Chapter 2: “Mathematical Approaches to the Concept of Time”

An Overview by Metod Saniga

In the introductory paper, *Metod Saniga* reviews his algebraic geometrical theory of pencil-spacetimes based on Cremona transformations between two projective spaces of three dimensions. These spacetimes exhibit an intimate connection between the extrinsic geometry of the time dimension and the dimensionality of space. Moreover, they seem to provide us with a promising conceptual basis for the possible reconciliation between two extreme concepts of time, viz. physical and psychological.

In the subsequent paper, *Jonathan Smith* gives a very readable and lucid account of the possible relationship between time in biological systems and physics. After reviewing what he calls *biological spacetimes*, he succinctly addresses such important topics as the demarcation between biology and physics, emergence of the universal time of physics, censorship theorems and causality, and the phenomenology of the “now;” he goes even further and proposes an answer to the famous question of Schrödinger, “What is life?”

A brief and concise analysis of the relation between real and imaginary time in physics is carried out by *George Jaroszkiewicz*. Introducing a number of illustrative scenarios from special relativity, general relativity, and quantum mechanics, the author points out that although imaginary time is formally a useful concept, there exists much controversy when it comes to the physical meaning of what is involved.

In an attempt to address the question of the “passage” of time, *Matej Pavšič* employs the concept of Clifford (or C-)spaces – a poly-dimensional continuum whose elements are, from the perspective of classical spacetime, not only points, but also loops, areas, volumes, etc. These objects are shown to have a natural geometrical representation in terms of a Clifford polyvector. The author argues that a specific part of the coordinate polyvector can well be taken as a crude model of our perception of time.

Employing further the notion of C-spaces, *Carlos Castro* makes an interesting approach to reformulate and extend Nottale’s scale relativity theory, with the Planck scale as another fundamental invariant of Nature. This “extended” relativity theory is shown to possess a number of unexpected features, for example, the existence of two distinct modes (“local” and “global”) of time dimension.

Based on time measurements from electronic oscillators, *Michel Planat* brings into evidence a possible connection between the (fine) structure of time dimensions and algebraic number theory. He stresses, in particular, that the properties of the $1/f$ power spectra of time variability in complex systems and the corresponding $1/f$ noise must have something to do with the distribution of prime numbers as well as the Riemann hypothesis.

Another important novel concept of complex systems, that of a time operator, is introduced and discussed in detail by *Ioannis Antoniou* and *Zdzisław Suchanecki.* The authors review constructions of the time operator for several systems and show how to distinguish between stable and unstable systems in terms of innovations. They also point out a striking analogy between some qualitative properties of the time operator and psychological time.

Quantum computing and its role in our understanding of time is examined by *Vito Di Gesù* and *G. Massimo Palma*. The authors pay particular attention to such conceptual problems as computability and complexity, the complexity of quantum algorithms, the halting problem, etc. and make it obvious that all of these touch the deepest issues of quantum mechanics, e.g., the EPR paradox and quantum non-locality.

An intriguing, “relational statistical view” of spacetime is introduced by *Vladimir Aristov*. The author comes up with a concept of “spatialized” time and – after introducing the rudiments of graph theory – he also discusses the possible discrete structure of spacetime at the Planck scale.

Last, but not least, there is a nice analysis by *Dimitrii Kucher* and *Alexandre Shkorbatov* of the time evolution of systems with memory, using cellular automaton models.