ON THE SPREAD FUNCTION FOR SOLAR STRAY LIGHT

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Abstract: Different types of mathematical expressions for the spread function are given. The normalization of the spread function is discussed. A comparison between two different analytical integrations and one numerical integration, giving the observed intensity is shown with a Gaussian spread function (b=2 min of arc). Typical aureoles at 3 different sites are shown.

In all papers dealing with solar stray light the spread function $\psi(r)$ is assumed to be symmetrical. Following Zwaan (1965), the spread function is usually split in two terms. The observed intensity $I'(d, \alpha)$ is then given by

$$I'(d, \alpha) = \iint_{-\infty}^{+\infty} I(\xi, \eta)\, \psi(r)\, \mathrm{d}\xi\, \mathrm{d}\eta = \\ = A'(d, \alpha) + B'(d, \alpha)\,. \quad (1)$$

When calculating the term $A'(d, \alpha)$, caused by the core of the spread function, the usual procedure has been to use Gaussians:

$$\psi(r) = (\pi b^2)^{-1} \exp(-r^2/b^2)\,. \quad (2)$$

The number of Gaussians have varied between one and four, but at least two are necessary to give the observed intensity distribution.

Usually a Lorentzian of the form

$$\psi(r) = A/(B^2 + r^2) \quad (3)$$

has been used when calculating the term $B'(d, \alpha)$, caused by the wing of the spread function. Brahde (1973) has also used the function

$$\psi(r) = A/(B^{2q} + r^{2q})\,. \quad (4)$$

Stepanov (1957) and Mattig (1971) used Gaussians for both the core and the wing of the spread function.

It is convenient to normalize the spread function so that

$$\int_0^\infty \psi(r)\, r\, \mathrm{d}r = 1\,. \quad (5)$$

When using Gaussians, there are no problems, because this normalization is correct with the limits shown in Eq. (5). But with Lorenzians it is not possible to use an infinite limit when determining the normalization constant A. Zwaan (1965) solved the problem by placing the sun in the zenith and integrating to the horizon. Brahde (1972) also integrated to the horizon but he kept the sun in its real place.

These two methods yield small differences in the value for A as long as the sun is not near the horizon. Hansen (1973) has shown that the discussion as to which method gives the best result, is unnecessary. A normalization is needed but the way to do it may be chosen freely when observing relative intensities. He suggested that one normalizes to render the observed intensity $I'(d=0)$ at the centre of the solar disk equal to 1. When using Zwaan's and Brahde's normalizations it is usual to put the true intensity $I(0)$ equal to 1.

In solving the integral, giving the computed intensities, analytically, there are two possible places for the origin. The limb and the centre of the disk are chosen in computations dealing with the core and the wing of the spread function, respectively. With numerical integration the origin is normally at the point for which one wants to get the intensity.

Two approximations for the limb darkening have been shown to be very useful in obtaining analytical expressions for the observed intensity (Staveland, 1972):

$$I(\varrho) = 1 - \alpha\,(\varrho/R_\odot)^2 - \beta\,(\varrho/R_\odot)^4 - \gamma\,(\varrho/R_\odot)^6 \quad (6)$$

$$I(x) = C + Dx^a \quad (7)$$

where $x = R_\odot - d$.

Table 1 shows a comparison between the three types of integrations with a Gaussian spread function. The parameter b, determining the width of the spread function, is 2 min of arc. The differences between the values are rather small. One may also take into account that a Gaussian with such a large value of b will have a contribution of the order 10^{-2}. The accuracies of the computed intensities $ID\,(d)$ and $Ix\,(d)$ are increased if the spread parameter b is larger than or less than 2 min of arc, respectively.

Three typical aureoles at different sites and with different instruments are shown in Figure 1. The observations at Izaña, the Canary Islands and at Kitt Peak, U.S.A., were made by Wöhl (1972).

Table. 1. Comparison between different integrations giving the observed relative intensity with a Gaussian spread parameter b=2 arc min for the wavelength λ 5790. $IB(d)$ is computed with numerical integration (Brahde, 1972). $ID(d)$ is omputed with an analytical expression, derived with the origin in the centre of the disk and using Eq. (6) for the Limb darkening (Staveland, 1973). $Ix(d)$ is computed with an analytical expression, derived with the origin at the limb and using Eq. (7) for the limb darkening (Staveland, 1972).

Distance from solar limb (min of arc)	Computed relative intensity		
	$IB(d)$	$ID(d)$	$Ix(d)$
4.00	0.800	0.800	0.805
2.00	0.637	0.637	0.649
1.00	0.478	0.483	0.494
0.50	0.389	0.390	0.397
0.25	0.342	0.341	0.346
0.00	0.288	0.294	0.297
−0.25	0.242	0.248	0.249
−0.50	0.199	0.205	0.204
−1.00	0.127	0.132	0.128
−2.00	0.039	0.041	0.037
−4.00	0.001	0.001	0.001

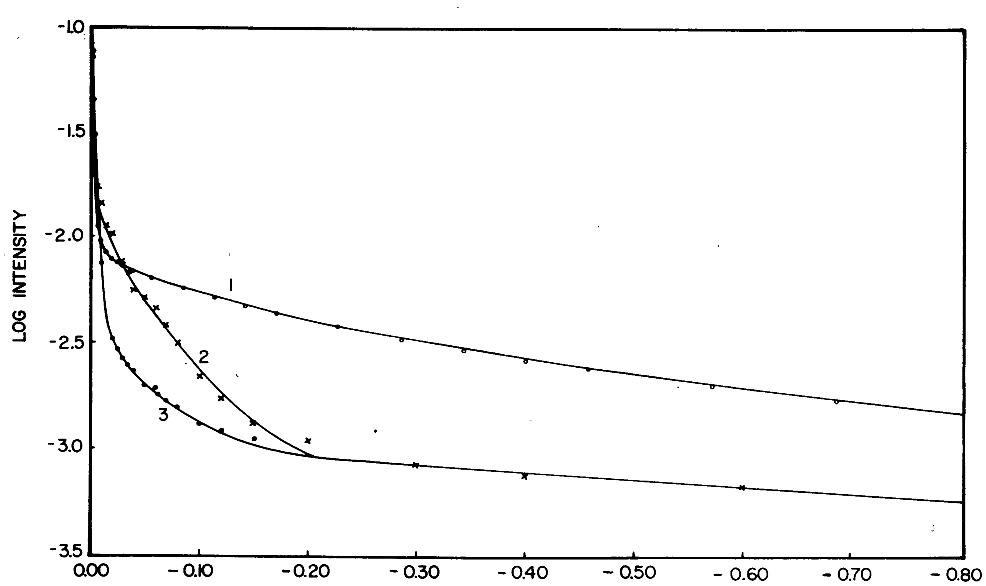

Fig. 1. Solar aureoles.
1. Oslo Solar Observatory, Harestua, Nov. 29, 1972, λ 5790. 2. Observatorio del Teide, Izaña, Aug. 8, 1972, λ 5500. 3. Kitt Peak National Observatory, May 16, 1972, λ 5287.

The spread functions to give these aureoles are given in Table 2. To describe the aureole from 1′ to 3′ at Izaña it was necessary to use a rather broad Gaussian and, therefore, a total of 5 Gaussians were used in the computer program. In this work the contributions from the Gaussians and from the Lorentzian are given by

$$\sum_i m_i + \varepsilon = 1\,. \quad (8)$$

The accuracy with which it is possible to deter-

Table 2. The parameters of the spread functions giving the observed aureoles at 3 sites

Site			Oslo		Izaña		Kitt Peak	
	b_1	m_1	0.7	0.160	0.7	0.055	0.7	0.164
Gaussians	b_2	m_2	3.3	0.735	3.4	0.842	2.5	0.238
(sec of arc)	b_3	m_3	10.2	0.024		0	6.6	0.529
	b_4	m_4	30	0.008	27	0.036	21	0.021
	b_5	m_5	132	0.005	107	0.028	107	0.008
Lorentzian			4.0		20.0		20.0	
(min of arc)			0.068		0.038		0.039	
Wavelength (Å)			5790		5500		5287	

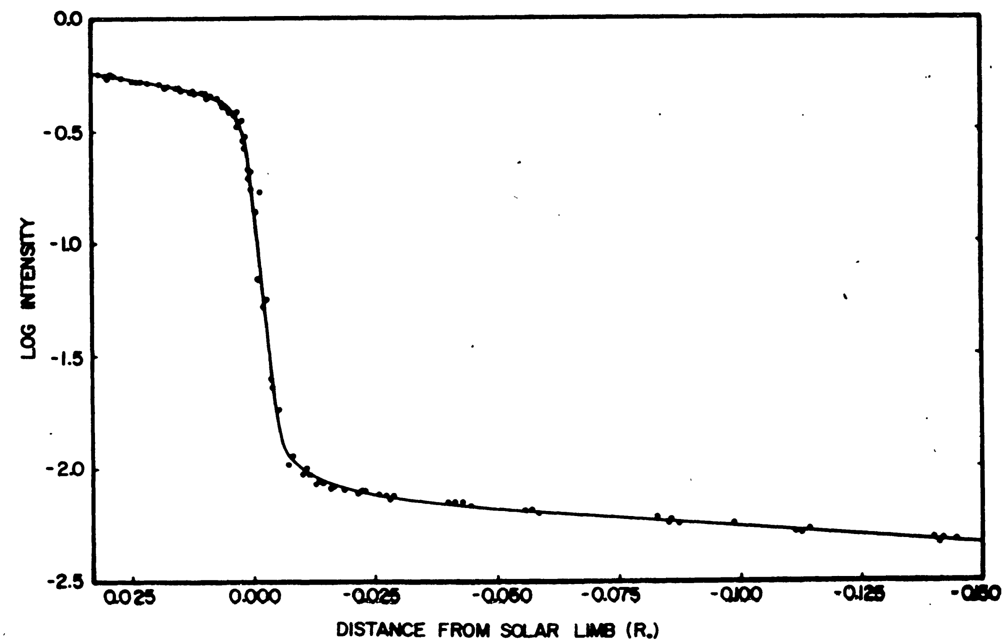

Fig. 2. Solar limb profile at Oslo Solar Observatory. The same observations as in Figure 1.

mine the core of the spread function depends strongly on the uncertainty in placing the limb. To get a good result with an uncertainty equal to or less than 0.5 sec of arc it is necessary to observe the limb profile both inside and outside the limb as shown in Figure 2.

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