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A further note on a formal relationship between the arithmetic of homaloidal nets and the dimensions of transfinite space-time

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Abstract

A sequence of integers generated by the number of conjugated pairs of homaloidal nets of plane algebraic curves of even order is found to provide an *exact* integer-valued match for El Naschie's primordial set of fractal dimensions characterizing transfinite heterotic string space-time.

1. Introduction

Recently [1], employing the algebra and arithmetic of self-conjugate homaloidal nets of planar algebraic curves, we have discovered a couple of integer-valued progressions that are found to mimic extraordinarily well the two El Naschie sequences of fractal dimensions characterizing transfinite heterotic string space-time. One of the progressions is demonstrated to provide an algebraic geometrical justification not only for all the relevant dimensions of the classical theory, but also for the other two dimensions advocated by El Naschie [2–5], namely the smallest value of the inverse of the non-supersymmetric quantum gravity coupling constant, $\bar{\alpha}_g = 42 + 2k \simeq 42.36067977$, and the inverse of fine structure constant, $\bar{\alpha}_0 = 2 \otimes (68 + 3k) \simeq 2 \otimes 68.54101967$. A non-trivial coupling between the two El Naschie sequences has also been revealed.

2. Conjugate pairs of homaloidal nets and primordial El Naschie's dimensions

In the present paper, we put forward another 'homaloidal' sequence that offers a nice fit to the primordial set of fractal dimensions introduced by El Naschie, i.e., to the set defined by [2–5]

$$D_q = [136 + 6\phi^3(1 - \phi^3)]\phi^q, \tag{1}$$

where q is a non-negative integer and ϕ is taken to be identical with the golden mean, $\phi = 1 - \phi^2 = (\sqrt{5} - 1)/2 \simeq 0.618033989$. The new sequence is borne by the numbers of conjugate pairs of homaloidal nets of a given order n, $\#_n^{\mathrm{pair}}$ (see [1,6–8] for the definitions, symbols and notation). From the properties of plane Cremona transformations [6–9] it is obvious that these numbers are given by the following simple formula:

$$\#_n^{\mathrm{pair}} = \frac{\#_n^{\mathrm{tot}} - \#_n^{\mathrm{sc}}}{2} + \#_n^{\mathrm{sc}} = \frac{\#_n^{\mathrm{tot}} + \#_n^{\mathrm{sc}}}{2}, \tag{2}$$

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Table 1

n	$\#_n^{\text{pair}}$	$\hat{\#}_n^{\text{pair}}$	D_q (El Naschie's primordial dimensions)
2	1	2	$D_{q=9} = 0 + 10k \simeq 1.803\ldots$
4	2	3	$D_{q=8} = 4 - 6k \simeq 2.917\ldots$
6	3	4	$D_{q=7} = 4 + 4k \simeq 4.721\ldots$
8	7	8	$D_{q=6} = 8 - 2k \simeq 7.639\ldots$
10	11	12	$D_{q=5} = 12 + 2k \simeq 12.360\ldots$
12	19	20	$D_{q=4} = 20 + 0k = 20$
14	31	32	$D_{q=3} = 32 + 2k \simeq 32.360\ldots$
16	51	52	$D_{q=2} = 52 + 2k \simeq 52.360\ldots$
18	84	85	$D_{q=1} = 84 + 4k \simeq 84.721\ldots$
20	137	138	$D_{q=0} = 136 + 6k \simeq 137.082\ldots$
22	202	203	No match ($D_{q=-1} = 220 + 10k \simeq 221.803\ldots$)

with $\#_n^{\text{tot}}$ standing for the total number of nets of order n and $\#_n^{\text{sc}}$ representing the number of those nets that are self-conjugate. As in the preceding paper [1], our homaloidal numerology relies almost exclusively on the 'data' found in an old, but very important, paper by Mlodziejowski [9], where the values of both $\#_n^{\text{tot}}$ and $\#_n^{\text{sc}}$ are given for the first 22 orders. Exploiting Eq. (2), we have obtained the values of $\#_n^{\text{pair}}$, n even, as listed in column two of Table 1. The sequence itself is generated by the 'augmented-by-one' quantities,

$$\hat{\#}_n^{\text{pair}} \equiv \#_n^{\text{pair}} + 1; \tag{3}$$

these are given in column three of the table. We see that this sequence is an *exact* integer-valued fit of the El Naschie sequence (introduced in an explicit form in the last column of the table); that is, the absolute value of the difference between $\hat{\#}_n^{\text{pair}}$ and its corresponding fractal dimension D_q is always less than *one*!

What strikes one most about the sequence $\hat{\#}_{n=2l}^{\text{pair}}$, $l = 1, 2, \ldots$, when compared with its 'self-conjugate' counterpart $D_l^{(2)}$ described in [1], are the following two facts: (1) it copies D_q within a *greater* interval of values of l ($1 \leqslant l \leqslant 10$, whereas in the case of $D_l^{(2)}$ it is for $1 \leqslant l \leqslant 6$ only [1]); and (2) the correspondence *terminates* at $D_{q=0} \simeq 137.082\ldots$ – the value that is believed to be very close to the inverse value of the fine structure constant [2,10]. It is especially the latter fact which may be regarded as a further justification of the prominent role this constant is supposed to play in all fundamental theories of stringy space-times and Cantorian space $\mathscr{E}^{(\infty)}$ as well [10].

3. Conclusion

The theory outlined in [1] and the present paper strongly suggests that we could gain some important physical insights into the nature of transfinite heterotic string space-time making use of the concepts of homaloidal nets and plane Cremona transformations. It is our hope that the two papers will elicit the interest of other physicists to explore its further possibilities so that we may soon know whether the extremely exciting prospects implicit in this rigorous arithmetic–algebraic approach are real or illusory.

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