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The generation and stability of magnetic fields in CP stars

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Abstract. A variety of magnetohydrodynamic mechanisms that may play a role in magnetic, chemically peculiar (mCP) stars is reviewed. These involve dynamo mechanisms in laminar flows as well as turbulent environments, and magnetic instabilities of poloidal and toroidal fields as well as combinations of the two. While the proto-stellar phase makes the survival of primordial fields difficult, the variety of magnetic field configurations on mCP stars may be an indication for that they are instability remnants, but there is no process which is clearly superior in explaining the strong fields.

Key words: stars: chemically peculiar – stars: magnetic fields – stars: evolution – magnetohydrodynamics

1. What do we have to explain?

A considerable fraction of chemically peculiar A and B stars (CP stars) show strong surface magnetic fields of about 500 G up to over 10 kG. The evolution of these fields from star formation to their current presence is unknown. This paper compiles a number of physical processes which may or may not play a role in the whole scenario of CP star magnetism.

The fraction of main-sequence stars with radiative envelopes which show magnetic fields was found to increase from F stars to late B stars (Wolff 1968). The result was recently confirmed by Power *et al.* (2007) who found an increase of mCP star fractions with stellar mass up to $3.6\,M_\odot$ for nearby stars. A further increase towards higher masses was hidden in the small number of higher-mass stars. While the average fraction of mCP stars among normal stars of the same mass range is generally quoted as being roughly 10%, the fraction is less than 2% in the solar neighbourhood of 100 pc radius (Power *et al.*, 2007). Why only a fraction of intermediate-mass stars shows strong magnetic fields is one of the key issues in the search for their origin. The other is that mCP stars form a distinct sub-group with slow rotation among A and B stars.

There are indications of a decrease of magnetic field strength with stellar age for $M > 3\,M_\odot$ (Kochukhov, Bagnulo 2006; Landstreet *et al.*, 2007). Although the increase of stellar radius may cause the reduction at constant magnetic flux, it appears that even the flux is decreasing during the main-sequence life of mCP

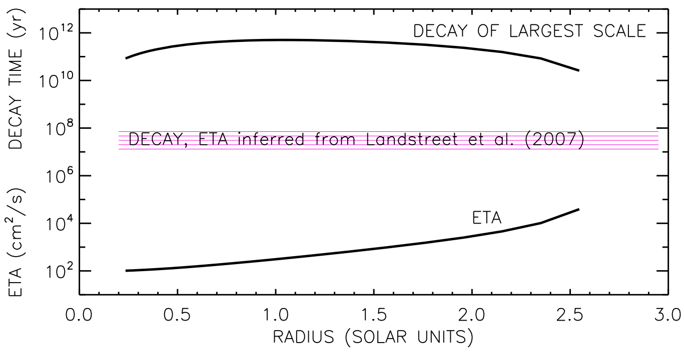

Figure 1. Magnetic diffusivities (lower part of diagram) and the corresponding decay times for magnetic structures (upper part of diagram). The diffusivity depends on the location in the star at radius r and so does the decay time of a structure as large as r. The "small-scale" curve refers to structures with a scale of $0.1r$ which decay 100 times faster. A rough estimate of the magnetic diffusivity according to the age dependence of magnetic field strengths of CP stars is given in middle.

stars. At first glance this decay may be attributed to ohmic decay in the radiative envelope of the star. The radial profile of the magnetic diffusivity $\eta(r)$ throughout the star according to Spitzer (1956) suggests that the decay time is of the order of 10^{11} yr everywhere in the radiation zone. The decay time inferred from Landstreet *et al.* (2007) is about 10^7 to 10^8 yr (numerically equal to η in $\mathrm{cm^2 s^{-1}}$, accidentally). Figure 1 shows η as a function of radius and the corresponding ohmic diffusion time $\tau_{\mathrm{diff}} = r^2/\eta$. The above decay time deduced from observations is also indicated – without radial dependence for obvious reasons. It appears as if the magnetic diffusivity is higher than the purely microscopic value hinting on very mild turbulence even without convection. Another puzzling fact is the decrease of obliquity of the magnetic field axis against the rotation axis with rotation period (Landstreet, Mathys 2000; Bagnulo *et al.*, 2002). MHD processes tend to prefer axisymmetric solutions for fast rotators, if a small degree of differential rotation is present in the star. This is the very opposite of what is observed. When comparing the rotational time-scale with the diffusive time-scale, even the slowest CP stars have rotation periods many orders of magnitude shorter than τ_{diff} and are fast rotators in that sense. Since in view of the time-scales normal A stars, and Ap stars are not vastly different, we tend to place the magnetic fields first – the reduced rotation is a consequence of their presence. In other words, the reason for mCP stars to be magnetic is not their initially slow rotation.

There is not only the gradual decrease of magnetic field strengths with the time which the $M > 3\,M_\odot$ stars spent on the main sequence, but also a possible late emergence of surface fields for stars with $M < 2\,M_\odot$. Hubrig *et al.* (2000)

found that these stars show magnetic fields only after they have spent about 30% of their life on the main sequence. Meanwhile, counter-examples were found, and the appearance of fields is certainly not as sharp as it seemed at an earlier research stage.

It should be noted that in general all these characteristics are based on quite scattered individual stars. The scatter is very likely not a result of observing uncertainties. Any physical process leading to the observed features must also allow for a certain bandwidth of results or sufficient input of stochastic behaviour.

2. The laminar dynamo

In a largely radiative environment, non-turbulent (laminar) flows may still be present, namely meridional circulation and differential rotation. These flows are driven radiatively (Eddington, 1929; Sweet, 1950; Kippenhahn, 1958). While turbulent convection zones provide very good prospects for dynamo action, one has to search for dynamo solutions in laminar flows, if an MHD dynamo is considered an option for the presence of magnetic fields on CP stars. Solutions for special flow constructions are known for more than 50 years (e.g. Bullard, Gellman 1954) and always provide non-axisymmetric solutions according to the theorem of Cowling (1934). A comprehensive compilation of spherical flows and their dynamo solutions was given by Dudley and James (1989, hereafter DJ). The Cowling theorem says that an axisymmetric magnetic field cannot be sustained by dynamo action. This is essential for laminar dynamos, while it is unimportant for turbulent dynamos where nonaxisymmetries will always be present in small-scale fields.

The rotation rate is typically normalized in terms of the magnetic Reynolds number $\mathrm{Rm} = R^2\Omega/\eta$ where R is the radius of the sphere and Ω is a typical angular velocity. DJ constructed a very simple flow which does provide a dynamo for fairly low Rm. All these dynamo models are kinematic; the equation of motion with the corresponding Lorentz force is not considered in any of them. Only the induction equation

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{u} \times \boldsymbol{B} - \eta \nabla \times \boldsymbol{B}), \qquad (1)$$

is used, with a prescribed $\boldsymbol{u}$. This is suitable for searches for the onset of dynamo action; the full nonlinear evolution of the dynamo also requires the momentum equation with a Lorentz force. All the dynamo and stability results obtained for this paper were computed with the MHD code by Hollerbach (2000).

The original DJ flow has an $\Omega(r)$ profile decreasing with radius and a meridional circulation which points towards the equator near the surface. Figure 2 shows the critical Rm for the onset of dynamo action as a function of the meridional flow speed. The amplitude of the meridional flow velocity is measured as a fraction of the maximum azimuthal velocity, $\epsilon = \max(u_\theta)/\max(u_\phi)$. The original DJ system for a full sphere is shown as a solid line. In an A star, we expect

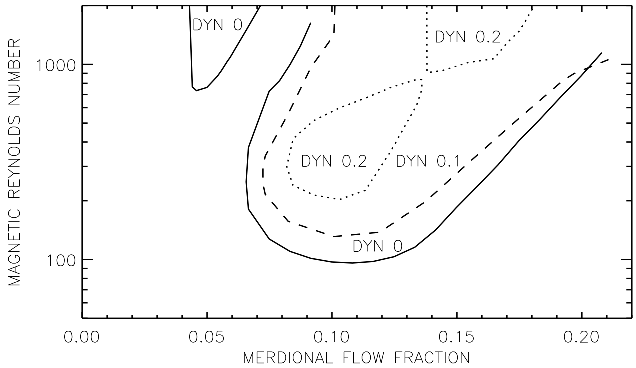

Figure 2. Dynamo action for the modified dynamo by Dudley and James (1989). The solid line refers to a full-sphere dynamo where the meridional flow covers the entire sphere, the dashed line is for a meridional going down to $0.1R_*$, and the dotted line is for a flow down to $0.2R_*$. The regions denoted with 'DYN 0' etc. show dynamo action for these three cases, respectively.

a convective core with very strong turbulent diffusion acting like a vacuum hole. We have modified the flow to obey a central hole and have plotted the critical Rm lines for core sizes of $0.1\,R$ and $0.2\,R$. The area in which dynamo action is found diminishes and is entirely gone for an inner hole of $0.24R$. Moreover, a meridional flow consistent with a negative Ω gradient would actually be poleward below the surface. There is no dynamo action for such a flow, but it is retained adding latitudinal differential rotation (Arlt, 2006). For another class of meridional flows without any differential rotation, Moss (2006) did find dynamo solutions, all with critical Rm beyond 10^4. Since all the solutions found so far consist of rather simple $m = 1$ modes, the diversity of magnetic field configurations observed can probably not be covered by such dynamos.

3. The stability of magnetic fields in radiative envelopes

There are essentially two types of instabilities which are relevant for magnetic fields in radiative envelopes. One is the magnetorotational instability (MRI; Velikhov, 1959; Balbus, Hawley 1991) which relies on a differential rotation decreasing with distance from the axis, and the other class is curent-driven instabilities for which rotation is only a modifier.

Especially in the beginning of the existence of the radiative envelope, the magnetorotational instability may be of interest for mCP sars. Only small magnetic fields are required for the onset of the instability. Since the energy source for the instability is the differential rotation instead of the magnetic field, perturbations may grow to a considerable strength in fairly short times. The stability analysis for nearly all magnetic-field configurations shows instability for field strengths which are not too strong (Balbus, Hawley 1998). How strong is that? The instability becomes significantly weaker if the scale of the dominant unstable mode is larger than the object size. In a CP star with a differential rotation of as small as 5%, the maximum field strength is as large as 20 kG; it is larger than 100 kG for 30% differential rotataion. Even though the MRI is a weak-field instability it has a huge range of applicability in a stellar context.

Arlt *et al.* (2003) have studied such a scenario in a spherical shell. The setup contained a magnetic field perturbation with both axisymmetric and non-axisymmetric parts, and without symmetry about the equator, in order to allow for a variety of modes to grow. The rotation rate decreases with axis distance. The model is simplified in that it employs an incompressible spherical shell with constant density, but allows for a subadiabatic temperature profile as is present in radiative zones (Boussinesq approximation). The equations now read

$$\frac{\partial \boldsymbol{u}}{\partial t} = \nu\nabla^2\boldsymbol{u} - (\boldsymbol{u}\cdot\nabla)\,\boldsymbol{u} - \nabla p - \alpha\Theta\boldsymbol{g} + \frac{1}{\mu_0\rho_0}(\nabla\times\boldsymbol{B})\times\boldsymbol{B}, \qquad (2)$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \eta\nabla^2\boldsymbol{B} + \nabla\times(\boldsymbol{u}\times\boldsymbol{B}), \qquad (3)$$

$$\frac{\partial \Theta}{\partial t} = \kappa\nabla^2\Theta - \boldsymbol{u}\cdot\nabla\Theta - \boldsymbol{u}\cdot\nabla T_{\rm s}, \qquad (4)$$

where ν is the kinematic viscosity, p is the pressure, α is the thermal expansion coefficient, Θ denotes temperature deviations from a purely conductive background profile $T_{\rm s}$, $\boldsymbol{g}$ is the gravitational acceleration, and κ is the thermal diffusivity. The constants μ_0 and ρ_0 are the magnetic permeability and the density, respectively.

The full three-dimensional and nonlinear evolution of an initial rotation profile and magnetic field is followed. The MRI sets in much faster than the diffusive time-scale, and the fluctuations of $\boldsymbol{u}$ and $\boldsymbol{B}$ start to redistribute angular momentum within the radiative zone by Reynolds stresses (correlation of velocity fluctuations) and Maxwell stresses (correlation of magnetic-field fluctuations).

The model was modified to include a background density stratification, sacrificing the temperature equation though. The time-scale for the redistribution of angular momentum remains the same as for the model with a stable temperture gradient. A snap-shot of a meridional section of the resulting flows is shown in Figure 3. At that point the angular momentum is already entirely redistributed towards a uniform rotation. Realistic magnetic Reynolds numbers are not accessible in a time-dependent numerical simulation. One always has to compute a set of scenarios and extrapolate the results such as magnetic field strength and decay time for the differential rotation to stellar parameters. The calculations presented here lead to a time-scale between 10 and 100 Myr for the decay of any initial differential rotation in mCP stars. The other class of insta-

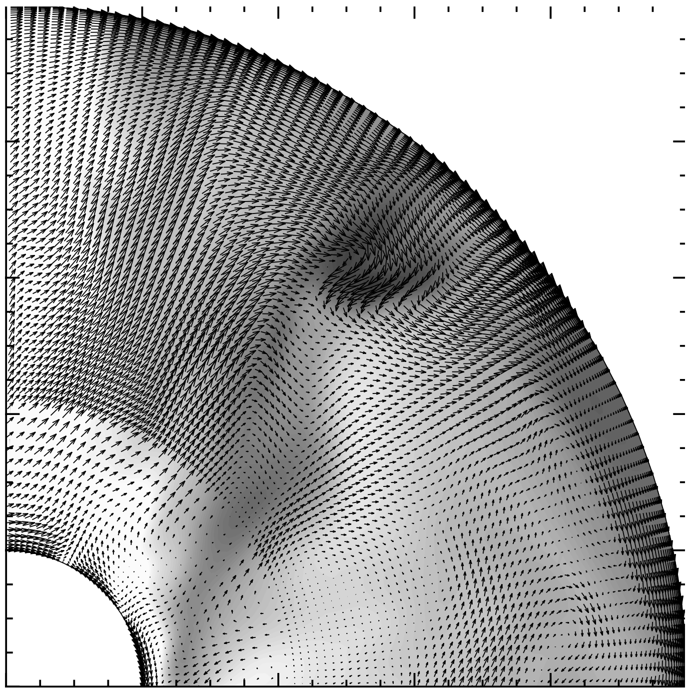

Figure 3. Velocity field after the onset of the magnetorotational instability in a stratified sphere. The graph is a vertical cut-out of the full-sphere simulation.

bilities is not based on the differential rotation, but on the presence of currents in the initial magnetic field. Nearly all poloidal magnetic fields become unstable when imposing a small non-axisymmetric perturbation (cf. Braithwaite, 2007, and references therein). The same holds for purely toroidal magnetic fields which were studied by Vandakurov (1972) and Tayler (1973).

Solid-body rotation has a stabilizing effect on poloidal as well as toroidal magnetic fields. For this review, the dependence of the stability of a purely poloidal field on the rotation rate was computed. The normalized rotation rate is again expressed by the magnetic Reynolds number. The stability limit versus Rm is plotted in Fig. 4. The Hartmann number is the dimensionless magnetic field strength; it needs to be converted to physical values for specific objects. At high Rm – the ones in which we are interested in the case of CP stars – the critical magnetic field strength becomes a power function of Rm. We can try to extrapolate the graph to the very high magnetic Reynolds numbers of CP stars. The result scales to a critical field strength of about 10 G for a rotation period of 1 day, and to about 1 G for a 100-day period. Maximum field strengths of up to 100 G were found for purely toroidal fields in the solar radiation zone

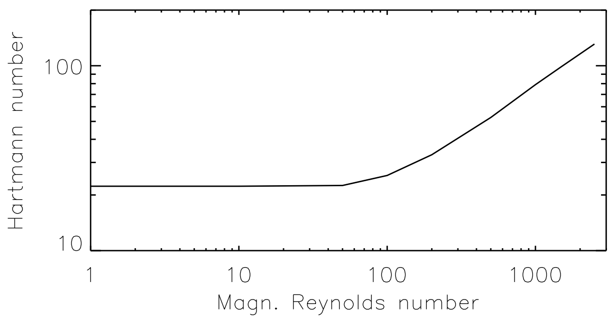

Figure 4. Stability of a poloidal magnetic field against a non-axisymmetric perturbation in dependence on the rotation rate in terms of Reynolds number.

(Arlt *et al.*, 2007). All these limits are much below the observed field strengths. It is possible construct a magnetic field of unlimited stability. Chandrasekhar and Kendall (1957) reported an entirely force-free field, and we confirmed its stability numerically. The construction requires the sphere to be embedded in a perfectly conducting medium. If vacuum is employed, the stability is lost, and surface forces emerge with unkown effect. Since the nonlinear simulations of Braithwaite and Nordlund (2006) lead to a configuration very similar to the force-free field, further studies are worthwhile in this direction.

4. An early-life dynamo

The early contraction phase of star formation may host a turbulent dynamo which can provide kG-fields. Once the star becomes radiative, the generated field could become frozen in and visible at the surface when the convective shell has vanished. The idea is supported by the findings of Dikpati *et al.* (2006) who showed that the oscillatory dynamo of the solar convection zone can lead to the build-up of stationary fields in the radiative solar interior. The following numerical experiment probes the applicability of such a mechanism for mCP stars. A similar approach was followed by Kitchatinov *et al.* (2001) for solar-type stars.

We start with a fully convective sphere in which the α-effect is believed to operate as a field generator. The α-effect is the description of the generation of poloidal magnetic fields from toroidal ones and vice versa in rotating, stratified turbulence. As time progresses, the convection zone becomes a shell, and the inner radius of the shell increases gradually. The core is radiative which is represented numerically by a vanishing α-effect and a magnetic diffusivity which is 1000 times smaller than the turbulent magnetic diffusivity $\eta_{\rm T}$ in the convection zone. The induction equation for a dynamo with a magnetic diffusivity

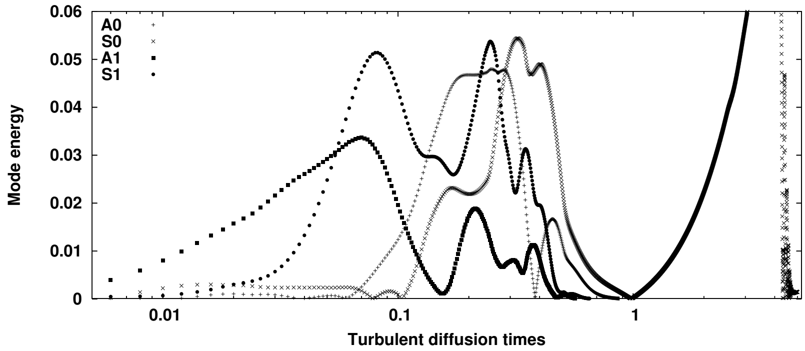

Figure 5. Dynamo solution based on a an evolution from a fully convective sphere to a radiative sphere with convective core. ‘A’ refers to equator-antisymmetry, ‘S’ to symmetry. ‘0‘ denotes axisymmetric $m = 0$ modes, ‘1’ non-axisymmetric $m = 1$ modes.

depending on the location in the star is

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times \left(\boldsymbol{u} \times \boldsymbol{B} + \alpha \boldsymbol{B} - \eta \nabla \times \boldsymbol{B} - \frac{1}{2}(\nabla \eta) \times \boldsymbol{B} \right), \tag{5}$$

where $\boldsymbol{u}$ can contain all large-scale velocities such as differential rotation and meridional circulation. The α effect is distributed such that it is proportional to $\sin^2\theta\cos\theta$ in the convective shell as well as in the inner convective core forming near the end of the simulation, where θ is the colatitude. We also introduced an anisotropy in the α-tensor as expected from rotating turbulence, such that the α-component parallel to the rotation axis vanishes (Rüdiger, Kitchatinov 1993).

The last term in Eq. (5) is due to the turbulence gradient and can be described as turbulent pumping since it has the effect of a velocity (Zel’dovich, 1957; Ruzmaikin, Vainshtein 1978). The term is often neglected in computations of turbulent dynamos with functions $\eta_{\rm T}(r)$, but plays a significant role in confining magnetic fields in radiative interiors.

The temporal evolution in Fig. 5 shows a short phase with dominance of the non-axisymmetric mode. This agrees with earlier findings that fully convective shells may provide $m = 1$ dynamo modes. When the radiative core becomes larger than $0.22R_\odot$, the $m = 0$ dominates and does so for the remaining time of the simulation. The storage of magnetic flux in the radiative interior appears to be chiefly axisymmetric, unfortunately. An initial convective dynamo may thus not provide the observed fields directly. The onset of a core dynamo does produce strong internal fields, but these weaken severely towards larger distances from the center. This was also found from direct numerical simulations by Brun *et al.* (2005).

Table 1. Four scenarios of CP star magnetism.

Background field amplified at star formation + random orientation + field strength decreases with age − convective phase or MRI produce small-scale field − contradicts possible late emergence of field	**Dynamo in radiative zone** + straight-forward mechanism − $m = 1$ modes cause regular obliquity − field strength does not decreases with age − rather independent of star formation → no distinction between A and Ap − dynamo difficult to excite
Early-life dynamo remnants + field strength decreases with age − fully convective phase questionable − non-axisymmetry only very shortly	**Early-life dynamo + field instab.** + random orientations at emergence + field strength decreases slowly with age

5. Fossil fields

An issue we have not addressed in this review so far is the survival of large scale magnetic fields from before the collapse of the star. Although we concentrated on dynamos and instabilities, a few words may be necessary for a comprehensive picture. A fully convective phase suggested to take place for stars with masses below $2.4M_\odot$ (Palla, Stahler 1993) could destroy these primordial fields in a rather short time of a few thousand years due to turbulent diffusion. Moss (2003) argued, however, that the exceptional strength of the fields in mCP star progenitors reduces turbulent diffusion and allows the survival of sufficient flux to be obervable after the star has reached the zero-age main-sequence. The exact behaviour of the diffusion reduction – known as η-quenching in mean-field electrodynamics – is not known though.

Even if it is not largely convective, the proto-stellar object will also be subject to differential rotation. Torques exerted by accreted material, magnetic coupling to the accretion disk and magnetic winds very likely cause differential rotation in the star (Stępień, 2000). Primordial fields of arbitrary direction will excite magnetorotational instability as long as the magnetic fields are weaker than $O(100)$ kG which is probably true. The instability is actually a good candicate for the diversity of field configurations afterwards; it also allows for amplification of the field, with the energy being taken from the shear.

6. Summary

A compilation of a few possible mechanism in CP star magnetism with their pros and contras are listed in Table 1. When searching for mechanisms explaining considerable magnetic fields in radiative envelopes, one always returns to the question why only a small fraction of A stars shows such fields. The effect of different star formation environments on the existence of strong fields on main-sequence stars now needs to be investigated. Bridging collapse and pre-main-sequence evolution is not a trivial task though.

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