AUTOCORRELATION ANALYSIS OF SOME SOLAR INDICES

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Abstract: Two solar indices have been used for the autocorrelation analysis. The first one was the daily sunspot area for the period 1883—1959, the second one the daily flare index *Q* for the period 1936—1949. In both cases data from the whole disc, as well as those from the northern and southern hemispheres have been investigated separately. The autocorrelation function gives information about the recurrence of the solar activity centres. Special attention has been given to the 27 day period of the solar rotation. The subsidiary peaks on the high-resolution power spectra near the 27-day period indicate a family of more rotational systems with different periods. Striking are three periods on the whole disc power spectrum of the flare index *Q*, viz. those of 25.3, 27.4 and 28.8 days. The occurrence of these subsidiary peaks can explain the origin of the similar structure of the power spectra of the geomagnetic disturbance indices, found by Shapiro and Ward (1966).

Introduction

Autocorrelation analysis can be used to an advantage for investigating the periodical variations of time series, particularly long ones. The indices of solar activity, derived from the observations of various solar activity phenomena, also form series of this type. The daily values of sunspot areas were used for the analysis, covering the period 1883—1959; the data were adopted from the Greenwich Heliographic Results. The sunspot areas for the northern and southern hemispheres as well as for the whole solar disc were investigated separately. The other index, the variation of which was investigated, is the daily flare index *Q*, the product of the importance and duration of the flare, published by Kleczek (1952) for the period 1936—1949. The data for the northern and southern hemispheres, as well as that for the whole disc were again investigated separately.

Analysis of the Sunspot Area

The sample contains 28,123 daily values of the sunspot area, covering the period from January 1, 1883 to December 31, 1959. Observations are lacking for a few days in the first years of observations in the last century. In these cases the data were interpolated and the gaps filled in. The large number of values in the sample allowed for the computation of the autocorrelation function, truncated at 1000 days, and also of a power spectrum with a high resolution. The resolution for the maximum lag of 1000 used is 0.73 days at a period of about 27 days. The computer program was only limited to 1000 lags by the capacity of the MINSK 22 computer memory.

Figure 1 represents the autocorrelation function for the whole disc with a maximum lag of $M = 1000$ days. The autocorrelation is positive throughout and its value decreases approximately exponentially. More of less marked oscillations, among which oscillations with periods of about 27 days may be found, are superimposed on this fundamental pattern. For the sake of clarity, arrows above the curve in the graph indicate multiples of the 27-day period. The autocorrelation function truncated at 1000 days, includes roughly 37 recurrences.

The regular damped oscillations, which appear at the beginning of the autocorrelation function and end with the 6th recurrence of the 27-day period, may be attributed to the recurrent groups in the first place. The autocorrelation function for the northern and southern hemispheres yield the same pattern as the function for the whole disc. Parts of these functions, truncated at 200 days, are shown in Figure 2. The oscillations for the northern hemisphere end at the 5th recurrence and for the southern hemisphere at the 6th recurrence.

In order to find out whether there is a relation between the amplitude of the oscillations and the number of the recurrent groups, use was made of the number of recurrent groups, determined from the Greenwich Heliographic Results by Eigenson (1940), Ringes (1964) and Mandrikina (1969). The period 1883—1923 was adopted from Eigenson, 1924—1953 from Ringnes, and 1954—1959 from Mandrikina.

If the amplitude of the oscillations is denoted by A_i and the number of recurrent groups by N_i for the recurrence i, the relative amplitude $a_i = A_i/\Sigma A_i$ and the relative number of recurrent groups $n_i = N_i/\Sigma N_i$. Log a_i vs. log n_i for the whole disc, as well as for both the hemispheres separately is shown in Figure 3 and the patterns are very similar in all three cases. This relation can be approximated by

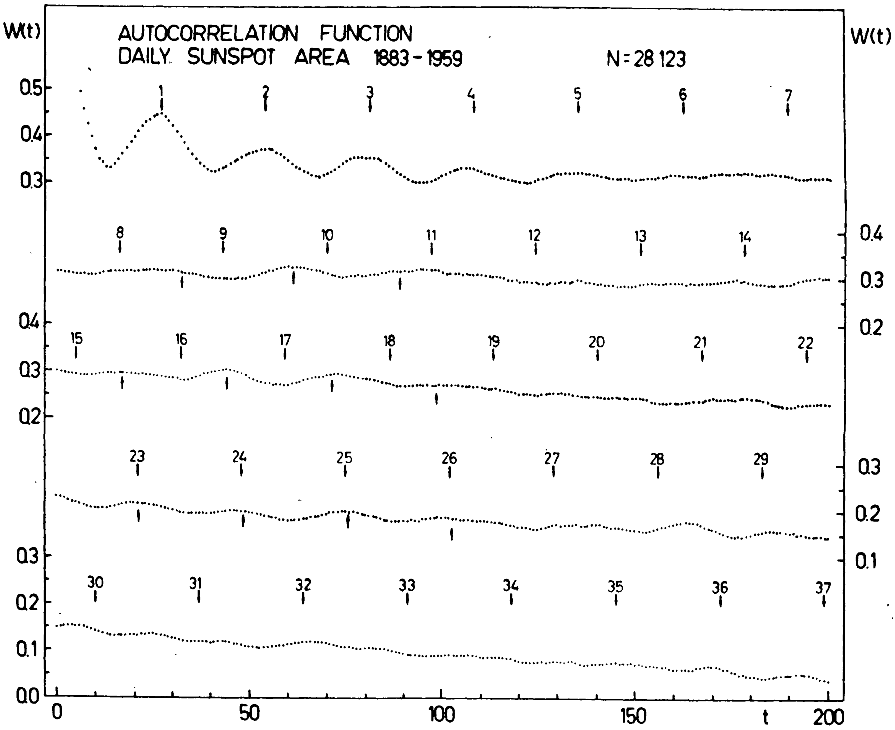

Fig. 1.

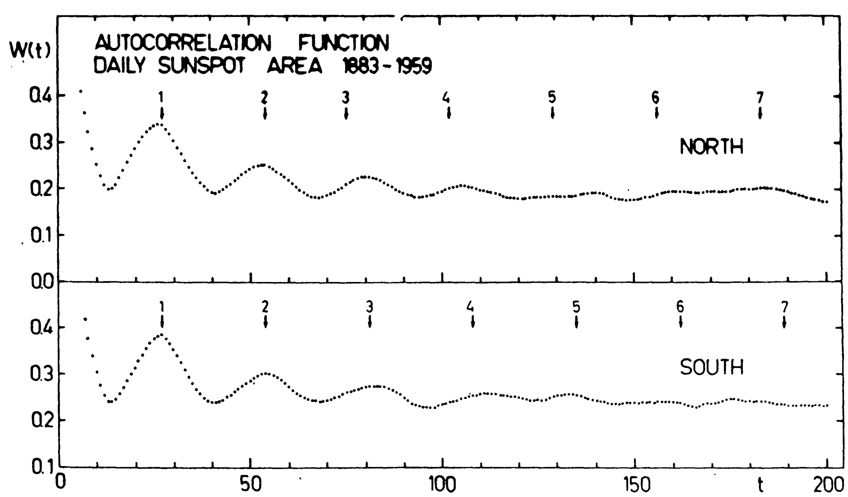

Fig. 2.

$$a_i = 0.56 n_i^{0.48}. \qquad (1)$$

The square of the amplitude is approximately equal to the number of recurrent groups. The similarity of the results for both hemispheres also hemispheres, because it does not display an NS asymmetry regardless of the NS asymmetry in the number of sunspot groups.

Further on the autocorrelation function pattern displays less markedly regular oscillations with

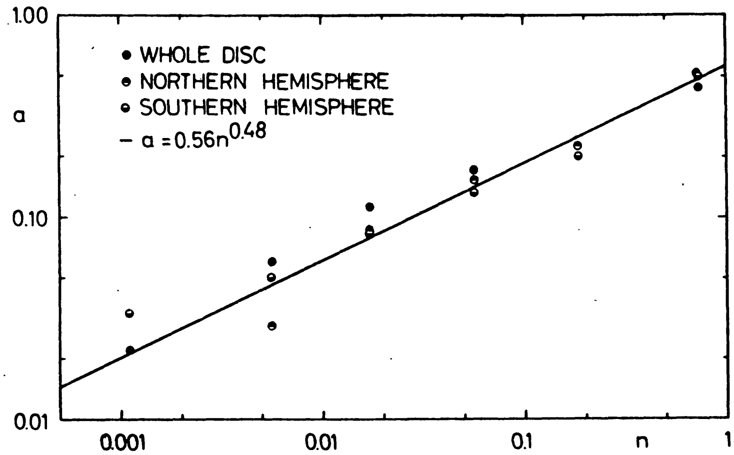

Fig. 3.

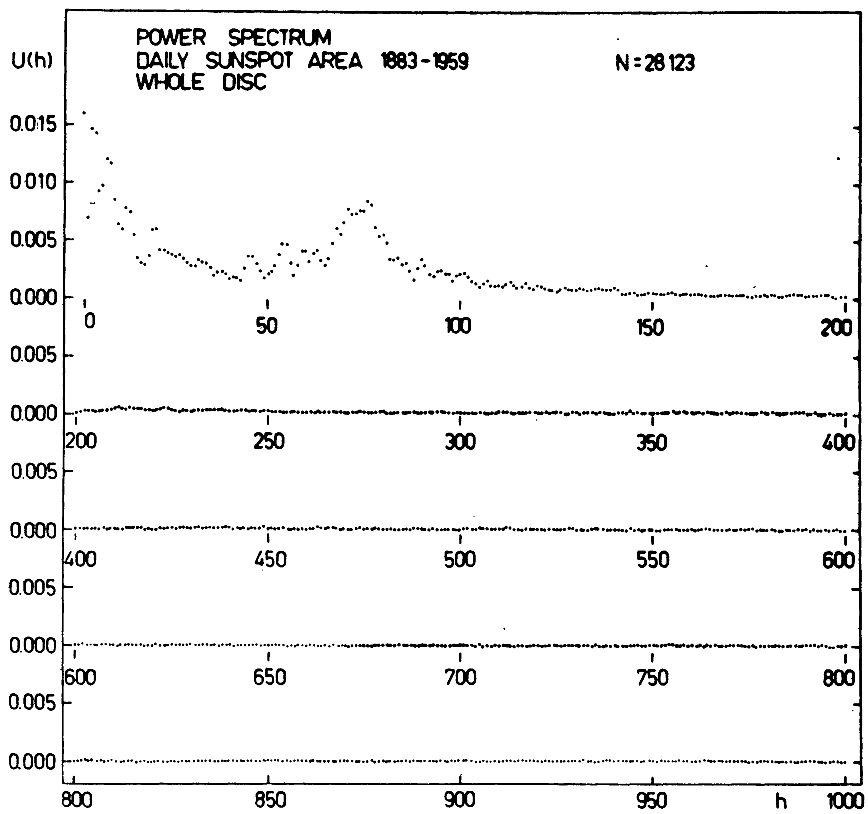

Fig. 4.

proves Kopecky's result (1967) that the average lifetime, determined by the frequency distribution of the groups with respect to the lifetime, is approximately the same in the northern and southern a period of approximately 27 days, which set on at the 8th, 15th and 23rd recurrence and last about 3 to 4 periods. In Figure 1 they are marked with arrows under the curves. Ambrož (1973b) found fluctuations with a duration of about 8—10 rotations the sunspot areas, the relative number, the areas of prominences, and the average surface density of Ca II plages covering the interval 1956—1964 in the 19th cycle of solar activity. Our analysis shows that over a long period fluctuations, repeated at approximately each 8 rotations, which are, however, damped, are observable. Roughly the same phenomenon can also be seen on the curves for the northern and southern hemispheres.

The Fourier transform of the autocorrelation function yields a power (or variance) spectrum, which presents all the periods, contained in the autocorrelation function, clearly. The resultant transform for the whole disc is shown in Figure 4. The expressive, relatively wide maximum for the lag of about 70 corresponds to the 27-day period of the rotation of the Sun. The spectrum is considerably damped for higher frequencies, and higher harmonics do not occur. This is due to the inherent 14-day running smoothing of the data for the whole disc (Shapiro, 1965). The power spectra for the northern and southern hemispheres have the same character. For this reason only their parts, truncated at a lag of 200, are shown in Figure 5.

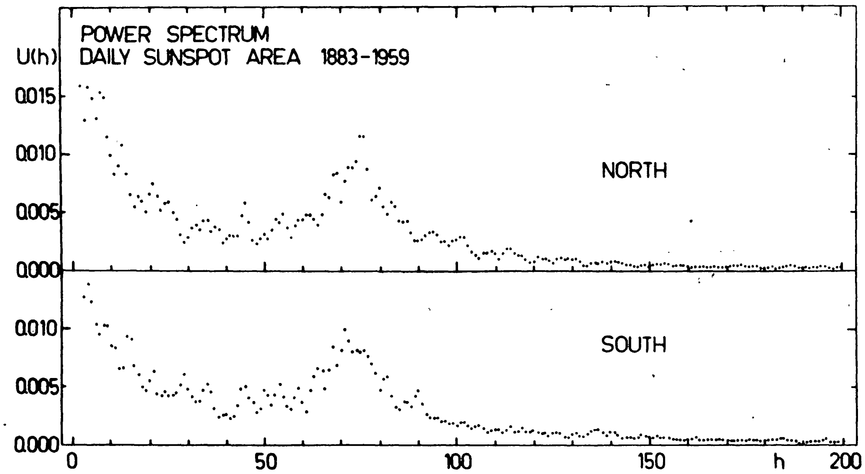

Fig. 5.

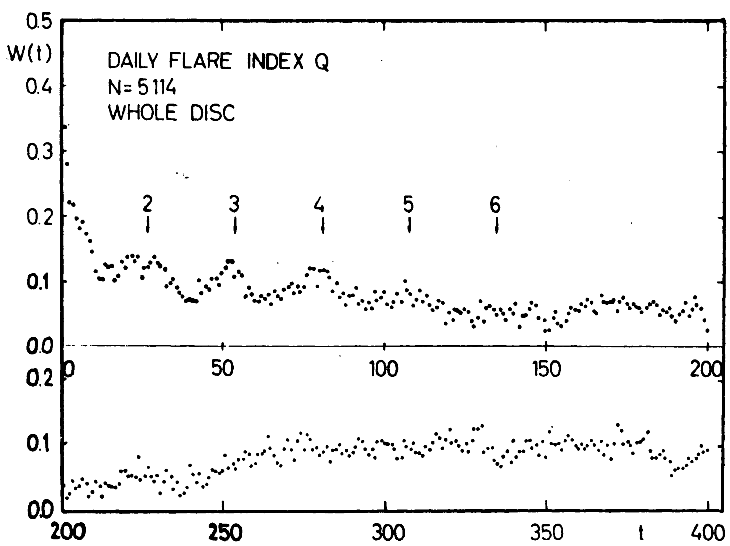

Fig. 6.

Analysis of the Flare Index Q

The sample contains 5114 daily values of the flare index Q from January 1, 1936 to December 31, 1949, which were adopted from Kleczek's catalogue (1952). The autocorrelation function was also truncated at a lag of 1000 days like in the analysis of the sunspot areas. Figure 6 represents part of the autocorrelation function, truncated at 400 days. The recurrence of the flaring regions can be observed upto the 6th rotation, and the damping of the oscillations of the autocorrelation function is similar to that with the sunspot area. The function also displays the variation with a period of about 1 year. This is a seasonal modulation effect, caused by most of the observing stations being located in the northern hemisphere of the Earth in the time interval considered, and the data used not being corrected for the period of observations. The autocorrelation functions for the northern and southern hemispheres have an analogous pattern.

The impulsive character and, to a certain extent, the discontinuity in the occurrence of flares, expressed by the Q-index, generate a quasi-random component, analogous to noise, in the pattern of this time series, which is reflected in the autocorrelation function, as well as in the power spectrum, where considerable variability appears at the higher frequencies. No filtration was used for the input data. The spectrum for the whole disc is shown in Figure 7. The higher harmonics of the 27-day period cannot be distinguished. Apart from the

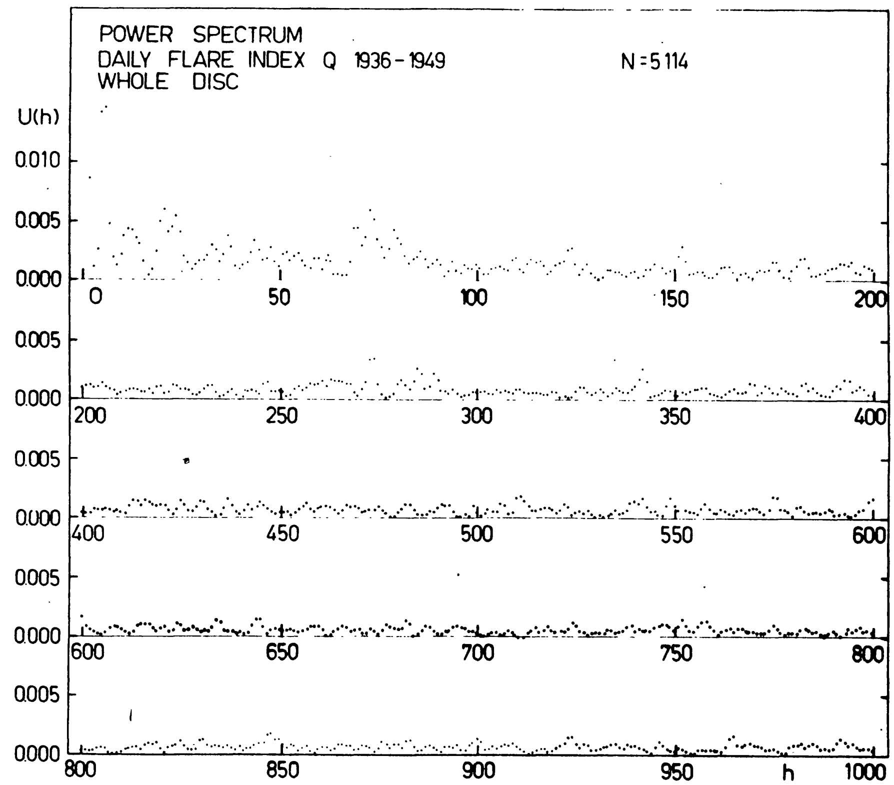

Fig. 7.

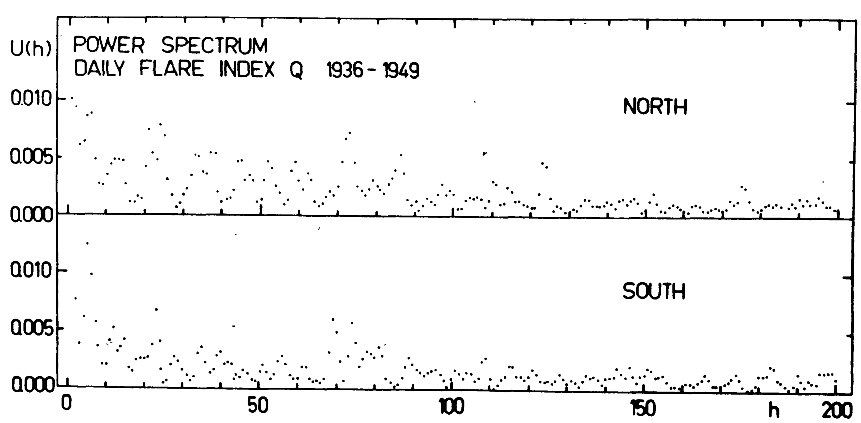

Fig. 8.

27-day period near the lag of 70 with 3 separate maxima, a period of about 90 days can be distinguished near the lag of 20 and an annual period near the lag of 5. Similar autocorrelation functions were also obtained for the northern and southern hemispheres. The parts of the spectra, truncated at 200, are shown in Figure 8.

Fine Structure of the 27-day Period

The most expressive peak in the power spectra of the sunspot area time series, which corresponds with the rotation of the Sun, is comparatively wide and displays a fine structure, formed by subsidiary peaks. The corresponding parts of the power spectra were replotted in Figure 9, in which also the period *P* scale is given below on the abscissa. The arrow in the upper part of the figure indicates Carrington's period. As can be seen, the spectral curves are composed of a whole series of partial peaks of various periods from about 22 to 33 days, which are superimposed on one another, particularly in the neighbourhood of Carrington's period.

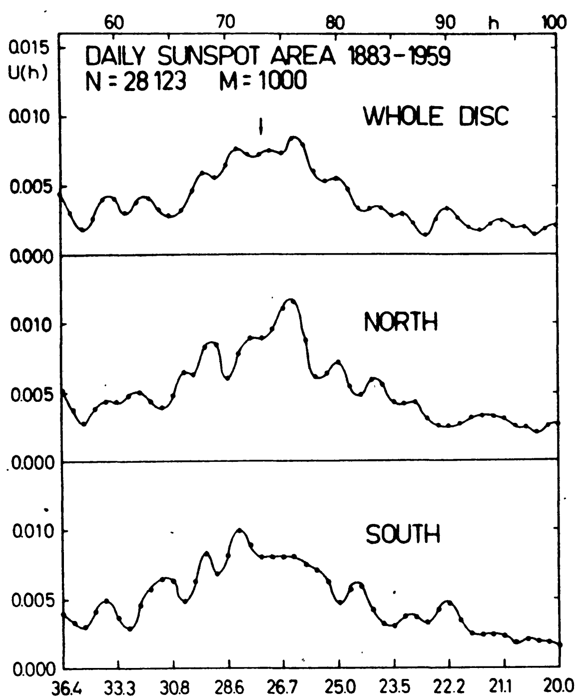

Fig. 9.

In the synoptic magnetic maps, mounted in chronological order and published by Bumba and Howard (1969), and in similar maps of Ca II plages, published by Ambrož (1973a), long-lived rows and streamers, corresponding particularly to active longitudes, can be seen. If the rows were not inclined to the vertical, it would indicate a rotation of exactly the Carrington's period. However, all of them are quite clearly at some angle, the inclination being positive as well as negative. Ambrož (1973a) derived periods of around 26.7, 27.2 and 29.2 days for the most expressive streamers of the Ca II plages in cycle No. 19.

The present analysis indicates that it would be possible to determine the existence of a larger number of periods, though less pronounced, than shown in the said chronologically arranged heliographic maps, from the much more extensive observational material of sunspot areas which encompasses approximately 7 solar cycles. The sets of periods in the northern and southern hemispheres differ quite distinctly. Their combination yields a spectrum for the whole disc.

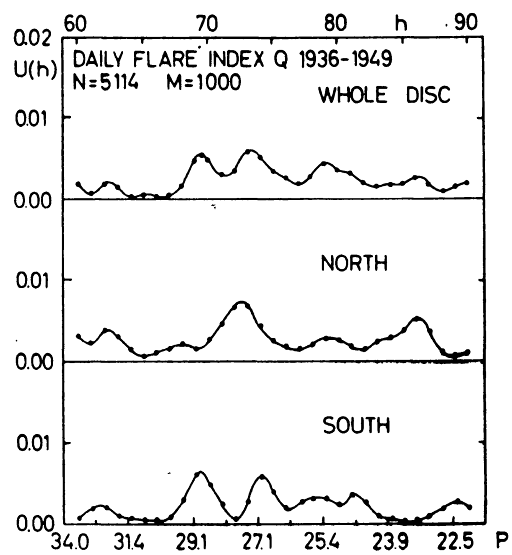

Fig. 10.

The sections of the *Q*-index power spectra, corresponding to the solar rotation period, are shown in Figure 10. Three separate peaks, corresponding to periods of 25.3, 27.4 and 28.8 days, can be distinguished in the spectrum for the whole disc. However, if one considers the hemispheres separately, one finds marked 26.9 and 28.9-day periods, as well as less pronounced periods of 24.5 and 25.5 days for the northern hemisphere. As regards the southern hemisphere, the only pronounced peak corresponds to a period of 27.6 days and periods of 25.3 and 29.5 days are only indicated very weakly. The combination of both spectra yields a spectrum for the whole disc, the individual maxima being possibly shifted as a result of the combination of adjacent peaks for the two hemispheres. The periods determined for the whole disc, with the exception of the shortest period of 25.3 days, agree relatively well with the periods, derived by Ambrož (1973a) for pronounced Ca II plage streamers. The shortest period of 26.7 days, determined by Ambrož, however, agrees with one the periods for the northern hemisphere. Therefore, the flare activity of the sunspot groups is apparently associated in the first place with pronounced streamers.

Shapiro and Ward (1966) found three peaks close to 27 days with periods around 26.0, 27.4 and 29.4 days, in their high-resolution autocorrelation analysis of C_i and K_p indices for the interval 1888—1959. On the other hand, for the sunspot number from the same interval they only found a single pronounced peak close to the period of 27.4 days. They devoted most attention to the longest of the periods found with the geomagnetic index, because they suspected that this was the result of a lunar modulation effect, corresponding to a synodic rotation of 29.54 days. However, as a result of their analysis they became inclined to the opinion that the subsidiary peaks were most likely the result, either direct or indirect, of solar activity. Figure 11 shows parts of our Q-index and sunspot area power spectra for the whole disc, and K_p-index and sunspot number spectra after Shapiro and Ward. The figure indicates a conspicuous similarity between the K_p- and Q-index spectra. This substantiates Shapiro's and Ward's hypothesis.

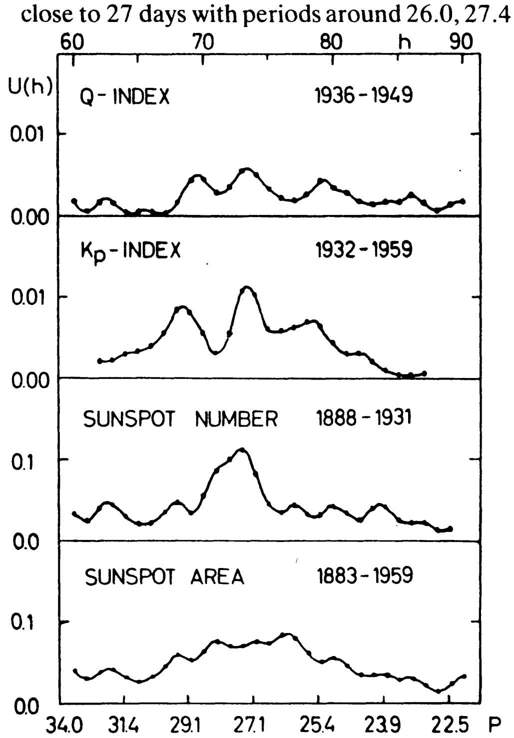

Fig. 11.

Revived Activity

The sunspot area as well as the Q-index power spectra display periods longer than the solar rotation, the expressiveness of wich differ appreciably. In order to guarantee a sufficiently large probability of existence, we only chose the periods which were common, i.e. the peaks of which coincide in the corresponding sunspot area and Q-index power spectra. The results are given in Table 1. The table shows a very interesting fact: most of these periods are a 2, 3, or 5-fold multiple of the periods which are associated with, or in the neighbourhood of the solar rotation period. These periods are marked $P'(N)$ in the table, where N indicates the multiple. The reality of the period, e.g., around 22 days,

Table 1. Common peaks in the power spectra of daily values of sunspot area 1883—1959 and of flare index Q 1936—1949

						Whole disc					
h	62.5	59.5	54.5		45.5/44.0	37.0	33.0		21.0/20.5	13.0	
P	32.0	33.7	36.7		44.0/45.5	54.1	60.6		95.2/97.6	154	
$P'(N)$					22.0/22.7(2)	27.0(2)	30.3(2)		31.8/32.5(3)	30.1(5)	
						North					
H	62.0	59.0			45.0	37.5	34.0	24.5	21.0	13.0	
P	32.3	34.0			44.4	53.3	58.8	81.7	95.2	154	
$P'(N)$					22.2(2)	26.7(2)	29.4(2)	27.2(3)	31.8(3)	30.1(5)	
						South					
h				51.0			35.0	28.5		14.5	5
P				39.2			57.0	72.8		148	400
$P'(N)$							28.5(2)	36.7(2)		29.7(5)	

observable in the sunspot area spectrum for the southern hemisphere, supports the occurrence of the period around 44 days, derived from the common peaks in the spectra of both solar indices for the northern hemisphere. In a similar fashion the period of around 37 days, given in Table 1 for the whole disc, seems to be quite realistic, because the double period of around 73 days occurs in common with the spectra for the southern hemisphere. However, the fundamental 37-day period can only be distinguished in the Q-index spectrum for the southern hemisphere, but does not appear in the same spectrum for the sunspot area. On the other hand it is present in the sunspot area spectrum for the northern hemisphere.

In investigating the frequency of great flare regions by means of the Q-index, Kleczek (1950) found a period of 88.76 days. As can be seen from the power spectra directly, as well as from Table 1, there are peaks in the power spectra which yield periods close to that derived by Kleczek. However, these periods are always a multiple of 3 of the existing periods, associated with the solar rotation period.

The analysis indicates that solar activity has a tendency to revive after 1, 2 or 4 rotations.

Discussion and Conclusions

The investigation of long time series of daily values of sunspot areas and of the flare Q-index by means of autocorrelation analysis has shown that this method can be used to determine the recurrency in the occurrence of longlived sunspot groups. It was also found that the activity revived after 1, 2 or more rotations, i.e. with a 2, 3-fold or multiple period. This revival displays a certain regularity. Since activity data, integrated over a wide longitude and latitude interval were used, the method is incapable of distinguishing directly between recurrency and revival of the individual groups, and a similar effect, generated by a change in the activity within a larger region on the Sun, regardless of whether the group in question is the same or not.

The high-resolution autocorrelation analysis of activity periods, associated with the solar rotation, has disclosed that there exists a whole family of periods over a comparatively wide range around the fundamental Carrington period. The occurrence of periods, more remote to the standard period of rotation of the Sun, which are realistic, cannot be fully explained just by the differential rotation and must be due to some other effects. The differences in the time patterns of solar activity in the northern and southern hemispheres, characterized by the NS asymmetry, are also closely associated with the occurrence of the individual periods in both hemispheres.

The flare activity, described by the flare Q-index, has three pronounced periods close to 27 days. Thus, it has been clearly proved that the subsidiary peaks in the power spectrum of the geomagnetic indices, found by Shapiro and Ward (1966), really have their origin directly on the Sun. It follows that the Q-index may be exploited to an advantage in statistical studies of geoactive and associated geophysical phenomena.

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