

MULTIDIMENSIONAL RADIATIVE TRANSFER IN ULTRAVIOLET RESONANCE LINES OF THE CHROMOSPHERIC FLASH SPECTRUM

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Abstract: The properties of the synthetic and analytic approaches to the study of astrophysical objects are briefly discussed. The general equation of radiative transfer and the general source function for two-level atom radiation are then utilized to describe resonance-line formation of the chromospheric flash

spectrum. An approximation of the local planeparallel layer is introduced to yield the solution of these complicated equations. Finally, the expression for the emergent intensity of radiation in the line is derived.

Introduction

Two general approaches can be used to determine the structure of the given astrophysical plasma from its emitted spectrum. One approach is to infer certain physical parameters by a direct inversion of the observed data; the other is to compute a theoretical spectrum from an assumed model of the object and adjust the model until the computed and observed data agree. The first, so-called *analytical approach*, which, in the case of planetaries and the solar corona, was studied by Hekela (1972a,b) and co-workers, can be extremely useful, but it is very elaborate and is subject to some limitations and mathematical uncertainties. The second one — *synthetic* — is more generally applicable, but it is not straightforward and, furthermore, it is not unambiguous. This means that there always exists an infinite number of ways of obtaining correspondence between computed and observed data by combining different actual initial quantities.

It is easy to see that the analytical approach is more efficient and applicable in getting new information about the physical state of the given object, but it is not possible to develop the analytical approach without a detailed knowledge of the solution (mainly the asymptotic behaviour of the solution) of the synthetic one. From this point of view, it is important to formulate firstly the synthetic and then the analytic approach for those astrophysical objects, for which the formulation of both approaches is as simple as possible. From the

point of view of the direct inversion of the observed data, so-called *extended sources*, e.g., planetary and diffuse nebulae, cometary atmospheres and, particularly, the solar chromosphere and corona are most suitable.

Statement of the Problem

The purpose of this paper is to derive equations for describing the resonance lines in the chromospheric flash spectrum in the synthetic approach formulation. We have not made any special assumptions about velocity fields and the atomic abundance distribution. In other words, we also allow horizontal variations of the model parameters in the chromosphere.

A complementary comment can be made now. There are two different possibilities in the synthetic approach formulation. The first is the exact theory of two-level atom radiation without simplified assumptions about geometry, homogeneity and velocity fields of the given medium. The second possibility is the theory, in which more realistic assumptions about the medium, containing the atoms, are made, but with restrictions as to the shape of the medium and other limiting assumptions about the homogeneity, velocity fields, etc.

We shall now consider the resonance lines in the chromospheric flash spectrum. The presented theory belongs to the first type of synthetic approaches, mentioned above. We adopt the simplest atomic model — the two-level atom with no

continuum. This is possible, because the radiation in some resonance lines is suitably expressed as two-level atom radiation.

Geometrically, the chromosphere here means the envelope bounded by two concentric spheres with radii R (solar radius) and $R + L$, where L is the thickness of the chromosphere.

General Equations

The equation of radiative transfer for the spectral line may be written in the form

$$\begin{aligned} & \frac{d}{ds} I(\nu, \vec{n}, \vec{r}) = \\ & = -k(\nu, \vec{n}, \vec{r}) [I(\nu, \vec{n}, \vec{r}) - S(\nu, \vec{n}, \vec{r})], \quad (1) \end{aligned}$$

where $I(\nu, \vec{n}, \vec{r})$ is the intensity of radiation of frequency ν at some position, \vec{r} , \vec{n} is the direction of the propagation of the photon, ds is the path element in the direction \vec{n} , $k(\nu, \vec{n}, \vec{r})$ is the absorption coefficient and $S(\nu, \vec{n}, \vec{r})$ the source function.

The absorption coefficient for the two-level atom is

$$\begin{aligned} & k(\nu, \vec{n}, \vec{r}) = \\ & = \frac{h\nu_0}{4\pi} \phi(\nu, \vec{n}, \vec{r}) [n_L(\vec{r}) B_{LU} - n_U(\vec{r}) B_{UL}], \quad (2) \end{aligned}$$

where ν_0 is the frequency of line centre, n_L and n_U the population densities of the lower and upper levels, $\phi(\nu, \vec{n}, \vec{r})$ is the normalized profile of the absorption coefficient and the B 's are the Einstein transition probabilities.

The formal solution of the transfer equation may be written in the form

$$\begin{aligned} & I(\nu, \vec{n}, \vec{r}^0 + s\vec{n}) = \int_a^{\vec{r}} S(\nu, \vec{n}, \vec{r}^0 + \\ & + s\vec{n}) e^{-\tau(\nu, \vec{n}, \vec{r}^0, s, s')} k(\nu, \vec{n}, \vec{r}^0 + s'\vec{n}) ds', \quad (3) \end{aligned}$$

where

$$\vec{r}^0 = \vec{r} - (\vec{r} \cdot \vec{n}) \vec{n} \quad (4)$$

and

$$\tau(\nu, \vec{n}, \vec{r}^0, s, s') = \int_{s'}^s k(\nu, \vec{n}, \vec{r}^0 + s''\vec{n}) ds'' \quad (5)$$

and the lower limit of integration a depends upon the concrete kind of medium studied.

Equation (3) describes the radiation passing along a given straight line, defined parametrically by the parameter s , direction \vec{n} and some point \vec{r}^0 in a tridimensional space.

The source function follows from the solution of the equation of statistical equilibrium (see, e.g., Jefferies, 1968) for a two-level atom. We have

$$\begin{aligned} & S(\nu\vec{n}, \vec{r}) = \\ & = [1 - \varepsilon(\vec{r})] \frac{4\pi}{\phi(\nu, \vec{n}, \vec{r})} \int_0^\infty \int_{4\pi} R(\nu', \vec{n}'; \nu, \vec{n}; \vec{r}) \cdot \\ & \cdot I(\nu', \vec{n}', \vec{r}) d\nu' \cdot d\Omega' + \varepsilon(\vec{r}) B(\vec{r}), \quad (6) \end{aligned}$$

where $R(\nu', \vec{n}'; \nu, \vec{n}; \vec{r})$ is the redistribution function (Hummer 1962, 1969) and ε and B are well-known quantities in the radiative transfer theory (Jefferies, 1968; Hummer, 1969).

A more detailed specification of the redistribution function depends on the kind of material, velocity fields and upon the assumptions about atom mechanism, involved in the scattering of the photon. A detailed discussion of this problem can be found for example in Hummer (1962, 1969).

The radiation field at any given point can be found by solving the system of equations (3) and (6) for every straight line in the given medium. Unfortunately, this problem is still practically unsolvable. Therefore, it is necessary to utilize the physical properties of the given astrophysical object in order to simplify the equations mentioned above.

Application to the Chromospheric Flash Spectrum

Let us introduce coordinates l and z in given plane, described by the position angle p (Fig. 1). We assume that only the hatched area affects the radiation passing along the straight line α (pointing at the observer). As the chromosphere is a very thin layer with respect to the solar radius ($L \ll R$), we can simplify the solution of equations (3) and (6) using the so-called local planeparallel layer. This means that at every point on the line of sight α we construct a normal to the chromospheric boundaries and between these two areas we assume a planeparallel layer. The intensity of radiation and the source function can be found by well-known methods (Hummer, 1969).

With the aid of the procedure mentioned above we can obtain a "planeparallel" source function and an intensity depending on the depth. Following easy geometrical considerations, it is possible to transform these quantities to values along the line of sight α .

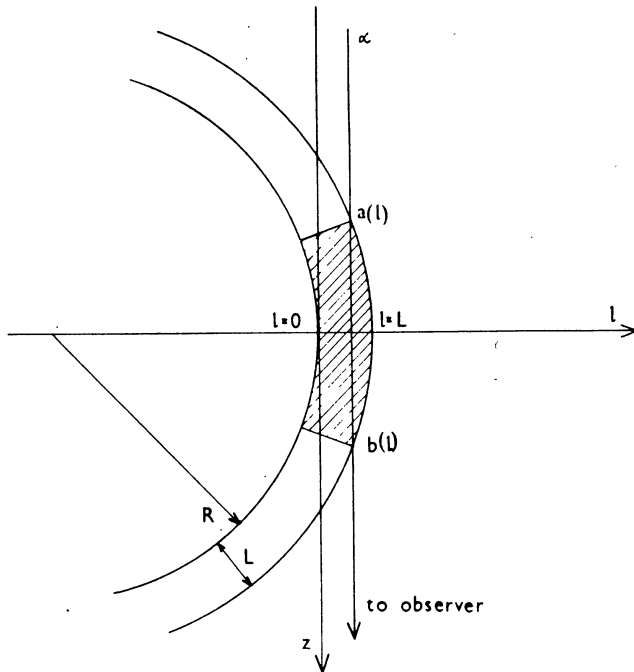


Fig. 1. Definition of the coordinates for the used approximation of the model of the solar chromosphere.

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From equations (3) and (6) we obtain

$$S(\nu, \vec{n}, \vec{r}) = [1 - \varepsilon(\vec{r})] \frac{4\pi}{\phi(\nu, \vec{n}, \vec{r})} \int_0^{\infty} \int_{4\pi} R(\nu', \vec{n}'; \nu, \vec{n}; \vec{r}) \cdot \int_a^{\vec{r}, \vec{n}} S(\nu, \vec{n}', \vec{r}^0 + s' \vec{n}') e^{-\tau(\nu', \vec{n}', \vec{r}^0, \vec{r}, \vec{n}, s')} \cdot k(\nu', \vec{n}', \vec{r}^0 + s' \vec{n}') ds' d\Omega' d\nu' + \varepsilon(\vec{r}) B(\vec{r}), \quad (7)$$

where in $S \vec{n}$ is taken as the direction to the observer (usually we choose the z -axis, $\vec{n} \equiv (0, 0, 1)$) and for S in the right-hand side values, obtained transforming the evaluated "local planeparallel" values.

If we introduce

$$\varphi(\nu, e, z, p) = S(\nu, \vec{n}, \vec{r}) \left| \begin{array}{l} \vec{n} = (0, 0, 1) \\ \vec{r} \rightarrow (e, z, p) \end{array} \right. \quad (8)$$

and similiary I, k, t , then

$$I(\nu, e, z, p) = \int_{a(e)}^z \varphi(\nu, e, z', p) e^{-\tau(\nu, a(e), z')} k(\nu, e, z', p) dz', \quad (9)$$

where $a(e)$ is the lower limit of integration over the line of sight (see Fig. 1). The emerging profile of the line, in terms of local absolute monochromatic energies (LAME — defined by Hekela, 1972a, b), is then given in the form

$$H_\nu(e, p) = \frac{\Delta\nu}{d^2} \int_{a(e)}^{b(e)} \varphi(\nu, e, z', p) e^{-\tau(\nu, a(e), z')} k(\nu, e, z', p) dz', \quad (10)$$

where $\Delta\nu$ is the measured frequency interval, d the distance to the observer, and $b(e)$ is the lower limit of integration over the line of sight α .

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