

Rapid X-ray variability and properties of compact stars

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Received: December 4, 2017; Accepted: March 28, 2018

Abstract. The Rossi X-ray timing explorer, which operated from 1995 to 2012, has provided a large amount of NS data. Timing analysis of the X-ray flux in more than a dozen NS systems reveals remarkable correlations between frequencies of two characteristic peaks present in the high-frequency part of the power-density spectra. We discuss a simple analytic relation that well reproduces these correlations. We outline a possible physical interpretation of the relation's parameters and explore the impact of the obtained results in a broader context.

Key words: X-Rays: Binaries – Accretion, Accretion Discs – Stars: Neutron

1. Introduction

Low-mass X-ray binaries (LMXBs) provide a unique opportunity to probe the effects associated with strong gravity in both black hole (BH) and neutron star (NS) systems (van der Klis, 2006). In the case of NS systems they may serve as a good tool for exploration of the supra-dense matter (Lewin et al., 1997).

The Proportional Counter Array aboard the Rossi X-ray timing explorer (PCA, RXTE Bradt et al., 1993; Jahoda et al., 1996), which operated from 1995 to 2012, has provided observations of NS sources that reveal the existence of two characteristic peaks present in the high-frequency part of the power-density spectra (twin-peak QPOs). In Figure 1a we illustrate correlations between twin-peak QPO frequencies in terms of the upper and lower QPO frequency, ν_U and ν_L , for a group of 14 sources including 8 atoll sources, 5 Z sources, and one millisecond X-ray pulsar. These 14 sources are listed in Table 1. At present there is no commonly accepted model that could explain the observed correlations. Nevertheless, based on various strong arguments, it is usually expected that the twin-peak QPOs are related to orbital motion in the vicinity of NSs. Miscellaneous concepts have been proposed to explain the observed phenomenon (Alpar & Shaham, 1985; Lamb et al., 1985; Miller et al., 1998; Psaltis et al., 1999; Stella & Vietri, 2001; Wagoner et al., 2001; Kluźniak & Abramowicz, 2001; Abramowicz & Kluźniak, 2001; Kato, 2001; Titarchuk & Wood, 2002; Abramowicz et al., 2003; Rezzolla et al., 2003; Kluźniak et al., 2004; Zhang, 2004; Pétri, 2005;

Bursa, 2005; Čadež et al., 2008; Wang et al., 2008; Mukhopadhyay, 2009; Bachetti et al., 2010; Dönmez et al., 2011; Stuchlík et al., 2013; Stuchlík & Kološ, 2014; Huang et al., 2016; Le et al., 2016; Germanà, 2017, and several others).

2. Orbital models

Various attempts to model the individual observed correlations with simple orbital models assuming test particle motion or oscillating pressure-supported fluid structures in the slender torus limit approximation have been rather unsuccessful (see, e.g., Lin et al., 2011; Török et al., 2010, 2012, 2016b, and references therein). In Figure 1a we include an example of correlation predicted by a particular geodesic model, the so-called relativistic precession model (Morsink & Stella, 1999; Stella & Vietri, 1998, 1999, in the following RP model). This correlation is compared to the prediction of a non-geodesic model recently proposed by Török et al. (2016a) to which we in the following refer as the CT (Cusp Torus) model. Both curves in the Figure are drawn for a non-rotating NS with gravitational mass $M = 1.7M_{\odot}$.

2.1. The RP model

The RP model in its usual form incorporates the assumption that the observed rapid X-ray variability originates in the orbital motion of hot inhomogeneities orbiting in the innermost parts of the accretion disc. Within this framework the frequencies of the two observed QPOs are related to the Keplerian frequency ν_K and the relativistic precession frequency ν_P of slightly perturbed circular geodesic motion that occurs at an arbitrary orbital radius r ,

$$\nu_U(r) = \nu_K(r), \quad \nu_L(r) = \nu_P(r). \quad (1)$$

The precession frequency equals to a difference between the Keplerian and the radial epicyclic frequency, $\nu_P(r) = \nu_K(r) - \nu_r(r)$.

It has been shown that relation (1) roughly matches the data of NS sources (e.g., Stella & Vietri, 1998, 1999; Belloni et al., 2007; Lin et al., 2011; Török et al., 2012, 2016b). It is however questionable whether the local motion of an individual spot can be responsible for the high amplitudes and coherence times of the observed QPOs (e.g., Barret et al., 2005a; Méndez, 2006; Barret & Vaughan, 2012).

2.2. The CT model

The CT model explored recently by Török et al. (2016a) deals with global modes of accreted fluid motion in an oscillating torus (Rezzolla et al., 2003; Abramowicz et al., 2006; Blaes et al., 2006; Šrámková et al., 2007; Ingram & Done, 2010; Fragile et al., 2016; Mishra et al., 2017; Parthasarathy et al., 2017; de Avellar et al., 2017). The torus is assumed to form a cusp by filling up the so-called

critical equipotential volume. The upper kilohertz QPO frequency is assigned to the Keplerian frequency of the fluid at the centre of the torus where both the fluid pressure and the density peak, and from which most of the torus radiation emerges. The lower kilohertz QPO frequency is assigned to the frequency of a non-axisymmetric $m = -1$ radial epicyclic mode, $\nu_{r,-1}$. It is expected that the torus at all times keeps its maximal possible size filling the ‘Roche-like’ lobe, and that its radial oscillations modulate the boundary layer accretion flow (Paczynski, 1987; Horák, 2005; Abramowicz et al., 2007; Parthasarathy et al., 2017).

The QPO frequencies predicted by the CT model are functions of the torus centre location r_0 ,

$$\nu_U \equiv \nu_K(r_0), \quad \nu_L \equiv \nu_{r,-1}(r_0). \quad (2)$$

Rather long analytic formulae that define $\nu_{r,-1}$ can be found in Straub & Šrámková (2009). Török et al. (2016a) numerically calculated the $\nu_L(\nu_U)$ correlation following from relation (2). Assuming this correlation, they obtained a good match of the data in the case of the atoll source 4U 1636-53 for NS mass of $M = 1.69M_\odot$ (see Figure 1b for illustration).

3. Simple analytical formula

In Török et al. (2017a) we suggested that the $\nu_L(\nu_U)$ frequency relations can be well described by the following formula

$$\nu_L = \nu_U \left(1 - \mathcal{B} \sqrt{1 - (\nu_U/\nu_0)^{2/3}} \right), \quad (3)$$

where ν_0 equals the Keplerian orbital frequency at the innermost stable circular orbit around a non-rotating NS with gravitational mass M_0 . This frequency can be expressed in the units of Hz as (e.g., Kluzniak & Wagoner, 1985; Kluzniak et al., 1990),

$$\nu_0 = \nu_{ISCO} = \frac{1}{6^{3/2}} \frac{c^3}{2\pi G} \frac{1}{M} = 2198 \frac{M_\odot}{M} = 2198 \frac{1}{\mathcal{M}}. \quad (4)$$

For $\mathcal{B} = 1$, relation (3) merges with the frequency relation implied by the RP model while the CT model prediction is well approximated by relation (3) for $\mathcal{B} = 0.8$. We illustrate this property of relation (3) in Figure 1b.

4. Data matching - main results

For 9 sources, namely 4U 1608-52, 4U 1636-53, 4U 1735-44, 4U 1915-05, IGR J17191-2821, GX 17+2, Sco X-1, Cir X-1 and XTE J1807.4-294 we find good agreement between the CT model and data. In all these sources the CT model

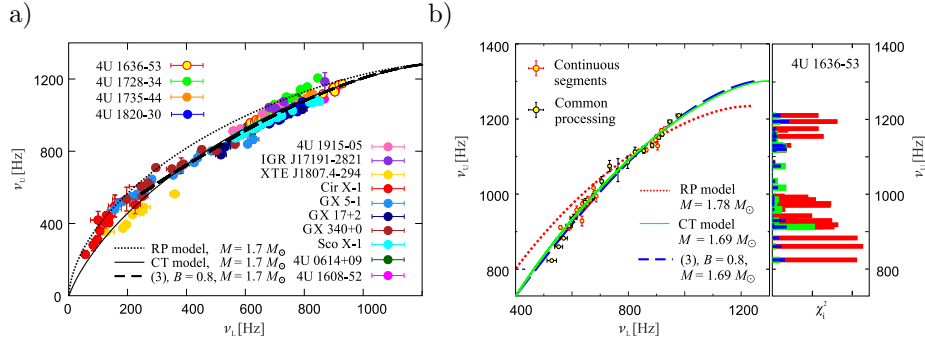


Figure 1. Correlations between the frequencies of twin-peak QPOs. a) The data of 14 sources and examples of the expected frequency relations. Detailed information on the individual sources along with the appropriate references is given in Table 1. b) The data of the atoll source 4U 1636-53 and their best fits. For the sake of clarity, the data-set which corresponds to the individual continuous observations is compared to the data-set associated with the common processing of all observations (see Török et al., 2016a). In both panels the expected frequency relations are drawn for a non-rotating NS.

matches the observed trend better than is done by the RP model (see Figure 2). For the other 5 sources, namely 4U 1728-34, 4U 0614+09, 4U 1820-30, GX 340+0 and GX 5-1, it is not possible to match the data due to the CT model limitations discussed by Török et al. (2016a).

Relation (3) well describes the data in each of the 14 sources (see Table 1). In some cases one does not achieve $\chi^2 \approx 1$ d.o.f, but no clear deviation of data from the expected trend is found. This can be seen from a direct comparison between the expected curves and data, which is for each source included in Török et al. (2017a,b) where we present detailed results.

5. Consideration of models of rotating NSs

In the series of works (Török et al., 2010, 2012, 2016b) the authors explored the implications of several orbital QPO models. They found that the estimation of NS parameters based on these models leads to the effective degeneracy of these parameters. For a given model, each combination of NS mass M , angular momentum j and quadrupole moment q corresponds to a certain value of a single generalized parameter, e.g., non-rotating NS mass. Indeed, Török et al. (2016a) noticed that the NS mass implied by the CT model for the atoll source 4U 1636-53 can be expressed as

$$M = M_0[1 + 0.7(j + j^2)], \quad M_0 = 1.69^{\pm 0.01} M_\odot. \quad (5)$$

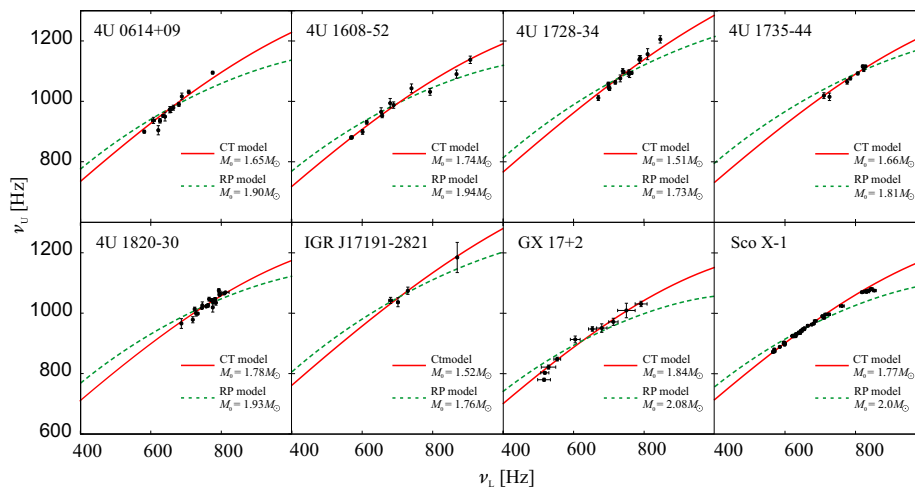


Figure 2. A comparison between the best fits obtained assuming the CT and RP models for data of the individual sources. The best fits for the atoll source 4U 1636-53 are shown in Figure 1b.

5.1. Neutron star spin, radius and equation of state

There is good evidence on the NS spin frequency of 4U 1636–53 based on the X-ray burst measurements (see the reference in Wang et al., 2017). The consideration of the X-ray burst models implies the NS spin to take the value of $\nu_s \doteq 580$ Hz. The CT model predictions can therefore be compared to the expectations given by models of rotating NSs.

Figure 3a (after Török et al., 2016a) displays several mass-angular momentum relations expected from the models of rotating NSs for the spin of 580 Hz. Inspecting this Figure one can see that there is an overlap between the relations given by the models of rotating NSs and the relation inferred from the CT model. The required NS mass is $M(580\text{Hz}) \approx 2M_\odot$.

Considering the above result, one can expect that similar consideration should also be valid for the other sources listed in Table 1. This is justified in Figure 3b, where we explore the CT model predictions for another atoll source, 4U 1608-52. Table 1 indicates that $\mathcal{M}(\mathcal{B} = 0.8) = 1.74$. The NS spin inferred from the X-ray burst measurements takes the value of $\nu_s \doteq 610$ Hz (see references in Wang et al., 2017). Analyzing Figure 3b one can see that the relation

$$M = M_0[1 + 0.6(j + j^2)], \quad M_0 = 1.74^{\pm 0.01} M_\odot \quad (6)$$

describes the CT model prediction and overlaps with the relations given by the models of rotating NSs around $M(610\text{Hz}) \approx 2.1M_\odot$.

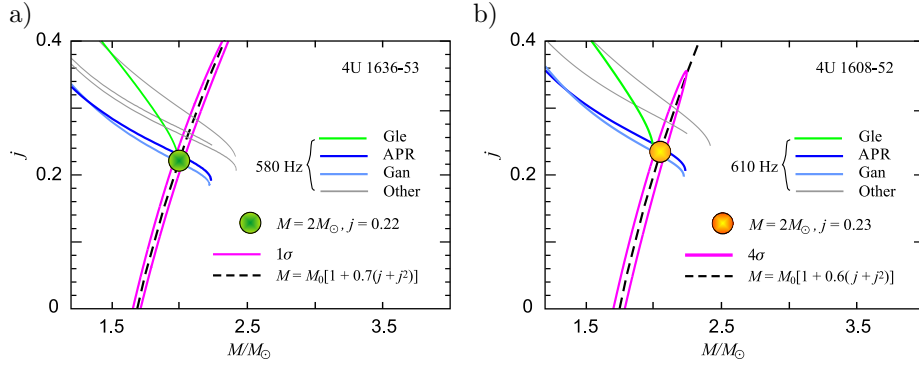


Figure 3. The mass-angular momentum contours obtained from the fitting of datapoints using the CT model vs. the mass-angular momentum relations predicted by models of rotating NSs. These are drawn for several NS EoS and the values of spin inferred from the X-ray burst measurements. For the models of rotating NSs we adopt the approach of Urbanec et al. (2013). The assumed EoS are as follows. Gle - Glendenning (1985); APR - Akmal et al. (1998); Gan - Gandolfi et al. (2010); other - Rikovska Stone et al. (2003); Urbanec et al. (2010). For the QPO model, the σ -confidence levels are calculated in accordance with the method used in Török et al. (2016a,b). a) The atoll source 4U 1636-53. b) The atoll source 4U 1608-52. In both panels the emphasized spot indicates where the QPO model and the EoS relations overlap.

6. Conclusions

The CT model as well as relation (3) for $\mathcal{B} = 0.8$ well reproduce the data of 9 sources. Assuming \mathcal{B} as a free parameter, we obtain good fits for each of the 14 considered sources. We suggest that larger deviations from the case of $\mathcal{B} = 0.8$ are caused by further non-geodesic effects acting on the torus formation. These can be induced by the influence of magnetic field.

The particular consideration of the CT model agrees with the general interpretation, in which the \mathcal{M} parameter represents the main parameter reflecting the spacetime geometry given by the NS mass and spin, while the \mathcal{B} parameter reflects the additional stable factors. Further determination of NS mass and spin is possible when implications of relation (3) are confronted with the results of NS modeling based on NS spin measurements. In the case of the CT model we have already obtained promising results in this research direction.

Acknowledgements.

We would like to acknowledge the Czech Science Foundation grant No. 17-16287S, the internal SU grant No. SGS/15/2016, and the INTER-TRANSFER project No. LTT17003. We are grateful to Marek Abramowicz and Omer Blaes for useful discussions. Last but not least, we would like to acknowledge the hospitality of University of

Table 1. The results of data matching for relation (3) assuming $\mathcal{B} = 0.8$ and \mathcal{B} as a free parameter. The goodness of fits is formally characterized by the χ^2 values. The uncertainties displayed here correspond to standard errors.

Source No./Type	Name	$\mathcal{M}(0.8)$	$\frac{\chi^2}{d.o.f.}$	$\mathcal{M}(\mathcal{B})$	\mathcal{B}	$\frac{\chi^2_{\mathcal{M}(\mathcal{B})}}{d.o.f.}$	Data- points
1/A	4U 1608-52	$1.80^{\pm 0.01}$	1.6	$1.79^{\pm 0.04}$	$0.79^{\pm 0.03}$	1.7	12
2/A	4U 1636-53	$1.70^{\pm 0.01}$	2.0	$1.70^{\pm 0.01}$	$0.8^{\pm 0.01}$	2.1	22
3/A	4U 1735-44	$1.69^{\pm 0.01}$	2.1	$1.48^{\pm 0.10}$	$0.61^{\pm 0.06}$	1.0	8
4/A	4U 1915-05	$1.58^{\pm 0.03}$	0.8	$1.65^{\pm 0.03}$	$0.82^{\pm 0.01}$	0.2	5
5/A	IGR J17191	$1.58^{\pm 0.02}$	0.6	$1.63^{\pm 0.20}$	$0.85^{\pm 0.2}$	0.8	4
6/Z	GX 17+2	$1.89^{\pm 0.02}$	1.2	$1.77^{\pm 0.07}$	$0.72^{\pm 0.04}$	0.8	10
7/Z	Sco X-1	$1.82^{\pm 0.01}$	1.0	$1.81^{\pm 0.01}$	$0.8^{\pm 0.01}$	1.0	39
8/Z	Cir X-1	$0.74^{\pm 0.10}$	1.2	$1.42^{\pm 0.5}$	$0.89^{\pm 0.06}$	1.1	11
9/P	XTE J1807.4	$2.61^{\pm 0.11}$	0.8	$2.85^{\pm 0.25}$	$0.86^{\pm 0.07}$	0.8	7
10/A	4U 1728-34	$1.57^{\pm 0.01}$	3.2	$1.35^{\pm 0.12}$	$0.65^{\pm 0.06}$	2.5	15
11/A	4U 0614+09	$1.71^{\pm 0.02}$	5.1	$1.39^{\pm 0.06}$	$0.62^{\pm 0.02}$	1.1	13
12/A	4U 1820-30	$1.81^{\pm 0.01}$	9.3	$1.53^{\pm 0.07}$	$0.58^{\pm 0.03}$	3.2	23
13/Z	GX 340+0	$1.62^{\pm 0.08}$	4.2	$2.23^{\pm 0.10}$	$1.10^{\pm 0.08}$	1.6	12
14/Z	GX 5-1	$1.65^{\pm 0.10}$	16.7	$2.31^{\pm 0.04}$	$1.11^{\pm 0.02}$	1.5	21

Source type: A - Atoll, Z - Z, P - Pulsar

References: (1)–(3), (10) – (12) - Barret et al. (2005b,c, 2006), (4) - Boirin et al. (2000), (5) - Altamirano et al. (2010), (6) - Homan et al. (2002), (7) - van der Klis et al. (1997), (8) - Boutloukos et al. (2006), (9) - Linares et al. (2005), (13) - Jonker et al. (2000), (14) - Jonker et al. (2002).

California in Santa Barbara, and to express our thanks to concierges of Mlýnská hotel in Uherské Hradiště, Czech Republic for their participation in organizing frequent workshops of Silesian University and Astronomical Institute of the Czech Academy of Sciences.

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