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On the Possibility of a Chern–Simons Physics on the Sun

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Abstract—It is argued that the observational evidence that most of the net magnetic flux through the solar photosphere is concentrated into small, isolated and intense flux tubes might indicate the presence of Chern–Simons physics in this lowermost layer of the solar atmosphere. We show, in particular, that the main effect of the Chern–Simons term is to associate with each 'lump' of uncompensated electric charge a flux tube which is carried around by the motion of the lump in a 'rigid way', resembling thus fairly well what one really observes. Copyright © 1996 Elsevier Science Ltd.

One of the most striking and unexpected discoveries of new solar physics in the early 1970s was undoubtedly the observation that the magnetic field penetrating the surface of the Sun outside sunspots exhibits the property of being concentrated into isolated, very small and intense flux tubes (e.g., [1]). These tubes have been found to possess rather strong fields (amounting to 1–2 kG) with characteristic dimensions lying beyond the best currently achieved spatial resolution of ground-based instruments, some 200–350 km. Although considerable MHD theoretical interest has up to now been focused on these structures (for a review see, e.g., [2]) it is by no means too exaggerative to say that their true physical nature still remains wrapped in a shroud of mystery. What has so far been achieved is an extensive but unassembled set of ideas rather than a compact physical description. Rephrased in Parker's words [3]

> the concentration is the rule rather than the exception and defies conventional notions of magnetic fields, whose behaviour in the laboratory is always to spread out under the influence of the isotropic pressure $B^2/8\pi$ to fill all the space allowed by the tension $B^2/4\pi$ along the lines of force. In a passive fluid the molecular diffusivity, and more important, the hydrodynamic exchange instability, disperse the field, permitting a trend toward degradation of the field energy and an approach to homogeneity. We might also expect turbulent diffusion to produce a similar trend toward reducing the extraordinary field energies of the concentrated state observed on the Sun. But nature would not have it so, apparently. In the one layer of the Sun we can observe—the photosphere—the opposite prevails.

It, therefore, appears to be both quite natural and of particular interest to consider an alternate to MHD approach where the tube-like pattern in the magnetic field distribution would follow from first principles.

So as to formulate the essentials of the problem, and yet to keep the latter tractable, we will adopt some kind of simplification and consider the photosphere as an effectively two-dimensional object. That such an assumption is fully justified follows from the fact that the thickness of the photosphere, amounting to several hundred km, is much smaller than a typical distance between the individual flux tubes in question, acquiring a few thousand km, and that on the latter scale also the effects of a curvature of the Sun's surface can be neglected. Thus, the physical processes taking part in the photosphere can be looked upon as being confined to the plane.

Such an 'effective' 3→2 reduction of spatial dimensions makes it, however, possible to consider here a non-standard electromagnetic theory whose kinetic action is just the so-called Chern–Simons term [4], i.e., the theory described by the Lagrangian density

$$\mathscr{L}_{\mathrm{CS}}^{\odot} = -A_\rho j^\rho + \frac{\Theta}{4}\varepsilon^{\mu\nu\rho}F_{\mu\nu}A_\rho; \tag{1}$$

here the Greek indices run from 0 to 2, A_ρ and j^ρ are the electromagnetic vector potential and electric current density, respectively, $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$, $\varepsilon^{\mu\nu\rho}$ stands for the Levi-Cività pseudotensor with $\varepsilon^{012} = \varepsilon_{012} = 1$, Θ represents a real-valued constant, and the speed of light c has been set equal unity. That such theory is not simply of academic interest but can also be relevant to physics is indicated by its successful application to describe the fractional quantum Hall effect as well as some features of high T_{C} superconductivity (e.g., [5]). In what follows we will outline that the Chern–Simons dynamics, when viewed at a purely classical level, might also have something to do with the phenomena observed on the Sun.

To this end we write down the equations of motion following from equation (1):

$$\frac{\Theta}{2}\varepsilon^{\mu\nu\rho}F_{\mu\nu} = j^\rho, \tag{2}$$

which tell us that the field strength deviates from zero only in the regions of a non-vanishing current density j^ρ; in particular, for $\rho = 0$ it means that the magnetic field strength, which has now only a single component F_{12}, is uniquely tied to the sites of the concentration of electric charge j^0, that is

$$\Theta F_{12} = j^0. \tag{3}$$

Next, because the typical dimensions of photospheric flux tubes lie—as mentioned at the beginning—below the best presently achieved spatial resolution these are observed as bright points [6]. Hence to model them we can take the current density in the form

$$j^\rho(x^i) = \sum_{n=1}^{N} e_{(n)}\frac{\mathrm{d}z^\rho_{(n)}(t)}{\mathrm{d}t}\delta^2(x^i - z^i_{(n)}(t)), \tag{4}$$

with δ standing for the Dirac function and i denoting a spatial index, which represents a set of N point-like particles, the nth particle being situated at the point $z^i_{(n)}(t)$ of the $x^1 - x^2$ (i.e., the photosphere's) plane and carrying the electric charge $e_{(n)}$. Inserting the last equation into equation (3) and performing spatial integration of the latter we arrive at

$$\Theta\Psi_{\mathrm{tot}} = \sum_n e_{(n)} \tag{5}$$

where Ψ_{tot} represents nothing but the total magnetic flux through the $x^1 - x^2$ plane; in other words, the nth particle carries the flux

$$\Psi_{(n)} = \frac{e_{(n)}}{\Theta}, \tag{6}$$

which thus explicitly reveals that "the main effect of the Chern–Simons term is to attach to each 'particle' a flux tube . . ., and that such flux tubes are carried around by the motion of the particles in a 'rigid' way" [7]. Rephrased in terms of solar physics it means that as long as the photospheric flux tubes are of a Chern–Simons origin they simply map the regions of uncompensated electric charge, or, equivalently, that any local departure from the equilibrium between the carriers of positive and negative electric charges appears as a magnetic flux tube perpendicular to the Sun's surface! Such a scenario gets considerable observational support, especially in the following two aspects. On the one hand, there seems to exist no physical reason for one region of the quiet photosphere to have more plausible conditions for producing the lumps of a net non-zero electric charge than the other, which thus accounts for the observed ubiquity of intense flux tubes. On the other, we also expect the equilibrium restoring forces to act in such a way that it is rather unfavourable for the sites of uncompensated charge to acquire a large-scale structure, and this implies the observed 'smallness' of the typical dimensions of flux tubes.

Apart from this still entirely speculative motivation, however, there is an excellent pedagogical reason for discussing this subject in relation to another well-known type of magnetic flux structures seen on the Sun—sunspots. The (geometric, magnetic and thermodynamic) structure of the latter shows a wide range of complexity and as far as dimensions are concerned they surpass, when fully developed, their 'quiet photosphere' counterparts by (at least) two orders. To grasp the essentials of sunspot physics (and overcome some of the crucial problems faced by MHD approaches) we have put forward a non-MHD, the so-called Abelian Higgs model [8, 9] represented by the Lagrangian density whose 'electromagnetic' sector reads

$$\mathcal{L}^{\odot}_{\mathrm{AH}} = -A_{\rho}j^{\rho} - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} \qquad (7)$$

and which is to be compared with Lagrangian density (1); the two show a substantial difference in the last terms and thus it should not be surprising that the physical configurations they are invoked to describe differ so profoundly from each other.

Summarizing this short letter we may say that we could gain some important physical insights about the nature of photospheric flux tubes by making further studies of the 'Chern–Simons' electromagnetism.

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