The CoMP-S@LSO: polarimetric calibration

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- Instrumental polarization
- Polarimetric calibration
- CoMP-S as polarimeter
- CoMP-S polarimetric calibration procedure
- CoMP-S polarimetric calibration results
- A proposal for a new CoMP-S polarimetric package

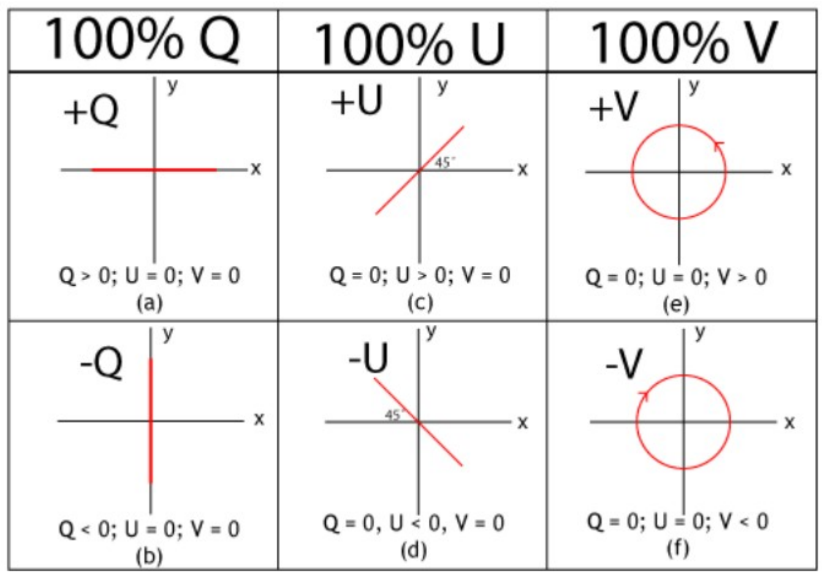
Polarimetry as a tool

- Polarimetry - measurements of the polarization properties of the light
- Polarization of the measured light – result of the magnetic field surrounding the atoms emitting the observed light:
  - Only some atom transitions
  - The Zeeman and Hanle effects
  - (other physical effects)
- We can not measure polarization, we can measure only intensity…
- Approaches for measurements:
  - Sequential measuring of different individual pol. states
  - Sequential measuring of different combinations of the individual pol. states

Polarimetry as a tool

- The polarized light descriptions:
  - The Poincare sphere, the Jones vector
  - **The Stokes vector [I,Q,U,V]:** the standard Cartesian basis (x, y), the Cartesian basis rotated by 45° (a,b), and a circular basis (l, r), E – the el. field amplitude

$$
\begin{aligned}
I &\equiv \langle E_x^2 \rangle + \langle E_y^2 \rangle \\
&= \langle E_a^2 \rangle + \langle E_b^2 \rangle \\
&= \langle E_r^2 \rangle + \langle E_l^2 \rangle, \\
Q &\equiv \langle E_x^2 \rangle - \langle E_y^2 \rangle, \\
U &\equiv \langle E_a^2 \rangle - \langle E_b^2 \rangle, \\
V &\equiv \langle E_r^2 \rangle - \langle E_l^2 \rangle.
\end{aligned}
$$



Wikipedia: https://en.wikipedia.org/wiki/Stokes_parameters

Instrumental polarization

- All optical parts of your instrument might alter (modulate) the polarimetric status of the light to be measured
- Your instrument as a whole: **O - modulation matrix** (n_meas x 4)

$$\mathbf{I}_{\mathrm{meas}} = \mathbf{O}\mathbf{I}_{\mathrm{in}} \quad \boldsymbol{I} \equiv (I, Q, U, V)^T$$

$$\mathbf{O} = \begin{pmatrix} I_{\mathrm{in}} \to I_{\mathrm{meas}} & Q_{\mathrm{in}} \to I_{\mathrm{meas}} & U_{\mathrm{in}} \to I_{\mathrm{meas}} & V_{\mathrm{in}} \to I_{\mathrm{meas}} \\ I_{\mathrm{in}} \to Q_{\mathrm{meas}} & Q_{\mathrm{in}} \to Q_{\mathrm{meas}} & U_{\mathrm{in}} \to Q_{\mathrm{meas}} & V_{\mathrm{in}} \to Q_{\mathrm{meas}} \\ I_{\mathrm{in}} \to U_{\mathrm{meas}} & Q_{\mathrm{in}} \to U_{\mathrm{meas}} & U_{\mathrm{in}} \to U_{\mathrm{meas}} & V_{\mathrm{in}} \to U_{\mathrm{meas}} \\ I_{\mathrm{in}} \to V_{\mathrm{meas}} & Q_{\mathrm{in}} \to V_{\mathrm{meas}} & U_{\mathrm{in}} \to V_{\mathrm{meas}} & V_{\mathrm{in}} \to V_{\mathrm{meas}} \end{pmatrix}$$

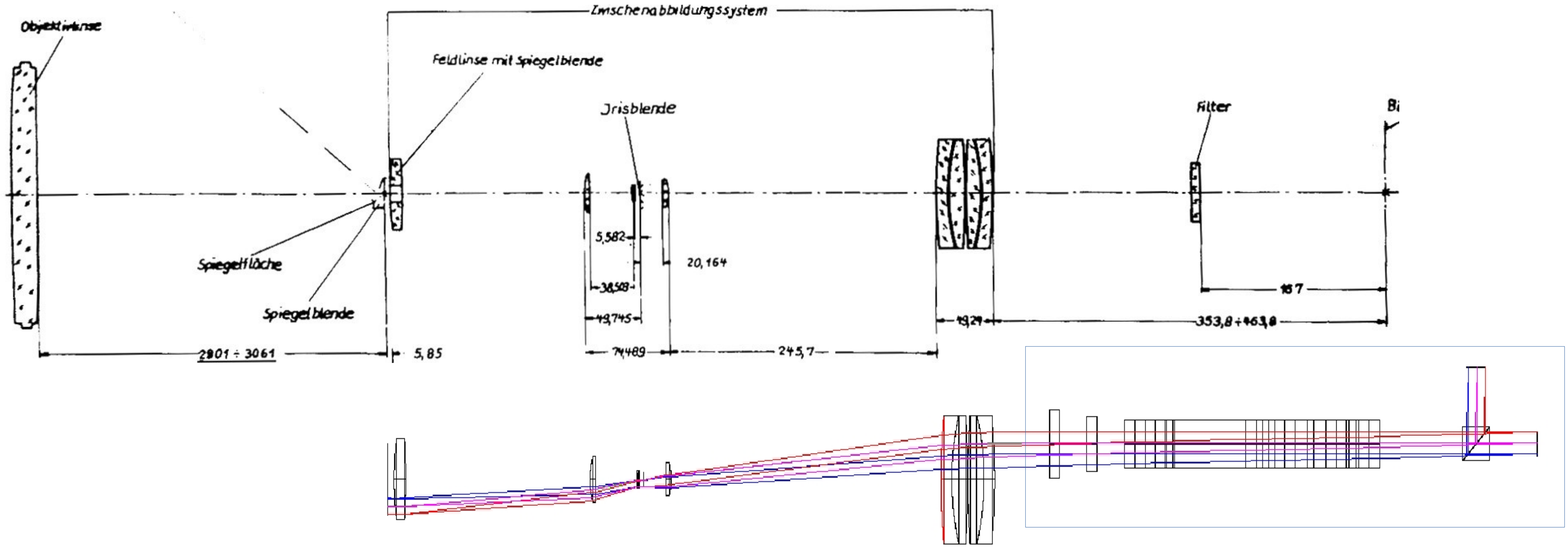
- This modulation of the light polarimetric status has to be determined in form of the **O** modulation matrix = the polarimetric calibration

Polarimetric calibration

- Method:
  - IN - known polarimetric status
  - OUT – measured polarimetric response to IN
  - O_{ij} – determination from IN and OUT using an instrument description
- Approaches for measurements:
  - Sequential measuring of individual pol. states (+Q,-Q,+U,...)
  - **Sequential measuring of different combinations of the individual pol. states**

CoMP-S as polarimeter

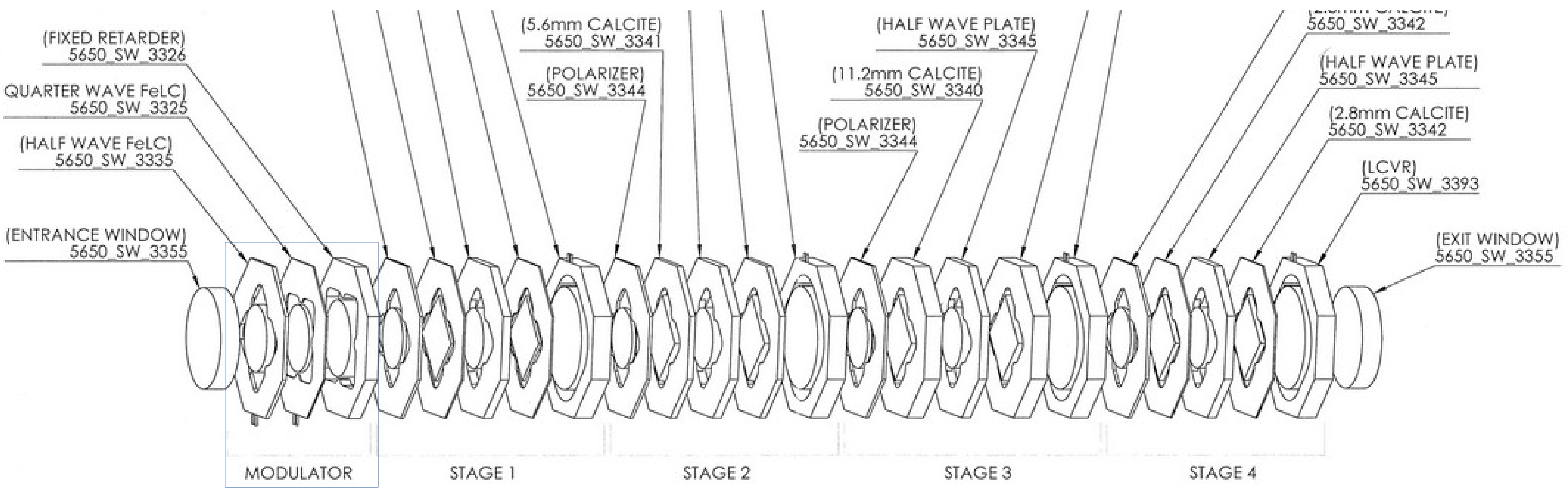
- The CoMP-S instrument and the ZEISS 200/3000/4000 coronagraph



Coronagraph & primary optics   Coronagraph secondary optics   & CoMP-S instrument + tertiary optics

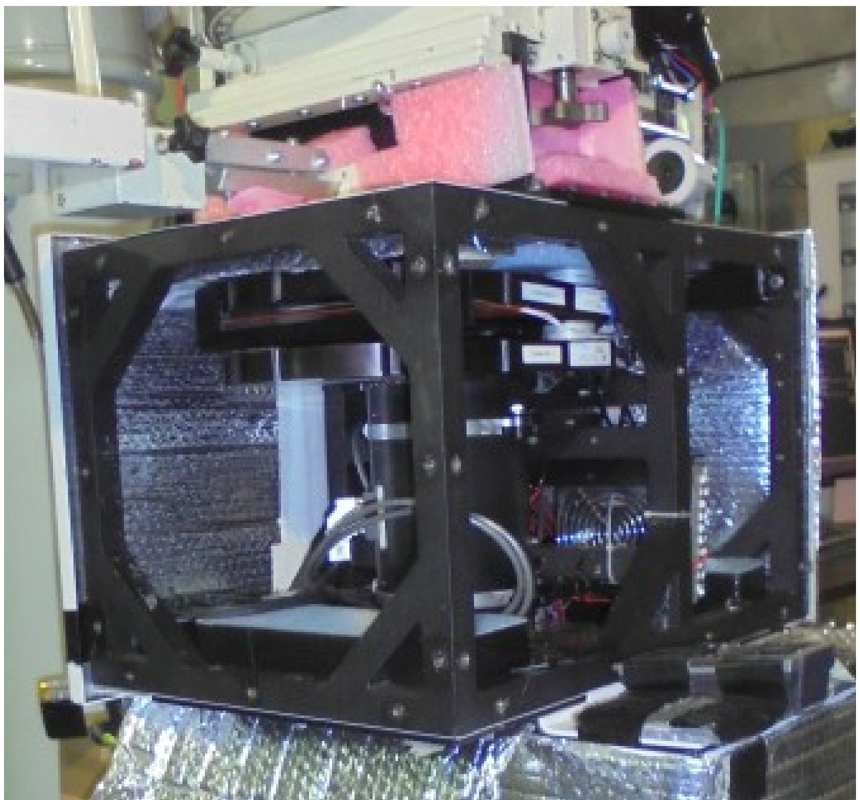
CoMP-S as polarimeter

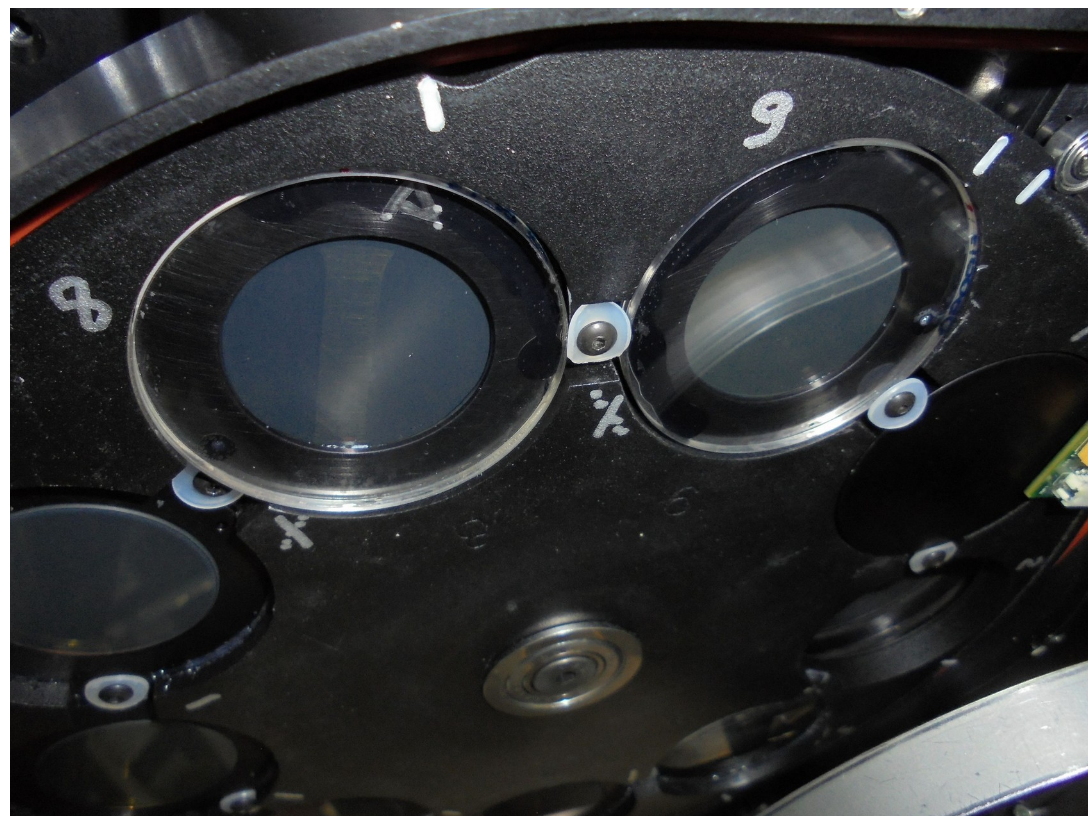
- The CoMP-S instrument: 4-stage tunable Lyot filter + polarimeter
- Polarimeter: a half-wave plate + a quarter-wave plate + a fixed retarder
- Wave plates: Ferroelectric liquid crystals (DISPLAYTECH, # LV2500), external el. field ±5 VDC → 4 alternations of the passing light polarization state change (retardation) is wavelength dependent



CoMP-S pol. calibration

- Polarimetric calibration: IN – light of known polarimetric status:
  - polarimetric optics in the calibration wheel: CLEAR, lin. polarizer @ 0°, 90°, 45°, -45°; lin. polarizer @ 0° + ¼ wave retarder @ +45°; lin. polarizer 0° + ¼ wave retarder @ -45°

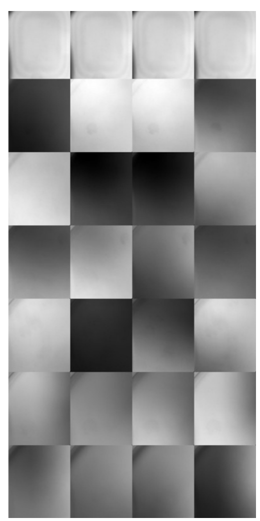




CoMP-S pol. calibration

- Polarimetric calibration: OUT – response to the light of known polarimetric status passing the instrument:
  - Horizontal: polarimeter - 4 FLC combinations in the CoMP-S
  - Vertical: calibration optics - CLEAR, lin. polarizer @ 0°, 90°, 45°, -45°; lin. polarizer @ 0° + ¼ wave retarder @ +45°; lin. polarizer 0° + ¼ wave retarder @ -45°
  - Example for 656.16 nm (FWHM ~0.05 nm): 2D images & 1 point

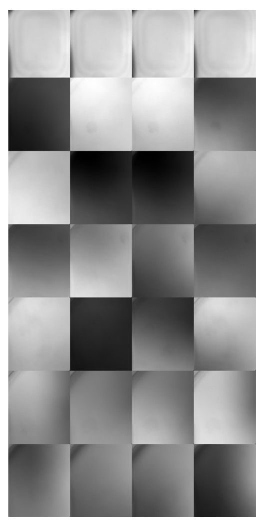
I_{meas} matrix:



1.00	1.00	1.00	1.00
0.28	1.18	1.17	0.50
1.04	0.16	0.16	0.82
0.50	0.80	0.56	0.45
0.82	0.55	0.79	0.87
0.67	0.59	0.62	0.50
0.37	0.93	0.83	0.30

CoMP-S pol. calibration

- Procedure for the determination of the **O** matrix from the measured response of the instrument $\mathbf{I}_{\mathbf{meas}}$ to the input light of the known pol. states is inevitable
- A more sophisticate approach: a polarimetric model of the instrument with some free parameters to be adjustable in order to describe the instrument as good as possible. Reasons for this approach: optical properties of the pol. parts (transmission & retardation ~ wavelength, FoV spatial variations)



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CoMP-S pol. calibration

- The polarimetric model of the CoMP-S instrument:
  - Parameters (initial guess):
    - Input light Stokes vector [I,Q,U,V] ($I_{\_mean}$,0,0,0)
    - linear polarizers (free: transmission (0.5), fixed: pol. ratio, orientation)
    - ¼ wave retarder (free: transmission (1.0), retardation (90. * $wave_{\_norm}$), fixed: orientation)
    - O matrix:

$$\left| \begin{matrix} I_{\_meas}(0,1)-I_{\_meas}(0,2))/2., (I_{\_meas}(0,3)-I_{\_meas}(0,4))2., (I_{\_meas}(0,5)-I_{\_meas}(0,6))/2. \\ I_{\_meas}(1,1)-I_{\_meas}(1,2))/2., (I_{\_meas}(1,3)-I_{\_meas}(1,4))2., (I_{\_meas}(1,5)-I_{\_meas}(1,6))/2. \\ I_{\_meas}(2,1)-I_{\_meas}(2,2))/2., (I_{\_meas}(2,3)-I_{\_meas}(2,4))2., (I_{\_meas}(2,5)-I_{\_meas}(2,6))/2. \\ I_{\_meas}(3,1)-I_{\_meas}(3,2))/2., (I_{\_meas}(3,3)-I_{\_meas}(3,4))2., (I_{\_meas}(3,5)-I_{\_meas}(3,6))/2. \end{matrix} \right| / I_{\_mean}$$

  - Definitions of the polarimetric calibration optics (type, orientation)
  - Calculation of the calibration optics impact on the passing light (in form of the Mueller matrices)

CoMP-S pol. calibration

- The polarimetric model of the CoMP-S instrument:
  - Fitting the model parameters to reach the minimum of the user-written function = sum of intensity differences $(I_{meas} – I_{model})^2$
  - The IDL procedure **POWELL**: based on powell code for minimization in N-dimensions *(section 10.5 of Numerical Recipes in C: The Art of Scientific Computing (Second Edition), Cambridge University Press, 1992)*
  - Typical IDL performance for our data: 19 parameters, 28 measurements, 15-50 iterations to reach the 10^{-6} limit, normalized chi^2 <0.01, runtime ~10^{-1} s
  - The parameters used to clarify appropriateness of the model:
    - normalized $(I_{meas} – I_{model})/I_{meas}$ intensity differences
    - reliability of the obtained values of the physical parameters: linear polarizer transmission, circular retarder transmission & retardation Stokes_input vector, efficiencies (and other additional parameters)

CoMP-S pol. calibration

- The polarimetric model of the CoMP-S instrument:
  - normalized **(I_{meas} – I_{model})/I_{meas}** intensity differences measure

    Horizontal: polarimeter - 4 FLC combinations in the CoMP-S

    Vertical: calibration optics - CLEAR, lin. polarizer @ 0°, 90°, 45°, -45°; lin. polarizer @ 0° + ¼ wave retarder @ +45°; lin. polarizer 0° + ¼ wave retarder @ -45°

I_{meas}

1.00	1.00	1.00	1.00
0.28	1.18	1.17	0.50
1.04	0.16	0.16	0.82
0.50	0.80	0.56	0.45
0.82	0.55	0.79	0.87
0.67	0.59	0.62	0.50
0.37	0.93	0.83	0.30

(I_{meas} – I_{model})/I_{meas}

0.001	0.001	0.004	-0.007
-0.014	-0.145	-0.023	0.104
0.036	0.065	-0.009	-0.110
-0.008	0.010	0.025	0.002
-0.013	0.034	0.011	0.002
0.044	0.187	0.002	-0.543
0.040	0.342	0.002	-0.561

CoMP-S pol. calibration

- The polarimetric model of the CoMP-S instrument:
  - Resulting O matrix:

1.00	-0.58	-0.07	0.28
1.00	0.72	0.73	-0.01
1.00	0.73	0.50	0.07
1.00	-0.29	-0.05	0.38

$$\begin{pmatrix} I_{\rm in} \to I_{\rm meas} & Q_{\rm in} \to I_{\rm meas} & U_{\rm in} \to I_{\rm meas} & V_{\rm in} \to I_{\rm meas} \\ I_{\rm in} \to Q_{\rm meas} & Q_{\rm in} \to Q_{\rm meas} & U_{\rm in} \to Q_{\rm meas} & V_{\rm in} \to Q_{\rm meas} \\ I_{\rm in} \to U_{\rm meas} & Q_{\rm in} \to U_{\rm meas} & U_{\rm in} \to U_{\rm meas} & V_{\rm in} \to U_{\rm meas} \\ I_{\rm in} \to V_{\rm meas} & Q_{\rm in} \to V_{\rm meas} & U_{\rm in} \to V_{\rm meas} & V_{\rm in} \to V_{\rm meas} \end{pmatrix}$$

CoMP-S pol. calibration results

- What is good and what bad? The measure is:
  - normalized **(I_{meas} – I_{model})/I_{meas}** intensity differences
  - with a help of the obtained values of the physical parameters: linear polarizer transmission, circular retarder transmission & retardation Stokes_input vector, efficiencies (and other additional parameters)
  - Not the O matrix components…
- Results could be dependent on:
  - Position within the 2D field of view
  - Wavelength
  - other effects (e.g., mechanical structure bending, optical parts clearness)
- Results for 1 wavelength & 1 position in the FoV in this presentation

CoMP-S pol. calibration results

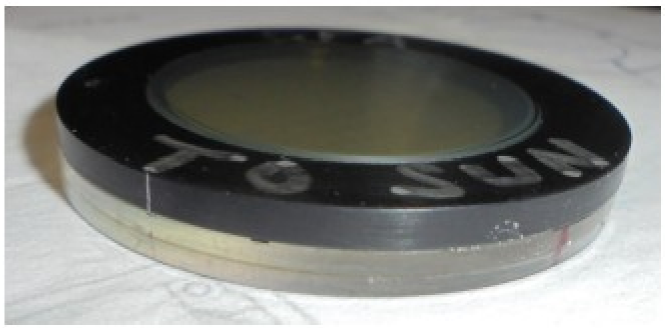
- Original CoMP-S setup & 656 nm: **$(I_{meas} – I_{model})/I_{mea}$**

0.00451119	0.000101250	5.81984e-06	-0.00473855
0.0213082	0.0573907	-0.0331494	-0.0306067
-0.0416938	-0.00697888	0.00598805	0.0341834
0.00717691	0.000137868	-0.00244441	-0.00689037
0.00280571	0.000276429	-0.00302491	-0.00267990
0.0914081	0.0443865	-0.0284551	-0.125710
0.141429	0.0337479	-0.0249009	-0.205270

- Something is wrong…
- Possible reason(s):
  - Reality: “wrong” optical parts ?
  - Model : insufficient description of the optical system ?
- Step by step checks/tests/analysis/re-considerations...

CoMP-S pol. calibration results

- Checks: calibration polarimetric optics: type, properties, orientation
  → lin. polarizer @ 0° + ¼ wave retarder @ +45°; lin. polarizer 0° + ¼ wave retarder @ -45°: orientation changes – unclear situation (really +/-45° ?)

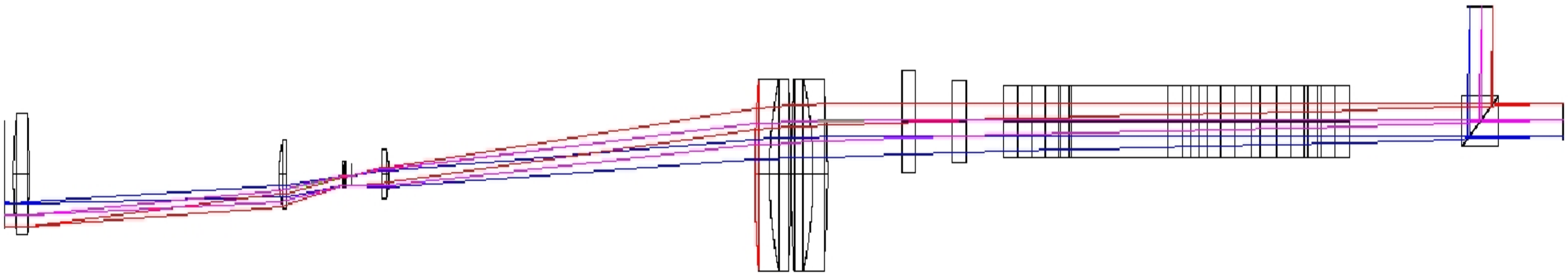


CoMP-S pol. calibration results

- Reconsiderations: CoMP-S “tertiary” optics - dichroich mirror @ 45° + polarizing beam splitting cube
- Tests: model apatation for these 2 optical elements:
  - The dichroic mirror MM analytical form (the free value of the refractive idex): *Collett, E., 1971, Am. J. Phys. 39, 517*
  - The polarizing beam splitter MM analytical form (birefringence and retardation – both linear and circular, depolarization, with few simplifications): *Song Zhang et al., 2018, J. Opt. 20, 125606*
- No significal inprovement of results $\mathbf{(I_{meas} – I_{model})/I_{meas}}$

CoMP-S pol. calibration results

- Reconsiderations: the ZEISS coronagraph “secondary” optics (SO)



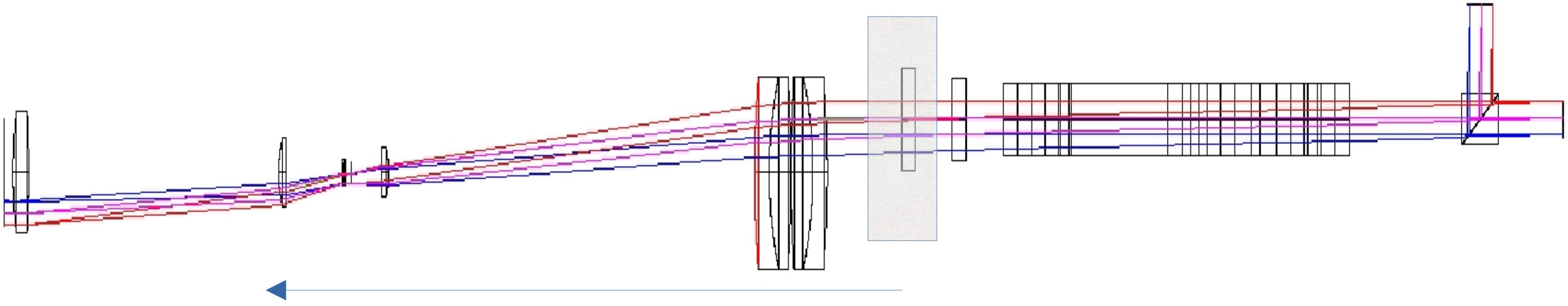
coronagraph secondary optics * CoMP-S instrument

field lens 3 lenses 4-lens objective cal.+ filt. wheels Lyot filter ter. optics

- Polarimetric effects of the single optical surface (without a symmetry of illumination): the Fresnel equations
- the quiet complex ZEISS coronagraph secondary optics changes the polarimetric status of the passing light

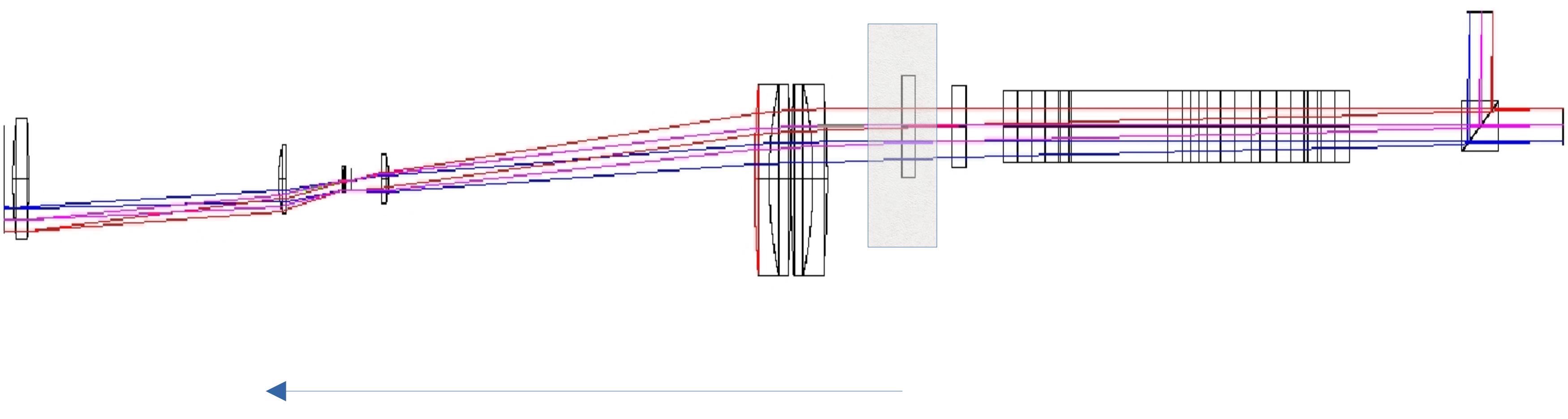
CoMP-S pol. calibration results

- **A conceptual problem of the instrument**: the complex ZEISS coronagraph secondary optics (SO) changes the polarimetric status of the passing light
- **So far:** an assumption of the completely unpolarized light [I,Q,U,V]=[1,0,0,0] entering the polarimetric calibration optics…
- **A realistic solution:** placing the polarimetric calibration optics in front of the SO, i. e., where the artificial moon is located !



CoMP-S pol. calibration results

- **The realistic solution:** placing the polarimetric calibration optics in front of the SO, i. e., where the artificial moon is located
- Assumption: the completely unpolarized light [I,Q,U,V]=[1,0,0,0] entering the polarimetric calibration optics is fulfilled
- Model has to be extended for the SO part – O_{SO} matrix (4x4):
  - observations: $O_{common} = O_{CoMP\text{-}S}$ # O_{SO}
  - pol. calibration measurements: $O_{common} = O_{CoMP\text{-}S}$ # O_{pol_cal} # O_{SO}



CoMP-S pol. calibration results

- Analytical problem: 28 measurements but 31 unknown variables to be derived (O_{SO} 4x3 + pol. optics 3 + Input 4 + $O_{CoMP\text{-}S}$ matrix 4x3)
- Numerical solutions:
  - a/ simplifications

    28 measurements but 31 unknown variables to be derived (O_{SO} 4x3 + pol. optics 3 + Input 4 + $O_{CoMP\text{-}S}$ matrix 4x3

    →

    28 measurements and at maximum 28 unknown variables to be derived (O_{SO} 4x3 + pol. optics 3 + Input 4 + $O_{CoMP\text{-}S}$ matrix at maximum 9 only)
  - b/ an iterative approach

CoMP-S pol. calibration results

- Preparation of a testing pol. calibration package #1:
  - Thorlabs lin. polarizer with variable orientation
  - lin. polarizer @ 0° + ¼ wave retarder @ +45°; lin. polarizer 0° + ¼ wave retarder @ -45° (HAO CPA/B parts)
  - in front of the SO – simplified approach - test data 2023/09/15
  - Results: $(I_{meas} - I_{model})/I_{meas}$ (optics: 0.67 & 0.84, 65.95)

-0.001	-0.002	0.002	0.001
-0.099	-0.008	-0.001	0.055
0.025	0.049	-0.035	-0.047
-0.006	0.008	0.019	-0.007
-0.004	0.011	0.014	-0.004
0.118	0.042	-0.012	-0.193
0.212	0.027	-0.009	-0.324

CoMP-S pol. calibration results

- Preparation of a testing pol. calibration package #2:
  - Thorlabs lin. polarizer with variable orientation
  - Thorlabs lin. polarizer @ 0° + ¼ wave retarder for 670 nm @ +/-45°
  - in front of SO – simplified approach - test data 2023/10/14
  - Results: $\mathbf{(I_{meas} - I_{model})/I_{meas}}$

-0.017	0.007	-0.005	0.016
0.025	-0.012	-0.017	-0.018
0.003	-0.072	-0.142	-0.006
0.165	0.039	-0.156	0.065
0.071	0.006	-0.077	-0.018
-0.056	0.016	-0.010	0.046
0.084	-0.041	0.035	-0.103

CoMP-S pol. calibration results

- Preparation of a testing pol. calibration package #2:
  - Thorlabs lin. polarizer with variable orientation
  - Thorlabs lin. polarizer @ 0° + ¼ wave retarder for 670 nm @ +/-45°
  - in front of SO – simplified approach - test data 2023/10/14
  - Results:
    - linear polarizes trans = 0.26
    - ¼ wave retarder trans/ret = 0.97/90.6
    - O matrices: $O_{CoMP\text{-}S}$ & O_{SO}

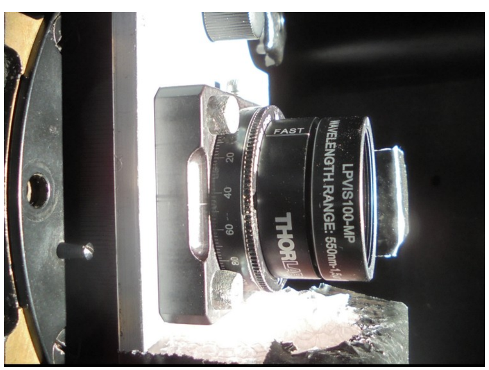
1.00	-0.58	-0.06	0.29
1.00	0.71	0.73	-0.02
1.00	0.72	0.49	0.08
1.00	-0.28	-0.06	0.34

1.00	0.53	0.10	0.10
1.00	-1.03	0.05	0.04
1.00	-2.69	1.00	0.10
1.00	-3.55	0.65	0.68

CoMP-S pol. calibration results

- A (preliminary) summary:

  It is needed to develop and fabricate an optimum achromatic pol. calibration package (500-1100 nm) which can be placed in front of the SO

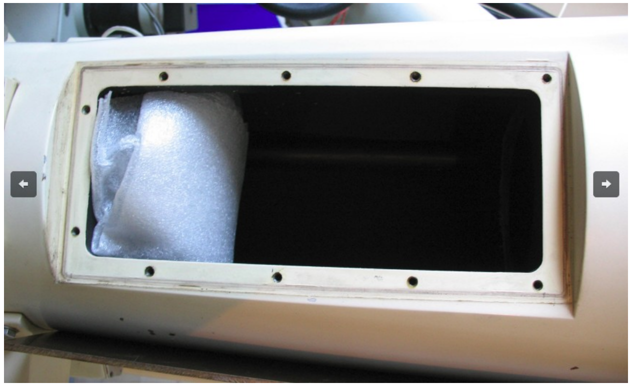


A new CoMP-S pol. package

- Requirements:
  - Placement in front the SO
  - Achromatic properties (500-1100 nm)
  - Versatile: in/out of the optical system allowing to exchange the artificial moon and to be placed to any location (position angle) around the solar limb
  - Computer controlled motorized operation
- Limitations:
  - Space & money
- Minimum content:
  - Achromatic linear polarizer - 4 orientations (0°, 90°, +45°, -45°)
  - Achromatic ¼ wave retarder - 2 orientations (+45°, -45°)

A new CoMP-S pol. package

- A preliminary concept:
  - Single linear polarizer rotatable to 4 orientations
  - Single achromatic ¼ wave retarder rotatable to 2 orientations
  - 2 rotatable platforms to place these pol. calibration optical parts to the required PA
  - Platform #1: 1 linear polarizer, #2: ¼ wave retarder + 1 hole
  - The whole package has to removable from the coronagraph
  - Computer controlled motorized operation


CoMP-S polarization plan

- The iterative approach to derive the $O_{CoMP\text{-}S}$ + O_{SO} matrices – JR (now in progress)
- The preliminary summary → the final decision – JR + the LSO group
- Polarimetric calibration and demodulation to the CoMP-S data pipeline - JR
- the new polarization package: project, realization, tests → regular usage – the LSO group

Who what?

- S. Tomczyk:
  - CoMP-S polarimeter design
  - model approach software: basics
  - some advice
- The LSO group:
  - discussions and suggestions: P. Schwartz, P. Gomory, ...
  - mechanical parts, help with handling optics: M. Trembac
  - operation of the LSO and its instruments CoMP-S + UJ2P + ...
- JR:
  - polarimetric test measurements
  - model approach software: extensions
  - data analysis → results
  - new pol. calibration package concept and the polarimetry plan

Thank you for your attention

CoMP-S pol. demodulation

- The solar spectroscopy gold standard since the year 2000: del Toro Iniesta and Collados, “Optimum modulation and demodulation matrices for solar polarimetry”, 2000, *Applied Optics* **39**, 1637

$$\mathbf{I}_{\mathrm{meas}} = \mathbf{O}\mathbf{I}_{\mathrm{in}} \quad \boldsymbol{I} \equiv (I, Q, U, V)^T$$

$$\mathbf{I}_{\mathrm{in}} = \mathbf{D}\mathbf{I}_{\mathrm{meas}}$$

$$\mathbf{D} = (\mathbf{O}^T\mathbf{O})^{-1}\mathbf{O}^T = \mathbf{A}^{-1}\mathbf{O}^T$$

CoMP-S polarization tests

- **List of the pol_cal (test) measurements:** the data analysis performed

Calibration optics type	Calibration optics location	Date	Position angle [°]
HAO CPA+CPB (CPA/B lin.pol. radially)	between SO and LF (calibration wheel)	(>27/04/2023) 2023/05/09 2023/05/10 2023/05/27	(~90) 105 105 105
HAO CPA+CPB (CPA/B lin.pol. radially)	in front of the SO (special holder)	(>19/07/2023) 2023/08/11 2023/09/06 **2023/09/15**	(90) 90 90 **90**
THORLABS lin_pol + wave/4	between SO and LF (calibration wheel)	-	-
THORLABS lin_pol + wave/4	In front of the SO (special holder)	**2023/10/14 1w** 2023/11/22 2w	**90** 90

LSO pipeline: done

- **HAO (ST) soft + LSO (JR) fitting to derive the pol_cal modulation:**
  - pol_cal calibration optics definition:

```
;pol=    [0,  1,  1,  1,  1,  1,  1] ;  pol determines whether polarizer is present
;ret=    [0,  0,  0,  0,  0,  1,  1] ;  ret determines whether retarder  is present
p_angle=[0., 0.,90., 45.,135.,  0.,  0.] ; pol angles
r_angle=[0., 0., 0.,  0.,  0., 45.,-45.] ; ret angles
```

  - Calculation of the calibration optics impact on the passing light:

```
if pol[i] eq 0 and ret[i] eq 0 then begin ; clear
   obs[*,i] = omx ## input
endif
if pol[i] eq 1 and ret[i] eq 0 then begin ; polarizer only
   obs[*,i] = omx ## mueller_partial_polarizer(ptrans,pp,p_angle[i]) ## input
endif
if pol(i) eq 1 and ret(i) eq 1 then begin ; retarder and polarizer
  obs[*,i] = omx ## mueller_retarder(rtrans,r_angle[i],delta) ## $
                    mueller_partial_polarizer(ptrans,pp,p_angle[i]) ## input
endif
```

LSO pipeline: done

- **Test data:** THORLABS lin.pol. + wave/4 @ in front of SO: 2023/10/14 & **model variants:** no adds, SO simplified, TO: PBS, TO: DM+PBS

```
no adds                        SO simplified                  TO: PBS                        TO:DM+PBS
(i_obs-model)/(i_obs)
 -0.045  0.023 -0.010  0.036  -0.034  0.010  0.001  0.022  -0.039  0.014  0.000  0.023  -0.036  0.014  0.001  0.022
  0.027 -0.010 -0.017 -0.016   0.082 -0.004  0.005 -0.003   0.085 -0.000  0.006  0.007   0.079 -0.003  0.004  0.006
  0.005 -0.061 -0.148 -0.007   0.018 -0.043  0.095  0.012   0.022 -0.019  0.113  0.021   0.019 -0.039  0.089  0.017
  0.174  0.037 -0.161  0.052   0.013  0.030 -0.068 -0.075   0.009  0.015 -0.076 -0.075   0.005  0.028 -0.071 -0.073
  0.061  0.011 -0.093 -0.007   0.017  0.034 -0.051 -0.031   0.009  0.039 -0.060 -0.029   0.014  0.030 -0.057 -0.029
 -0.035  0.010  0.002  0.025  -0.015 -0.026  0.030  0.009  -0.008 -0.032  0.028  0.008  -0.015 -0.024  0.029  0.010
  0.052 -0.034  0.021 -0.052   0.029 -0.040  0.034 -0.026   0.010 -0.030  0.032 -0.012   0.022 -0.039  0.031 -0.014
Pol.trans, Ret.trans Ret:
   0.53  0.97   90.62          0.57   0.86   81.94          0.57   0.86   81.56          0.57   0.86   81.78
Input Stokes Vector:
   1.26   0.04  -0.44  -0.82   1.23   0.22  -0.33  -0.52    1.25   0.23  -0.31  -0.48    1.36   0.25  -0.35  -0.53
O Matrix:
   1.00  -0.58  -0.07   0.28   1.04  -0.71  -0.10   0.25    1.01  -0.69  -0.12   0.23    0.93  -0.64  -0.09   0.22
   1.00   0.72   0.73  -0.01   0.87   0.56   0.60   0.00    0.84   0.54   0.57   0.01    0.78   0.50   0.54   0.00
   1.00   0.73   0.50   0.07   0.83   0.59   0.34   0.06    0.80   0.57   0.31   0.06    0.74   0.53   0.30   0.05
   1.00  -0.29  -0.05   0.38   1.00  -0.41  -0.09   0.38    0.97  -0.40  -0.11   0.37    0.89  -0.37  -0.09   0.34
Efficiencies:
   0.108  0.157  0.053  0.037  0.173  0.195  0.076  0.053   0.198  0.192  0.081  0.058   0.162  0.176  0.069  0.049
Efficiencies MAX:
  1.000  0.606  0.445  0.240   0.938  0.577  0.352  0.228   0.907  0.561  0.336  0.220   0.841  0.518  0.313  0.205
Chisq:     0.00108556          0.000362365                    0.000368211                    0.000357809
d_mx ## [1.,1.,1.,1.]
   1.00   0.00  -0.00  -0.00   1.14   0.23  -0.19  -0.17    1.16   0.24  -0.16  -0.11    1.25   0.25  -0.18  -0.13
I                             I                             I                             I
   1.01   0.01  -0.03  -0.03   1.15   0.24  -0.21  -0.19    1.16   0.25  -0.18  -0.13    1.26   0.27  -0.20  -0.15
Q                             Q                             Q                             Q
   0.06   1.33  -0.03  -0.05   0.25   1.37   0.02  -0.07    0.26   1.40   0.04  -0.05    0.28   1.52   0.04  -0.05
U                             U                             U                             U
  -0.67  -0.19   1.84   0.53  -0.43  -0.04   1.54   0.14   -0.38  -0.04   1.52   0.06   -0.47  -0.05   1.71   0.13
V                             V                             V                             V
  -0.31  -0.03   0.51   2.30  -0.07   0.01   0.11   1.69   -0.03   0.01   0.02   1.58   -0.07   0.01   0.10   1.82
o_mx ## input_stokes_vector
  1.040  0.979  1.016  0.961  1.029  0.993  1.005  0.974   1.034  0.988  1.006  0.973   1.031  0.988  1.005  0.974
```