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Light-time effect as a valuable tool in stellar astrophysics

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Abstract. The light-time effect (LITE) is a well-known phenomenon and traditional tool for study of multiple stellar systems. Its role in modern stellar astrophysics is briefly reviewed. Several examples of LITE and its combination with other effects are demostrated in $O-C$ diagrams of three early-type eclipsing binaries.

Key words: eclipsing binary – stars: individual: AH Cep, AO Mon, ζ Phe – stars: fundamental parameters – stars: early types

1. The light-time effect in eclipsing binaries

The *light-time effect*, (hereafter LITE), is a well-known phenomenon with many interesting applications in stellar astrophysics. The eclipsing pair moves around the barycentre of a wider triple system, which produces periodic variations in the mid-eclipse times. The eclipses act as an accurate clock detecting variations in the distance to the object. It was originally Chandler (1888), who first mentioned the LITE as a possible reason for apparent period changes of Algol. The suitable formulae were derived by Irwin (1959), the necessary criteria were given by Frieboes-Conde & Herczeg (1973): (i) secondary minima behave identically with primary, (ii) a reasonable value of the mass function, (iii) corresponding variation in the systemic velocity, and (iv) an appropriate amount of the third light in the light-curve solution.

The theory of the third body motion and the LITE analysis in eclipsing binaries was reviewed several times in the literature, see e.g. Mayer (1990). The light travel time is given by

$$O-C = \frac{A}{\sqrt{1-e_3^2\cos^2\omega_3}}\left[\frac{1-e_3^2}{1+e_3\cos v}\sin(v+\omega_3)+e_3\sin\omega_3\right],$$

where e_3 is the eccentricity of the third-body orbit, ω_3 the longitude of periastron and v the mean anomaly. The observed semi-amplitude A of the light-time curve (in days) is

$$A = \frac{a_{12}\sin i_3}{173.15}\sqrt{1-e_3^2\cos^2\omega_3},$$

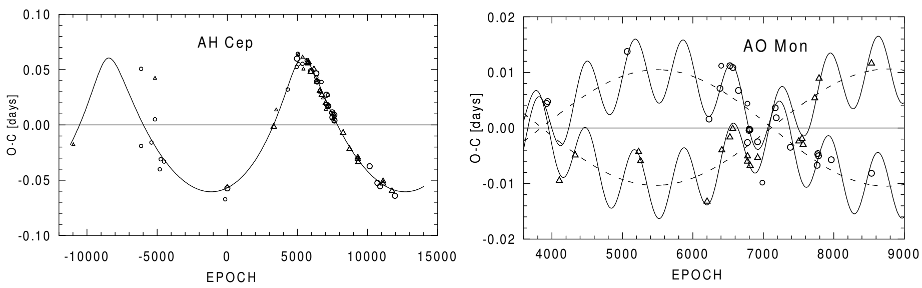

Figure 1. The current $O-C$ diagrams for two early type eclipsing binaries AH Cep and AO Mon. The individual primary and secondary minima are denoted by circles and triangles, resp. Sinusoidal curves represent a solution of the LITE (full curves). **AH Cep** demonstrates pure LITE with the period of 67.2 yr and a large amplitude of 87 min. In the case of **AO Mon** the apsidal advance with the period of 33.8 yr (dashed curves) is modulated by a very short period of the third body (full curves, 3.56 yr, Wolf et al. 2010).

where a_{12} is the semi-major axis of the relative orbit of the eclipsing pair around the common center of mass (in AU), i_3 is the inclination of the third-body orbit, e_3 is the eccentricity and ω_3 the longitude of periastron of the third-body orbit. There are 7 independent variables including the linear ephemeris to be determined in this procedure: $(T_0, P_s, A, T_3, P_3, e_3, \omega_3)$. The solution of LITE is usually presented on the $O-C$ diagram, see the Brussels workshop (Sterken 2005). The derived parameters of the third body orbit allow us to determine the mass function $f(M)$ of the triple systems

$$f(M) = \frac{M_3^3 \sin^3 i_3}{(M_1 + M_2 + M_3)^2} = \frac{a_3^3 \sin^3 i_3}{P_3^2} = \frac{1}{P_3^2} \left[\frac{173.15\, A}{\sqrt{1 - e_3^2 \cos^2 \omega_3}} \right]^3,$$

where P_3 is the period of the third-body (in years) and M_i are the masses of components. The systemic radial velocity of the eclipsing pair has an amplitude

$$K = \frac{A}{P_3} \frac{5156}{\sqrt{(1 - e_3^2)\,(1 - e_3^2 \cos^2 \omega_3)}}.$$

Assuming a coplanar orbit ($i_3 = 90°$) we can obtain lower limits for the mass of the third component $M_{3,min}$.

For the short third-body period the LITE demonstrates other effects of celestial mechanics similar to the Earth-Moon-Sun system (the binary-orbit precession). Recently, the LITE was used on triple star candidates among KEPLER binaries (Rappaport et al. 2013).

Table 1. Light-time effect parameters for three different early-type systems with the similar orbital period.

	AH Cep	AO Mon	ζ Phe
Spectrum	B0.5V+B0.5V	B3V+B5V	B6V+B9V
P_s [day]	1.77474242 (3)	1.88476297 (7)	1.6697772 (13)
T_0 [HJD]	24 34989.4614 (3)	24 40588.3277 (8)	24 41643.7382 (8)
P_3 [years]	67.2 (0.1)	3.56 (0.03)	220.9 (3.5)
e_3	0.544 (0.012)	0.0 (0.1)	0.37 (0.8)
A [day]	0.0605 (3)	0.0061 (2)	0.0808 (7)
ω_3 [deg]	79.1 (0.1)	54.0 (1.0)	97.1 (2.2)
T_3 [HJD]	24 44260	24 52818	24 36800
U_{aps} [years]	–	33.8 (0.5)	109.7 (1.2)

Examples: IU Aur, IM Aur, FZ CMa, TX Her, and AH Cep (67.2 yr, Fig. 1).

2. LITE and apsidal motion

Apsidal motion is the precession of the orbit due to a gravitational quadruple moment induced by tidal distorsion in a binary star. The rate of that motion is dependent on the density distribution in the stellar interior, and so is predictable from models, one of the few ways of conforming model computations. The periastron position ω at epoch E is defined by the linear equation

$\omega = \omega_0 + \dot{\omega}\ E,$

where $\dot{\omega}$ is the rate of periastron advance, and the position of periastron for the zero epoch T_0 is denoted as ω_0. The relation between the sidereal and the anomalistic period, P_s and P_a, is given by

$P_s = P_a\,(1 - \dot{\omega}/360°),$

and the period of apsidal motion by

$U_{aps} = 360° P_s/\dot{\omega}.$

The apsidal motion allows us to determine the internal structure constant (ISC) $k_{2,obs}$, which is an important parameter of stellar evolution models and is related to the variation of density within the star. The observed average value of $\overline{k}_{2,\rm obs}$ is given by

$$\overline{k}_{2,\rm obs} = \frac{1}{c_{21}+c_{22}}\,\frac{P_a}{U} = \frac{1}{c_{21}+c_{22}}\,\frac{\dot{\omega}}{360},$$

where c_{21} and c_{22} are functions of the orbital eccentricity, fractional radii, the masses of the components, and the ratio between rotational velocity of the stars

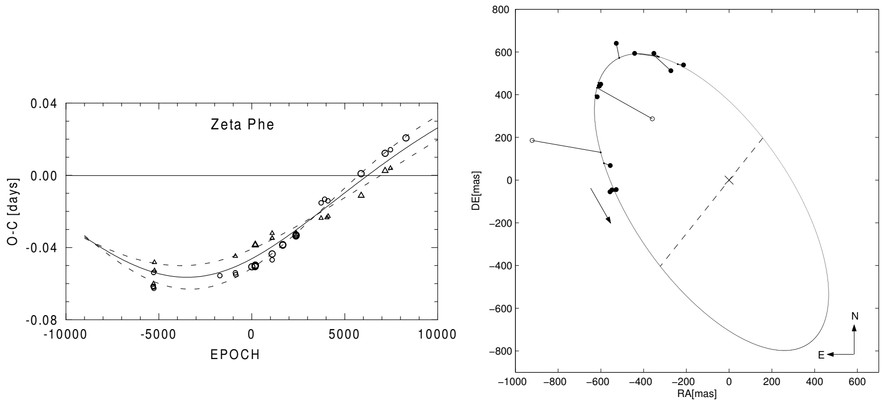

Figure 2. The current $O-C$ diagram for ζ Phe (left panel). This multiple system shows a similar combination of the apsidal motion with the period of 109.7 yr (dashed curves) superimposed with the LITE period of 221 yr, as a consequence of a visual orbit of the third component. The right panel shows the relative orbit of ζ Phe on the plane of the sky. The points represent individual observations, the cross indicates the position of the eclipsing binary. The distance to the system is 85.8 pc.

and Keplerian velocity (Kopal 1978). The rotation of the stars was assumed to be synchronized with the maximum angular orbital velocity achieved at periastron.

The combination of apsidal motion and LITE is relatively frequent in many eclipsing binaries with an eccentric orbit. On the $O-C$ diagram the significant sinusoidal period change due to apsidal motion is modified by a next sinusoidal change caused by presence of an additional orbiting body in the triple system. For apsidal-motion analyses the method of Giménez & García-Pelayo (1983), which is a weighted least-squares iterative procedure, can be used. There are **10** independent variables to be determined, 7 for the LITE and 3 for the apsidal motion. Superposition of the apsidal advance in the eccentric orbit and LITE caused by a third star is usually assumed:

$$(O-C)_{obs} = (O-C)_{aps} + (O-C)_{LITE}.$$

Examples: V539 Ara, CG Aur, GL Car, RU Mon, U Oph, YY Sgr, AO Vel, DR Vul, and AO Mon (see Fig. 1, Wolf et al. 2010).

3. LITE and astrometry

Combining astrometry and LITE in visual and eclipsing binaries is a next powerful method for study of multiple stellar systems. Methods given by Ribas et al. (2002) or Zasche & Wolf (2007) can be used. There are **10** parameters to be determined simultaneously, 7 for LITE and 3 for the astrometric orbit. The least

squares method and simplex algorithm are usually used. The astrometric orbit allows us to determine the distance to the system, the radius of the outer orbit and the precise mass of the third body. The final result of this method is very sensitive to the quality of the input data, especially if the method is used for determining the distances of these binaries. It can only be applied successfully in those cases where the astrometric orbit and the LITE in the $O-C$ diagram are well defined by existing observations.
Examples: QS Aql, 44i Boo, SZ Cam, QZ Car, VW Cep, V2388 Oph, V505 Sgr, HT Vir, or ζ Phe (see Fig. 2).

4. Conclusions

LITE in eclipsing binaries and its combination with other effects of celestial mechanics is a very efficient tool for study of triple and multiple stellar systems. Its usefulness in stellar astrophysics is comparable with the traditional methods of the light curve and radial-velocity curve analyses. It can only be applied successfully in those cases where the expected period of the third body is well defined by existing observations. LITE would be also very powerful in data processing of recent and upcoming photometric and astrometric space missions.

The LITE analysis does not have to be necessarily applied to eclipsing binaries, any periodic event could be potentially useful to detect stellar or sub-stellar companion. This includes pulsating stars and transiting exoplanets.

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