Analysis of light curves of magnetic CP stars by advanced PCA methods

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Abstract: We present a new method of phenomenological modelling of light curves based on ideas of the principal component analysis that enables a realistic description of light variations of a variable object with a minimum of free parameters. The application of the method is demonstrated on the light curve analysis of magnetic CP stars HD 90044 and HD 125248.

One of the most frequent problem of the astrophysics of variable stars is the as simple as possible quantitative description of observed light curves of a variable star taken, as a rule, in several colours of various photometric systems. This task can be optimally solved by application of a **principal component analysis** (PCA) (e.g. Jollife 2002). The PCA is a multivariate procedure suitable for constructing linear transformation of variables in a way that new variables are uncorrelated with each other arranged in the order of decreasing variance. The PCA reduces the redundancy contained in data by finding a new set of orthonormal axes in the Q-space such that the projection of the data vector points is maximized. The projection on the remaining axes (components) maximizes the residual variances. As the PCA does not assume any physical model or a model of an expected behavior of the object studied, turns to be an effective tool for revealing hidden relationships between data, and reduces the dimension of the problem in an optimum way.

Light curves and improvement of the period of HD 125248

HD 125248 = HR 5353 = CS Vir (A9 SrEuCr type) is a notorious magnetic, spectroscopic and photometric variable and the first star for which the oblique rotator model was proposed to describe its observed periodic variations (Stibbs 1950).

The list of photometric observations of this cool magnetic CP star is rich: instrumental colour (blue region) - Stibbs (1950); Maitzen & Rakosch (1970), Maitzen & Moffat (1972) - 11-colour photometry; Wolff & Wolff (1971) and Pyper & Adelman (1985) - *uvby*, Catalano et al. (1992) *uvbyJHK*; ESA (1997) - *Hp*.

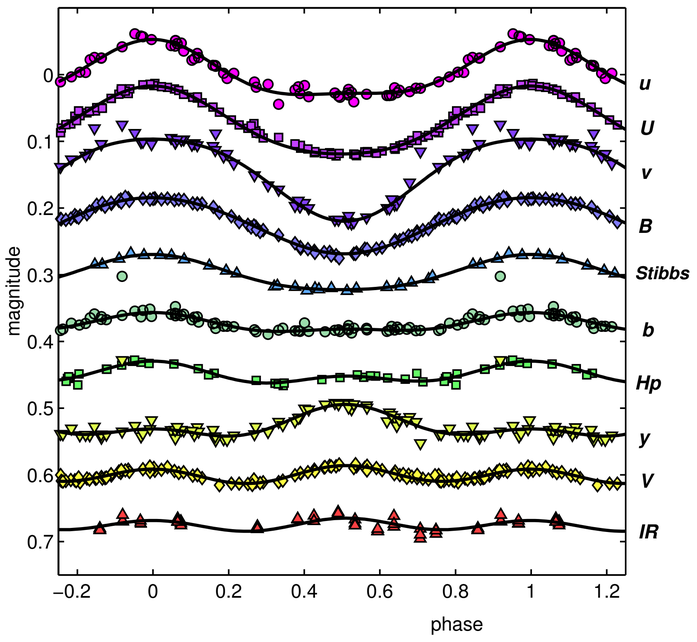

Fig. 1: Light variations of HD 125248 in various colours plotted versus phase according to the ephemeris of Leone & Catanzaro (2001). The IR colour contains observations in J, H and K of the enlarged Johnson photometric system, and Stibbs (1950) observations (effective wavelength 455 nm).

A majority of these measurements is depicted on Fig. 1, where the phase is calculated using the ephemeris: $M_0 = 24331103.95$, $P = 9.295545$ d (Leone & Catanzaro 2001) determined on the basis of Stibbs', B and v data. Using the PCA we concluded that the observed light variations in all colours can be suitably well represented by linear combination of two basic mutually orthogonal normalized light curves $F_1(\varphi), F_2(\varphi)$ displayed on Fig. 2a.

$$m(\varphi, c) = \overline{m(c)} + A_1(c)F_1(\varphi) + A_2(c)F_2(\varphi). \qquad (1)$$

The basic function $F_1(\varphi)$ is described by 3 parameters (the function is normalized), the function $F_2(\varphi)$ is described by 2 parameters (the function is normalized and orthogonal to $F_1(\varphi)$), having thus altogether 5 parameters.

The dependence of both the semiamplitudes of HD 125248 on the wavelength is represented by a more or less smooth function what allows a plausible prediction of variation in any colour in the optical region. Note the striking dissimilarity between light curves in V and y, and the apparent resemblance between infrared and V light curves already noticed by Catalano et al. (1992). The PCA approach remarkably decreases the number of parameters necessary to the description of the set of the light curves. While using the standard approach with Fourier decomposition up to the first harmonic we need 81 free parameters, using the PCA we only need 50 ones. This saving comes in useful e.g. when one wants to improve the ephemeris of the star.

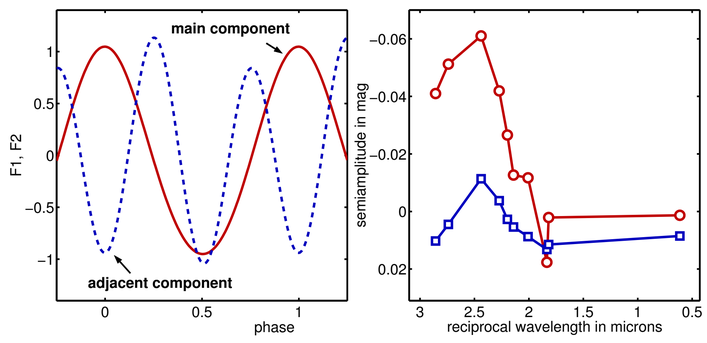

Fig. 2: (**a**) Phase diagram of the main and adjacent basic function for HD 125248. (**b**) Dependence of its semiamplitudes on the wavelength. Uncertainties of particular points are smaller than the dimension of their markers.

The spectroscopic variability of this star was established by Deutsch (1947) as equal to 9.295 d. The period was identified with the rotational period and improved step by step on the basis of various type of variation, including the photometric ones. Babcock (1958), from his magnetic measurements, inferred a value 9.2954 days, Renson (1975), on the basis of light variation, found a value 9.29541(7) d. Blanco et al. (1978), from their own UBV photometry, elicited a somewhat controversy value - 9.29477(5) d as well as Catalano et al. (1992) - 9.29571(18) d, or Hipparcos team 9.2870 d (!). The period by Leone & Catanzaro (2001) - 9.29545 d has already been mentioned above.

We attempted to improve the period on the basis of all available photometric data representing 592 photometric observations of HD 125248 covering a time interval of 43 years (1710 revolutions). The data were obtained by 7 authors observing in 6 photometric systems and 13 colours. As the shapes of the light curves in different colours are dissimilar (see Fig. 1) it is not possible to transform measurements done in various colours to one scale straightforwardly. Assuming every light curve can be expressed as a linear combination of two basic functions (Eq. 1) we arrived at the following result:

$$JD(\mathrm{MMC}) = 2440698.313(12) + 9.295450(30) \cdot (E - 817). \qquad (2)$$

Anyhow the new ephemeris resembles to the Leone & Catanzaro's (2001) one, its mathematical background is explicitly defined, the results are supplied with their uncertainties. The zero phase was set at the moment of the maximum of the main principal component, the epochs are counted analogically as in Leone & Catanzaro (2001). All the data used were weighted according to their uncertainties and the program employs robust procedures which are able to eliminate the influence of outliers (Mikulášek et al. 2003). The solution is an iterative process which converges relatively rapidly.

Light curves and improvement of the period of HD 90044

HD 90044 = HR 4082 = 25 Sex = SS Sex, was classified as the B9p Si(Sr,Cr) type CP star by Cowley et al. (1969). The spectrum variability of the star was discovered by Bonsack (1974), who found variations of SrII and CaII lines. The light variability of HD 90044 was studied by Manfroid & Renson (1983) who found the period to be 4.37(4) d. In the period 1982-88 the star was observed in *uvby* in the frame of *Long-Term Photometry of Variable at ESO* (Manfroid et al. 1991, Sterken et al. 1993). Renson & Manfroid (1991) from them and several own observations derived the following ephemeris: $M_0(\mathrm{min}) = 2445659.00$; $P = 4.37900(4)$ d.

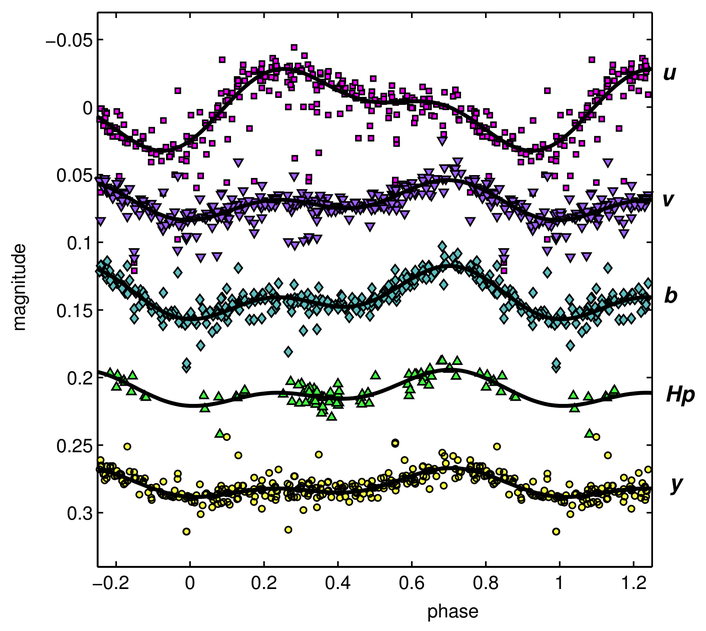

Fig. 3: Light variations of HD 90044 in *uvby* and *Hp* colours.

The ephemeris was adopted by Adelman (1997) who added his own 109 *uvby* measurements obtained in 1992-95. Catalano & Leone (1993) enlarged the number of observations by 12 *uvby* observations and established the value: $P = 4.37894(15)$ days. The star was observed by Hipparcos (ESA 1997), too (83 measurements in *Hp*). Catalano et al. (1998) revealed single wave light variations in the near infrared.

Our ephemeris based on the 1091 individual measurements in 5 colours covering the period of 12 years (= 1034 cycles) was found by our method under suggestion of the validity of Eq. 1. The beginning was set at the phase of the main principal function ($F_1(\varphi)$) minimum.

$$JD(\text{main comp. min.}) = 2448006.454(8) + 4.378967(22) \cdot E. \qquad (3)$$

The light minimum in u colour occurs at the phase -0.0076, in v at -0.006, in b at 0.005, in *Hp* at 0.010 and in y at 0.019 (Fig. 3). The weighted standard deviation of the fit is 0.0054 mag, the worst data with a huge fraction of outliers are in the u colour.

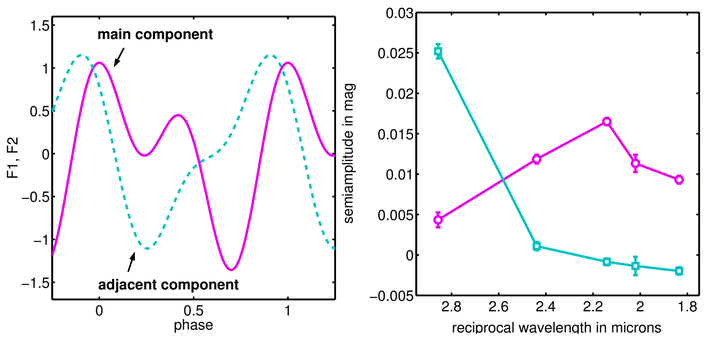

Fig. 4: (**a**) Phase diagram of the main and adjacent basic function for HD 90044. (**b**) Dependence of its semiamplitudes on the wavelength.

Conclusions

The method based on the PCA approach is a very efficient tool of the data processing; but one should keep in mind the method is not able to substitute a real physical or geometrical modelling of the variability of a particular variable object. Nevertheless, for its simplicity and consistency it should become an unavoidable tool for those dealing with the classification and diagnostics of variable stars. The whole method in details will be described in next papers.

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