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# Highly magnetized super-Chandrasekhar white dwarfs and their consequences

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**Abstract.** Since 2012, we have been exploring possible existence of highly magnetized significantly super-Chandrasekhar white dwarfs with a new mass-limit. This explains several observations, e.g. peculiar over-luminous type Ia supernovae, some white dwarf pulsars, soft gamma-ray repeaters and anomalous X-ray pulsars, which otherwise puzzled us enormously. We have proceeded to uncover the underlying issues by exploiting the enormous potential in quantum, classical and relativistic effects lying with magnetic fields present in white dwarfs. We have also explored the issues related to the stability and gravitational radiation of these white dwarfs.

**Key words:** white dwarfs – supernovae: general – stars: magnetic fields – pulsars: general – gravitation – x-rays: general

## 1. Introduction

In the last five years or so, we have initiated modeling highly magnetized super-Chandrasekhar white dwarfs (B-WDs). We initiated our exploration by considering a simplistic spherically symmetric Newtonian model to study the quantum mechanical effects of a strong magnetic field on the composition of the white dwarf (Das & Mukhopadhyay, 2012). This in turn led to the discovery of a new mass-limit significantly larger than the Chandrasekhar limit (Das & Mukhopadhyay, 2013), thus heralding the onset of a paradigm shift. Note that although the possibility of super-Chandrasekhar white dwarfs has been mentioned in the literature (e.g. Ostriker & Hartwick, 1968), there was no prediction of a new (super-Chandrasekhar) mass-limit. Rather the existing idea was mostly to establish magnetized white dwarfs of larger size, which need not be the case. After our preliminary success with impressive results, we have progressed to a general relativistic framework, however, still within the assumption of spherical symmetry (Das & Mukhopadhyay, 2014a) and again showed the existence of magnetized super-Chandrasekhar white dwarfs. Finally, we have constructed a sophisticated general relativistic magnetohydrodynamic (GRMHD) model with self-consistent departure from spherical symmetry, due to the presence of strong magnetic field as well as rotation, which confirmed our previous results by yielding highly super-Chandrasekhar B-WDs (Das & Mukhopadhyay, 2015; Subramanian & Mukhopadhyay, 2015). It has also been shown that B-WDs, depending on their field geometry, could be much smaller or bigger in size compared to the standard white dwarfs following Chandrasekhar's theory (C-WDs). More so, we have argued for a possible evolutionary scenario from C-WDs with weaker magnetic fields to B-WDs, by accretion (Das et al., 2013).

In this proceedings, we highlight the basic tenets of B-WDs – the observational motivations, main theoretical results and its multiple astrophysical implications. The various notations used here, if not defined, have their usual meaning and follow from our past work mentioned here.

## 2. Observational motivations

At least 10% of white dwarfs are known to be highly magnetized with surface fields $> 10^6$ G (Ferrario & Wickramasinghe, 2005) and, hence, they are likely to harbor much stronger interior fields. More interestingly, magnetized white dwarfs tend to be more massive (mean mass $0.784 \pm 0.4\mathrm{M}_{\odot}$) than their non-magnetized counterparts (mean mass $0.663 \pm 0.136\mathrm{M}_{\odot}$) (Ferrario et al., 2015). Thus, it is an important question in itself to study the effect of a stronger interior magnetic field on the structure and properties of white dwarfs. The theory of weakly magnetized white dwarfs was explored by a few authors (e.g. Ostriker & Hartwick, 1968; Suh & Mathews, 2000), however, we have initiated the investigation into strong magnetic fields in white dwarfs. The most compelling motivation behind the initiation of super-Chandrasekhar B-WDs, however, was the recent discovery of several peculiar, highly over-luminous type Ia supernovae (SNeIa), which invoke highly super-Chandrasekhar white dwarfs in the mass range $2.1 - 2.8\mathrm{M}_{\odot}$ as their most plausible progenitors (Scalzo et al., 2010).

## 3. Newtonian model of B-WDs and new mass-limit

An interior field $> B_c = 4.414 \times 10^{13}$ G introduces the quantum mechanical effect of Landau orbital, which stiffens the equation of state (EoS) of the underlying electron degenerate matter in the high density regime. In order to focus solely on this effect, we have assumed a simple Newtonian framework, a fluctuating (or constant) magnetic field and spherical symmetry. This led to significantly super-Chandrasekhar white dwarfs having mass $> 2\mathrm{M}_{\odot}$. The length scale over which the field is fluctuating is assumed to be large compared to the underlying electron Compton wavelength for quantum mechanical effects to work. However, this length scale is small enough to reveal an averaged magnetic field that does not produce any significant Lorentz force on the stellar matter. The magnetic field in this scenario affects only EoS, hence the emergence of super-Chandrasekhar B-WDs in this framework is due to the modified EoS alone.

Following Das & Mukhopadhyay (2013) the EoS of degenerate electron gas at high density (e.g. central region of the star) is approximately given by $P = K_m(B)\rho^\Gamma = K_m(B)\rho^{1+1/n}$, when $\Gamma = 2$. The spherical B-WD obeys the conditions for magnetostatic equilibrium and the estimate of mass, given by

$$\frac{1}{\rho+\rho_B}\frac{d}{dr}\left(P+\frac{B^2}{8\pi}\right) = F_g + \left.\frac{\boldsymbol{B}\cdot\nabla\boldsymbol{B}}{4\pi(\rho+\rho_B)}\right|_r, \quad \frac{dM}{dr} = 4\pi r^2(\rho+\rho_B). \tag{1}$$

As argued above, the magnetic terms could be neglected in the present framework and following Lane-Emden formalism (Das & Mukhopadhyay, 2013), the scalings of mass and radius with central density $\rho_c$ are obtained as

$$M \propto K_m^{3/2}\rho_c^{(3-n)/2n}, \quad R \propto K_m^{1/2}\rho_c^{(1-n)/2n}, \quad K_m = K\rho_c^{-2/3}. \tag{2}$$

Clearly $n = 1$ ($\Gamma = 2$) corresponds to $\rho_c$-independent $M$ (unlike Chandrasekhar's case when $K_m$ is independent of $B$ and the limiting mass corresponds to $n = 3$).

Substituting for the proportionality constants appropriately, we obtain the limiting mass

$$M_l = \left(\frac{hc}{2G}\right)^{3/2}\frac{1}{(\mu_e m_H)^2} \approx \frac{10.312}{\mu_e^2}\mathrm{M}_\odot, \tag{3}$$

when the limiting radius $R_l \to 0$. For $\mu_e = 2$, $M_l = 2.58\mathrm{M}_\odot$. This may establish the aforementioned peculiar, over-luminous SNeIa as new standard candles for cosmic distance measurement. Note that this result is in the spirit of original Chandrasekhar-limit, which also corresponds to zero radius at infinite density. Realistically, at a finite but high density and a finite magnetic field – e.g., $\rho_c = 2\times10^{10}$gm/cc and $B = 8.8\times10^{15}$G when $E_{Fmax} = 20m_ec^2$ – one obtains $M = 2.44\mathrm{M}_\odot$ and $R = 650$km, as opposed to $M = 1.39\mathrm{M}_\odot$ for the $B = 0$ case. Note that these $\rho_c$ and $B$ are below their respective upper limits set by the instabilities of pycnonuclear fusion, inverse-$\beta$ decay and general relativistic effects (Das & Mukhopadhyay, 2014b).

## 4. General relativistic model of B-WDs with rotation

To self-consistently account for generalized cases when the magnetic fields need not be fluctuating, we have constructed equilibrium models of B-WDs in a general relativistic framework, both with and without rotation, using the well-tested codes *XNS* and *Lorene*. Even if we restrict the ratios of magnetic-to-gravitational and rotational-to-gravitational energies following Ostriker & Hartwick (1968); Braithwaite (2009) (in order to assure stable-equilibria), the mass range turns out to be $\sim 2-2.5\mathrm{M}_\odot$ depending upon the field geometry. The equilibrium sequences shown in Fig. 1 reveal the shape of B-WDs obtained by *XNS*. Larger the fields and/or rotation, larger is the mass. Note that we did a thorough investigation by invoking all possible rotation profiles in B-WDs, including uniform and differential. Here, we report only the differentially rotating cases. However,

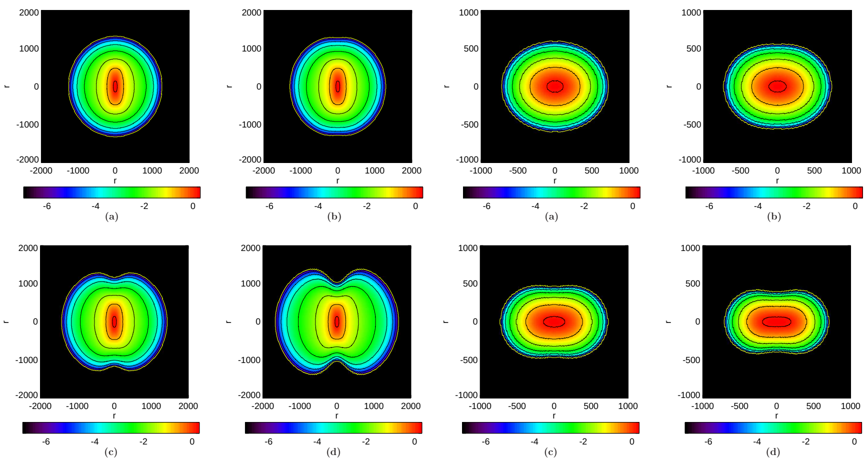


**Figure 1.** *Left panels*: Sequence of differentially rotating configurations with a purely toroidal magnetic field with maximum field $\approx 3.1 \times 10^{14}$ G fixed. The panels are density ($\rho$) contour plots of $\log\left(\frac{\rho}{\rho_0}\right)$, with $\rho_0 = 10^{10}$ gm/cc, corresponding to the values of central angular velocity in $\mathrm{s}^{-1}$ (a) 0.003, (b) 8.112, (c) 18.252, (d) 28.392. *Right panels*: The same, but for a purely poloidal magnetic field and values of central angular velocity in $\mathrm{s}^{-1}$ (a) 2.028, (b) 12.168, (c) 24.336, (d) 32.448. See, Subramanian & Mukhopadhyay (2015) for other details.

the magnetic braking effect might eventually cause the B-WDs to rotate uniformly. It is important to check the timescale over which differential rotation could be maintained in B-WDs, which needs further detailed exploration.

## 5. Astrophysical implications of B-WDs

B-WDs have many important astrophysical implications.

### 5.1. B-WDs as sources of SGRs/AXPs

B-WDs appear to be ideal candidates to explain SGRs/AXPs, as they resolve several discrepancies in the magnetar model (Duncan & Thompson, 1992) and also the weakly magnetized C-WD model (Paczynski, 1990). Some problems (Mereghetti, 2013) with the magnetar model are (1) no observational evidence has been found so far for strongly magnetized neutron stars, as required by the magnetar model to work, (2) Fermi observations are inconsistent with high energy gamma-ray emissions in magnetars, (3) inferred upper limit of surface field, e.g. for SGR 0418+5729, is quite smaller than the field required to explain the observed X-ray luminosity. Weakly magnetized C-WDs are challenged by the observed short spin periods and low UV-luminosities ($L_{UV}$). The advantage of

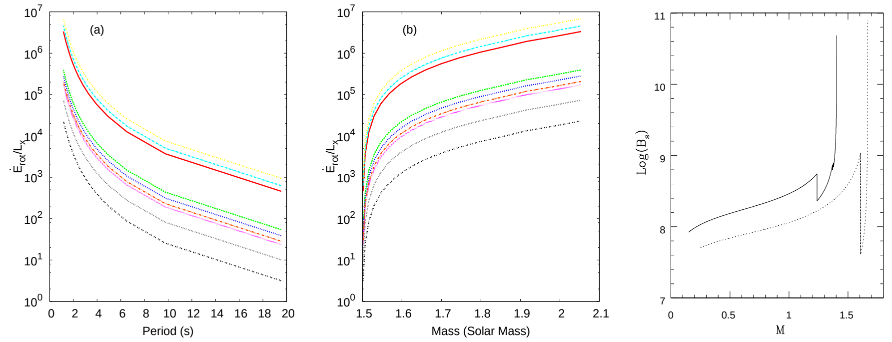


**Figure 2.** *Left*: The ratio of rate of rotational energy release to observed X-ray luminosity as a function of (a) spin period, (b) mass, for B-WDs when from the top to bottom various curves correspond to 1E 1547-54, 1E 1048-59, SGR 1806-20, SGR 1900+14, SGR 0526-66, SGR 1822-1606, 1E 1841-045, SGR 0418+5729 and 1E 2259+586. For details, see Mukhopadhyay & Rao (2016). *Right*: Time evolution of surface magnetic field in G as a function of mass in units of solar mass. Other parameters are accretion rate $\dot{M} = 10^{-8}\mathrm{M}_\odot\,\mathrm{yr}^{-1}$, the angle between rotational and magnetic axes $\alpha = 10^\circ$ and stellar radius $R = 10^4$ km at $t = 0$. See Mukhopadhyay et al. (2017) for details.

B-WDs chiefly lies in their intermediate size between neutron stars and weakly magnetized C-WDs, as well as a strong enough magnetic field. This magnetic field is, however, much weaker than that of neutron stars. Hence, B-WDs assure low $L_{UV}$.

We explore the possibility to explain the high energy phenomena in SGRs and AXPs by rotationally powered magnetic energy ($\dot{E}_{rot}$) of B-WDs – there is no need to invoke extraordinary, yet observationally unconfirmed, sources of energy. Figure 2 (left panels) shows that $\dot{E}_{\mathrm{rot}}$ computed based on B-WD model, with a fixed inclination angle between rotation and magnetic axes, is several orders of magnitude larger than the observed X-ray luminosity $L_x$ for nine sources.

More sources are to be observed by the Indian satellite AstroSat's wide band spectroscopic capabilities, with cyclotron resonance energy $E = 11.6\ (B/10^{12}G)$ keV in the spectrum. This would confirm B-WDs' surface field strength. Also other features such as, a wide band spectral shape (like a tail or second peak), can be examined in the context of beamed emission from the pole of a B-WD having X-ray luminosity similar to SGRs/AXPs ($\sim 10^{36}$ erg s$^{-1}$).

### 5.2. B-WDs as descendents of white dwarf pulsar AR Scorpii

Recent observation of the first white dwarf radio pulsar AR Scorpii (AR Sco) justifies the existence of strongly magnetized white dwarfs in nature. Although based on the current observational evidence AR Sco does not appear to have fields as strong as a B-WD could have, we have proposed in our latest work (Mukhopadhyay et al., 2017) a possible mechanism by which AR Sco can evolve to a B-WD eventually, due to binary interaction. This essentially involves repeated, alternating cycles of accretion-induced spin-up and increase in magnetic field due to flux freezing, followed by spin-down during radio emission when accretion is inhibited due to a strong magnetic field.

Figure 2 (right panel) reveals by a couple of possible evolutions of magnetic field with mass (which is varying with time) that initial smaller surface field $B_s$ is seen to increase with accretion, but drops during the spin-powered phase (when accretion stops and hence there is no change in mass), followed by a phase with increasing trend again. Also, there is a sharp rise ($B_s \sim 10^{11}$ G) at the last cycle. This corresponds to the increase of central field $B_c$ as well, leading to a B-WD. Of course, these are just representative samples and they may depend on many other factors. Hence, they may not match exactly with what is expected to happen in AR Sco itself.

### 5.3. B-WDs as sources of gravitational waves

Figure 1 shows that B-WDs are deformed due to the strong magnetic field as well as rotation, and they assume a triaxial shape. If their rotation axis is misaligned with the magnetic axis, then there is a time-varying quadrupole moment that leads to gravitational radiation with amplitude (Palomba et al., 2012)

$$h_+(t) = h_0 \left( \frac{1+\cos^2\alpha_0}{2} \right) \cos\Phi(t),\ \ h_\times(t) = h_0 \cos\alpha_0 \sin\Phi(t), \tag{4}$$

where $\alpha_0$ is the inclination of the star's rotation axis with respect to the line of sight and

$$h_0 = \frac{4\pi^2 G}{c^4} \frac{I_{zz}\epsilon}{P_s^2 d}, \tag{5}$$

$\Phi(t)$ is the signal phase function, $I_{zz}$ is the moment of inertial about z-axis, $\epsilon$ is the measure of ellipticity of the star and $d$ is the distance of the star from the detector. In our latest work (Mukhopadhyay et al., 2017), we have justified B-WDs as potential sources for gravitational wave to be detected. A B-WD with mass $\sim 2\mathrm{M}_\odot$, polar radius $\sim 700$ km, rotational period $P_s \sim 1$ s (Subramanian & Mukhopadhyay, 2015), $\epsilon \sim 5 \times 10^{-4}$ and at $\sim 100$ pc away from us would produce $h_0 \sim 10^{-22}$, which is within the sensitivity of the Einstein@Home search for early LIGO S5 data (Palomba et al., 2012). If the B-WD's polar radius is $\sim 2000$ km, $P_s \sim 10$ s and other parameters intact as above, a firm confirmation of gravitational wave emission can be provided by DECIGO/BBO (Yagi & Seto, 2011) with $h_0 \sim 10^{-23}$. Nevertheless, high magnetic field rotating white dwarfs approaching a B-WD would be common and it is possible that such white dwarfs of radius $\sim 7000$ km, $P_s \sim 20$ s and $d \sim 10$ pc will have a $h_0 \geq 10^{-22}$, which is detectable by LISA.

## 6. Discussion and conclusion

A lot of groups have adopted and pursued our model of B-WDs: exploring polytropes for anisotropic matter (Herrera et al., 2014), explaining SGRs/AXPs (Belyaev et al., 2015), constructing magnetic compressible fluid stars (Federbush et al., 2015), to construct ungravity-inspired model (Bertolami & Mariji, 2016), possible exploration of third family of compact stars (Sotani & Tatsumi, 2017), for emission of gravitational wave (Franzon & Schramm, 2017), establishing central magnetic field of a magnetic white dwarf (Shah & Sebastian, 2017), to mention a few.

However, there have also been concerns regarding instability to neutronization and pycnonuclear reactions, general relativistic instability and MHD instabilities (Chatterjee et al., 2017; Bera & Bhattacharya, 2017). We have, however, argued/established that these are non-issues. First, our simplistic models (Das & Mukhopadhyay, 2012, 2013) were constructed in the spirit of Chandrasekhar's calculations, whose actual mass-limit also corresponds to infinite density and zero radius – an ideal case. Moreover, these models assumed a fluctuating (or constant) magnetic field, as mentioned above, such that the net averaged field does not contribute significantly to the Lorentz force, but affects the white dwarf properties on a quantum mechanical scale. In this scenario the above concerns do not hold. Next, all these concerns are automatically resolved in our fully self-consistent GRMHD model (with and without rotation), which invokes magnetic field strengths that satisfy all constraints from general relativity and neutronization. It is additionally important to note that the rates for pycnonuclear reactions are extremely uncertain, and if the chosen rates (Chatterjee et al., 2017) were to be accurate, then that poses a concern for even the weakly magnetized white dwarfs. As far as the underlying magnetic field configurations and related stability are concerned, purely toroidal and purely poloidal field configurations have been long known to be subjected to MHD instabilities (Tayler, 1973), as we pointed in our work (Das & Mukhopadhyay, 2015; Mukhopadhyay et al., 2017). So arguments based on solely these configurations (Bera & Bhattacharya, 2017) are too premature and may be misleading. One needs to construct mixed field configurations which are more likely to occur in nature (Ciolfi & Rezzolla, 2013), for further stability analysis.

B-WDs have enormous astrophysical significance – they can be ideal progenitors for the recently discovered peculiar, highly over-luminous, SNeIa; they can explain the origin of SGRs and AXPs; they are potential sources of gravitational waves for future detectors; their new mass-limit altogether offers a new standard candle – to list a few. Hence, more explorations are needed, both theoretical and observational, for thoroughly understanding the physics behind B-WDs, in order to fully utilize them for explaining a wide range of astrophysical phenomena.

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