hubble radius to the Planck scale. Dordrecht: Kluwer Academic Publishers; 2002. p. 507–10.
[10] Saniga M. Geometry of time and dimensionality of space. In: Buccheri R, Saniga M, Stuckey WM, editors. The nature of time: geometry, physics and perception (NATO ARW). Dordrecht: Kluwer Academic Publishers; 2003. p. 131–43. Available from <physics/0301003>.
[11] Battersby S. Einstein on acid. New Scientist 2003/2004;180(2426):40–3.
[12] Saniga M. On an intriguing signature-reversal exhibited by Cremonian space-times. Chaos, Solitons & Fractals 2004;19:739–41. Available from <physics/0303012>.
[13] Saniga M. Cremonian space-time (s) as an emergent phenomenon. Chaos, Solitons & Fractals 2005;23:645–50. Available from <physics/0402097>.
[14] Saniga M. On an observed-related unequivalence between spatial dimensions of a generic Cremonian universe. Chaos, Solitons & Fractals 2005;23:1935–9. Available from <physics/0403074>.
[15] Saniga M, Buccheri R. The psychopathological fabric of time (and space) and its underpinning pencil-borne geometries. J Mind Behav, in press. Available from <physics/0310165>.
[16] Hudson HP. Cremona transformations in plane and space. Cambridge: Cambridge University Press; 1927.
[17] Cremona L. Sulle trasformazioni razionali nello spazio. An Mat Pura Appl 1871;II(5):131–62.
[18] Levy H. Projective and related geometries. New York: The Macmillan Company; 1964.
[19] Semple JG, Kneebone GT. Algebraic projective geometry. Oxford: Clarendon Press; 1956.
[20] Childs LN. A concrete introduction to higher algebra. Heidelberg: Springer; 1995.
[21] El Naschie MS. The concepts of E-infinity: an elementary introduction to the Cantorian-fractal theory of quantum physics. Chaos, Solitons & Fractals 2004;22:495–511.
[22] Sidharth BG. The nature of quantum space-time and the Cantorian $\mathscr{E}^{(\infty)}$ proposal. Chaos, Solitons & Fractals 2002;14:1325–30.
[23] Czajko J. On Cantorian spacetime over number systems with division by zero. Chaos, Solitons & Fractals 2004;21:261–71.