

## Atlas of H $\alpha$ emission lines and V light curves of 30 carbon Miras

Z. Mikulášek<sup>1</sup> and T. Gráf<sup>2</sup>

<sup>1</sup> *Institute of Theoretical Physics and Astrophysics, Faculty of Science, Masaryk University Brno, Kotlářská 2, 611 37 Brno, The Czech Republic, (E-mail: mikulas@ics.muni.cz)*

<sup>2</sup> *Observatory and Planetarium of J. Palisa, 17. listopadu 15, 708 33 Ostrava-Poruba, The Czech Republic, (E-mail: tomas.graf@vsb.cz)*

Received: November 9, 2004; Accepted: May 4, 2005

**Abstract.** This paper presents an atlas of V light curves and H $\alpha$  emission lines of 30 carbon Miras observed in various photometrical phases. The visualization of both photometric and spectral variations allowed us to reveal a strong correlation between the equivalent widths of the H $\alpha$  emission of carbon Miras and their V brightnesses as a function of the photometric phase.

**Key words:** Carbon stars – Carbon Miras – H $\alpha$  emission – light curves

### 1. Introduction

Carbon stars are cool objects of the asymptotic giant branch in an advanced stage of their evolution when outer layers of stars have been polluted by products of their inner shell helium nuclear burning. All carbon stars exhibit light variations on the time scale of tens and hundreds of days, apparently caused by pulsations and propagations of shock waves through the stars. Light variations of part of carbon stars are semiregular or maybe multiperiodic (Dušek & Mikulášek, 2003), other carbon stars vary more or less regularly with a single well defined period of several hundred days.

The variable stars called *carbon Miras* exhibit distinguished light variations of a few magnitudes in the V region, which make them popular targets for visual observers. Carbon Miras show also distinct spectroscopic variations that can give us a hint of mechanisms of variability of these stars. For example, it was found that apparently all carbon Miras display strongly variable H $\alpha$  emissions, which vary with the period of light variations (Barnbaum, 1994; Gráf, 2003).

The aim of this paper is to visualize and investigate the relation between variations of the equivalent width of H $\alpha$  emission lines and photometric V variations for 30 carbon Miras.

**Table 1.** The basic characteristics of studied carbon Miras. The star names; the spectral classifications according to SIMBAD; osculating or SIMBAD julian date of the basic maximum  $\widetilde{M}_0 = M_0 - 2\,400\,000$ , osculating or SIMBAD period  $P$  in days, SIMBAD ephemeris are those in brackets. The  $V_{\min}$  denotes mean minimum  $V$  magnitudes of individual stars and  $A$  their mean amplitudes,  $A = V_{\min} - V_{\max}$ , in mags.  $N_{\text{HP}}$  and  $N_{\text{vis}}$  are the numbers of Hipparcos measurements and visual estimates of brightness used for the light curve analysis, respectively.

name	sp. type	$\widetilde{M}_0$	$P$	$V_{\min}$	$A$	$N_{\text{HP}}$	$N_{\text{vis}}$	Fig.
V374 Aql	C7.3	49 985±10	466±3	13.2	1.5	0	89	2
S Aur	C4-5	47 908±5	590.4±0.7	12.6	2.1	0	2 966	6
UV Aur	C6.2-C8.2Jep	48 124±5	397.7±0.7	10.3	1.5	72	2 663	5
AU Aur	C6-7.3e	48 074±3	396.8±0.8	13.8	1.6	73	152	3
AZ Aur	C7.1e-C8.2-3	48 270±2	414.3±0.3	13.2	3.3	0	1 571	4
S Cam	C7.3e	47 950±2	326.2±0.2	10.4	1.8	163	5 511	8
R CMi	C7.1Je	48 080±2	330.8±0.4	11.0	2.8	120	4 278	7
W Cas	C7.1e	47 880±1	403.6±0.2	12.0	2.7	256	9 870	9
X Cas	C5.4e	48 210±2	427.7±0.3	12.4	2.0	108	3 097	10
HV Cas	C4.3-5.4e	(38 120)	(527.1)			0	0	12
S Cep	C7.4e	48 176±2	485.5±0.5	10.2	2.3	121	5 140	11
V CrB	C6.2e	47 688±1	357.3±0.2	11.6	3.0	141	9 300	13
U Cyg	C7.2e-C9.2	47 792±2	462.3±0.4	10.7	3.3	117	11 892	16
V Cyg	C5.3e-C7.4e	47 786±2	419.6±0.2	13.7	3.8	81	3 592	14
RS Cyg	C8.2e	48 154±2	427.5±0.3	8.9	1.5	129	14 336	15
WX Cyg	C8.2JLi	47 848±2	410.8±0.3	13.3	2.9	131	3 541	18
T Dra	C6.2e-C8.3e	47 762±2	424.7±0.3	12.8	3.3	0	2 505	19
R For	C4.3e	48 178±2	386.5±0.3	13.3	2.8	0	1 087	17
VX Gem	C7.2e-C9.1e	48 069±3	383.1±0.6	13.0	4.0	88	953	20
ZZ Gem	C5.3e	47 881±3	316.1±0.4	11.3	2.1	0	1 209	22
V Hya	C6.3e-C7.5e	47 943±4	530.2±0.7	9.5	2.0	98	1 774	21
CZ Hya	Ce	(27 546)	(442)			0	0	24
R Lep	C7.6e	48 187±2	439.1±0.4	10.2	2.5	131	6 030	25
U Lyr	C4.5e	48 033±3	450.7±0.5	12.3	1.8	0	3 563	23
CL Mon	C6.3e	48 008±9	497±2	13.9	3.5	0	93	27
V Oph	C5.2-C7.4e	47 789±2	294.9±0.3	10.6	2.6	84	2 113	28
RZ Peg	C9.1e(Tc)	47 856±2	437.0±0.2	12.6	4.2	142	3 764	29
SY Per	C6.4e	48 021±5	476.4±0.9	11.6	1.8	83	1 369	30
RU Vir	C8.1e	47 783±2	442.6±0.4	13.6	2.9	84	1 906	26
SS Vir	C6.3e	47 887±4	360.4±0.3	9.5	2.1	0	5 591	31

## 2. Observations

The spectra used in the atlas were obtained by Barnbaum (1994) with the 3-m Shane Telescope at the Lick Observatory. The Hamilton Echelle spectrograph at coudé produces spectra (0.013 nm at 610 nm) with a linear reciprocal dispersion ranging from 0.28 nm mm<sup>-1</sup> in the blue to 4.3 nm mm<sup>-1</sup> in the red. The detector, a TI 800 × 800 CCD, measures only 12 mm. The setup results in 40 spectral orders per observation. The range with the H $\alpha$  line is order 87.

Barnbaum's list of observed cool stars comprises 87 individual objects. For the purpose of this paper we selected only the objects with H $\alpha$  line in emission.

Light variations of particular carbon Miras of our list were analyzed on the basis of observational material obtained from  $JD=2\,445\,000$  to  $2\,451\,000$  (from 1982 to 1998). To this end we used databases of the Hipparcos satellite (<http://simbad.u-strasbg.fr/>, 19 stars from 30), AAVSO (<http://www.aavso.org/>, 28 stars), AFOEV (<http://cdsweb.u-strasbg.fr/afoev/english.htm>, 25 stars) and VSOLJ (24 stars) companies of visual variable stars observers. In the case of Hipparcos observations we have used only the  $H_p$  measurements because they are much more accurate than the observations done in its pseudo  $V$  or  $B$  colors. In the total we have used 109 955 individual visual estimates of the stars' magnitudes and 2 222  $H_p$  measurements for this purpose. Both Hipparcos  $H_p$  and visual observations were merged into normal points with different weights corresponding to the number and reliability of measurements.

Using all the above mentioned observational data we were able to construct more or less reliable light curves of 28 from 30 carbon Miras listed in our catalogue. Only two stars from our list (HV Cas and CZ Hya) were not observed frequently enough so that we were not able to determine the course of their  $V$  light curves and predict reliably the magnitudes of the stars at the moment of their spectroscopic observations.

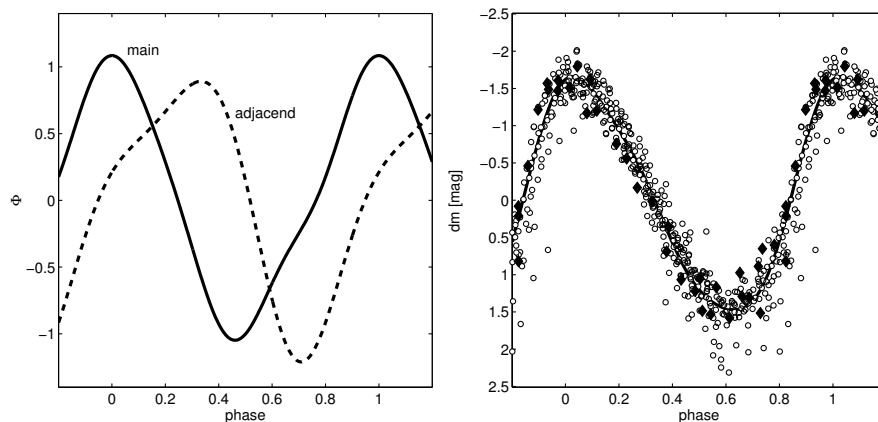
### 3. Light variations

After a careful inspection of the courses of  $V$  and  $H_p$  light curves of the studied carbon Miras, we have concluded that these are strictly periodical functions whose mean magnitudes and amplitudes are modulated by secular variations. The periods of the studied 30 carbon Miras lie in the range of 295 to 590 days with the median value of 426 days. The color indices ( $H_p - V$ ) of carbon Miras turned out to be roughly linearly proportional to  $V$ , so  $H_p$  magnitudes were transformed into  $V$  ones by a simple linear relation described by two parameters:  $\overline{(H_p - V)}_i$  and  $h_i$ , the latter expressing the ratio of the  $H_p$  amplitude and  $V$  amplitude, specific for each star. So, we have concluded that variations of each studied carbon Mira in  $V$  and  $H_p$  colors can be satisfactorily well described by the model for the first time formulated in Mikulášek & Gráf, (2005):

$$V_i(t) = \bar{V}_i + \sum_{j=1}^{p_i} b_{ij} S_j(T) + \left[ 1 + \sum_{k=1}^{q_i} a_{ik} S_k(T) \right] F(\varphi), \quad (1)$$

$$H_{p_i}(t) = \bar{V}_i + \overline{(H_p - V)}_i + h_i \left\{ \sum_{j=1}^{p_i} b_{ij} S_j(T) + \left[ 1 + \sum_{k=1}^{q_i} a_{ik} S_k(T) \right] F(\varphi) \right\}, \quad (2)$$

$$\varphi = \text{frac} [(t - M_{0i})/P_i]; \quad T = (2t - t_e - t_b)/(t_e - t_b), \quad (3)$$



**Figure 1.** (a) Phase graph of the first and second principal functions  $\Phi_1(\varphi)$  and  $\Phi_2(\varphi)$ . (b) The periodic component of the light curve of V CrB, one of the carbon Miras from our list. Open circles are normal points created from visual estimates, full diamonds denote *H<sub>p</sub>* observation transformed into *V*. The graph was constructed on the basis of 141 Hipparcos observations and 2300 individual visual estimates.

where  $\overline{V}_i$  is the average magnitude and  $\overline{(H_p - V)}_i$  is the mean color index of the  $i$ -th star,  $b_{ij}$  and  $a_{ik}$  are coefficients describing long-term variations of the mean magnitude and relative variation of the amplitude of that star, respectively.  $t$  is the time of the individual observation,  $t_b$  and  $t_e$  are the times of the beginning and end of the time interval covered by the observational data.  $S_i$  is an  $i$ -th term in a series of appropriate functions defined on the interval  $\langle -1, 1 \rangle$ . Legendre polynomials used in Mikulášek & Gráf, (2005) have been replaced by a sequence of quasi-orthogonal trigonometric functions with inflection points at the borders of the interval:  $\{1; T; \cos(\pi T/2) - 2/\pi; \sin(\pi T); \cos(3\pi T/2) + 2/(3\pi); \sin(2\pi T); \cos(5\pi T/2) - 2/(5\pi); \sin(3\pi T); \dots\}$  introduced firstly in a rudimentary form in Mikulášek et al. 2001. This series fits the observed data more evenly and does not diverge at the borders of the interval.

$F(\varphi, A_{mi})$  is a representation of the periodical term of the light curve of  $i$ -th star, described by a set of parameters  $\{A_{mi}\}$  with the meaning of an amplitude. All parameters (including  $P_i$  and  $M_{0i}$ ) of this non-linear model of light curves of carbon Miras were found by the gradient method of robust regression effectively suppressing the influence of outliers (see Mikulášek et al., 2003). As an initial estimate of ephemeris we adopted those given in Mikulášek & Gráf, (2005).

We tried to find the function  $F(\varphi, A_{mi})$  in a form valid for all photometrically studied 28 carbon Miras. The function was found by means of a generalized weighted principal component analysis. We have revealed that the function can be favorably expressed through the linear combination of only two principal,

**Table 2.** The list of spectrograms, photometric characteristics and equivalent widths of H $\alpha$  emission for 30 carbon Miras. The GCVS name of the star; the designation of the spectrogram,  $\tilde{JD} = JD - 2\,400\,000$ , where  $JD$  is the Julian date of the exposure; the photometric phase in respect to the oscillating light elements given in Table 2;  $\Delta m$  is the difference of the predicted V magnitude of instantaneous light curve minimum and the predicted V magnitude of the star in the moment of the spectrum exposure in mags; equivalent width of H $\alpha$  emission  $EW$  found by the IRAF technique in Ångströms.

star	$\tilde{JD}$	phase	$\Delta m$	$EW$	star	$\tilde{JD}$	phase	$\Delta m$	$EW$	star	$\tilde{JD}$	phase	$\Delta m$	$EW$
V374 Aql	a 47662	0.019	1.5	4.5	U Cyg	a 47723	0.850	2.2	17.8	R Lep	a 48194	0.017	2.5	9.9
	b 48141	0.046	1.4	3.1		b 48140	0.752	1.7	7.8		a 47661	0.174	1.2	3.9
S Aur	a 47871	0.937	1.9	1.9	V Cyg	a 47723	0.850	1.9	2.2	U Lyr	b 48049	0.035	1.7	5.7
	b 48175	0.451	0.0	1.2		b 48140	0.844	1.8	2.2		c 48140	0.237	0.9	3.1
	c 48222	0.531	0.3	1.7	RS Cyg	a 47662	0.850	1.5	4.0	CL Mon	a 47960	0.904	3.0	6.7
UV Aur	a 47870	0.362	0.5	1.9		b 48141	0.970	1.5	2.6		b 48175	0.337	0.8	0.6
	b 48175	0.125	1.3	12.9	WX Cyg	a 47723	0.695	2.0	11.9		c 48194	0.375	0.4	1.1
AU Aur	a 48175	0.255	0.8	7.2							d 48205	0.397	0.2	1.1
AZ Aur	a 47870	0.035	3.3	24.8	T Dra	a 47312	0.941	3.0	8.4		e 48222	0.431	0.1	0.7
	b 48194	0.817	1.4	10.4		b 47662	0.765	0.6	1.6	V Oph	f 48234	0.455	0.0	1.0
	c 48222	0.531	0.3	1.7		c 48049	0.676	0.1	1.0		a 47661	0.565	1.0	5.0
S Cam	a 47870	0.756	1.5	7.5	R For	d 48140	0.890	2.2	7.8		b 47723	0.775	2.4	8.7
	b 48234	0.872	1.8	5.4							c 48047	0.874	2.5	9.8
R CMi	a 47870	0.366	0.1	2.9		a 48141	0.905	2.4	10.4	RZ Peg	a 47723	0.696	0.1	4.8
	b 47960	0.638	1.7	12.6		b 48519	0.883	1.9	4.7		b 47871	0.035	4.2	17.6
	c 48194	0.345	0.3	1.9	VX Gem	a 47960	0.715	0.5	4.1		c 48194	0.774	0.8	6.9
	d 48205	0.378	0.1	1.3	ZZ Gem	a 47870	0.966	2.1	10.3	SY Per	a 47870	0.684	0.5	5.0
	a 47870	0.975	2.6	7.2		b 48194	0.991	2.2	7.8		b 48140	0.251	0.9	1.7
	b 48205	0.805	2.3	15.1	V Hya						c 48205	0.387	0.2	3.1
X Cas	a 47870	0.204	1.3	4.4		a 47312	0.810	1.6	6.8	RU Vir	a 47312	0.936	0.2	12.0
	b 48141	0.838	1.5	8.2		b 47661	0.468	0.0	2.1		b 47661	0.725	0.7	3.1
	c 48175	0.917	1.8	9.8		c 47870	0.863	1.7	6.0		c 47870	0.197	2.0	11.2
	a 47870	(0.498)				d 47960	0.020	1.9	5.6		d 47960	0.401	0.3	1.9
HV Cas	a 47870					e 48049	0.200	1.2	3.2		e 48234	0.020	2.4	14.8
	a 47723	0.064	1.9	6.5		f 48222	0.526	0.3	4.3					
	b 48049	0.738	1.1	5.2		g 48229	0.540	0.4	5.0	SS Vir	a 47312	0.404	0.1	1.8
	c 48140	0.925	2.2	8.8	CZ Hya	h 48234	0.549	0.4	5.8		b 47960	0.202	1.4	2.3
	a 47312	0.947	2.9	8.2		i 48254	0.587	0.6	2.9		c 48048	0.446	0.0	1.7
	b 47661	0.924	2.5	11.6		a 47312	(0.719)		13.3		d 48234	0.962	2.0	8.0
	c 47723	0.097	2.8	14.9		b 47662	(0.511)		3.8					
	d 47960	0.760	0.5	5.1		c 47870	(0.982)		16.2					
	e 48047	0.004	3.0	14.2		d 47960	(0.186)		4.0					
						e 48222	(0.778)		9.6					
						f 48234	(0.805)		14.6					

mutually orthogonal normalized functions  $\Phi_1(\varphi)$  and  $\Phi_2(\varphi)$ :

$$F(\varphi, A_{1i}, A_{2i}) = A_{1i}\Phi_1(\varphi) + A_{2i}\Phi_2(\varphi). \quad (4)$$

The first principal component reflects common properties of all periodical parts of light curves of studied carbon Miras, so we have centered the zero phase to the maximum of  $\Phi_1(\varphi)$  (see Fig 1). The second, usually smaller component expresses the observed variety of a periodical part of light curves of carbon Miras through a certain “distortion” of the main light curve’s component.

The amount of a periodic component of the variability in the time interval of spectroscopic observations is represented by the average value of the difference between the minimum and maximum magnitude of the star at that moments  $A$  (see Tab. 1). The minimum, median and maximum amplitudes of  $V$  light variations amount to 4.2, 2.6 and 1.5 magnitudes, respectively.

All figures of light curves of individual stars given in the atlas are of course only narrow segments of the whole treated time intervals, which cover dates of spectroscopic observations. The normal points of visual observations are depicted by open circles, Hipparcos normal points are denoted by full diamonds.

#### 4. H $\alpha$ emissions and their variations

The appearance of H $\alpha$  emissions in spectra of carbon Miras is quite disparate, their profiles are as a rule complicated - if they do not have several prominent components, then they are at least conspicuously asymmetrical. Equivalent widths of all H $\alpha$  emissions of particular stars are strongly time dependent - sporadically we can encounter variations of one order (UV Aur, R CMi, RU Vir or CL Mon). The time variability is well documented by the spectrograms depicting H $\alpha$  emissions reduced to their close vicinity (from 655 to 658 nm) to 1. Figures in the atlas are arranged according to Julian dates of exposures of the particular spectrum, the moments of which are given in Table 2. In the same table you can find equivalent widths obtained by the standard IRAF procedure. The minimum, median and maximum equivalent widths are 0.6, 5.4 and 24.8 Å, respectively.

The following atlas of H $\alpha$  emission lines includes 89 spectrograms of 30 stars at various photometric phases. The emission spectra in similar phases are comparable and a strong photometric phase dependency is obvious. This rough visual assessment of the emission appearances served as a starting-point for further thoughts about the nature of the dependency and for efforts to express the dependency in more detail (Gráf, 2003).

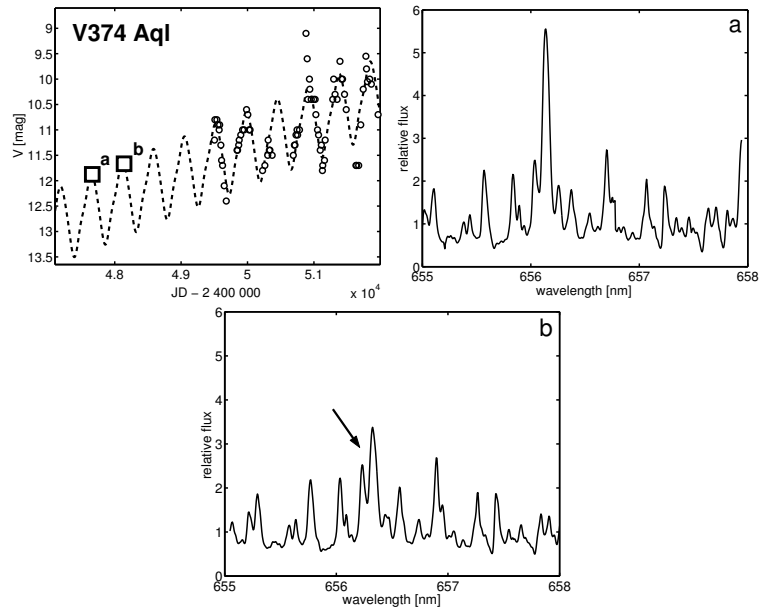


Figure 2. Light curve and H $\alpha$  emissions of V 374 Aql.

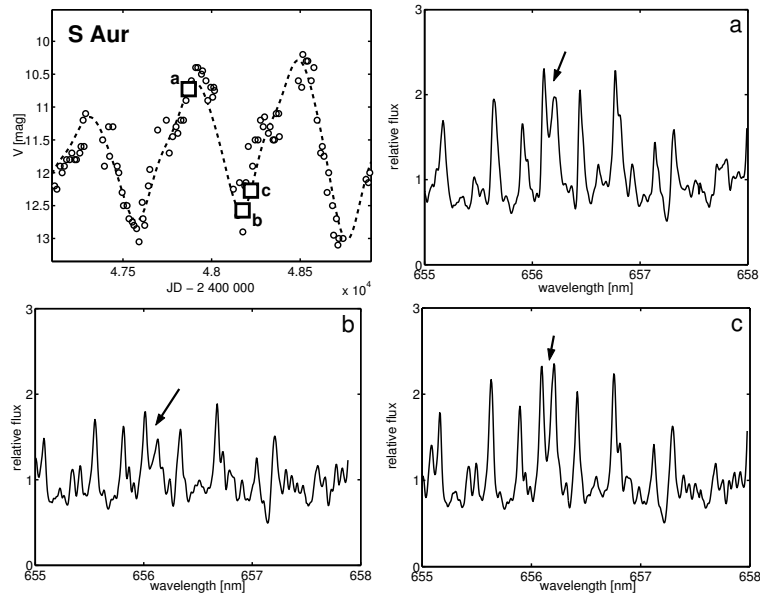


Figure 3. Light curve and H $\alpha$  emissions of S Aur.

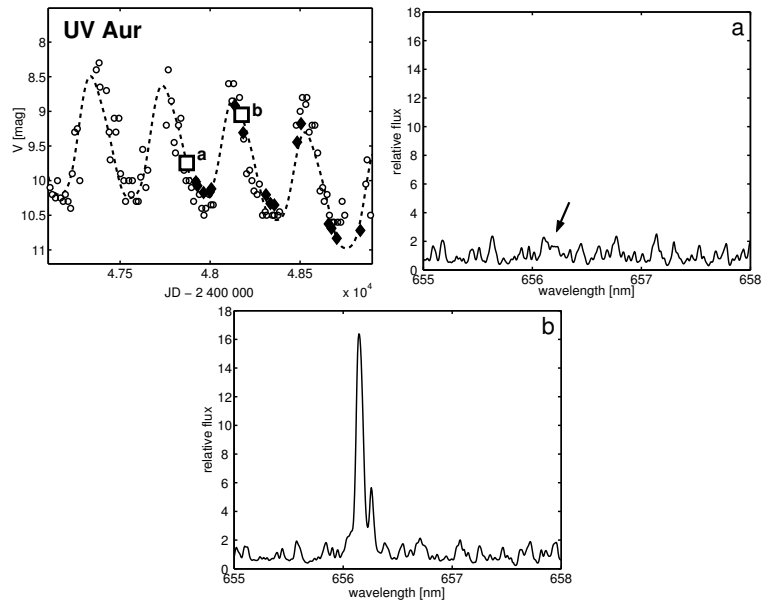


Figure 4. Light curve and H $\alpha$  emissions of UV Aur.

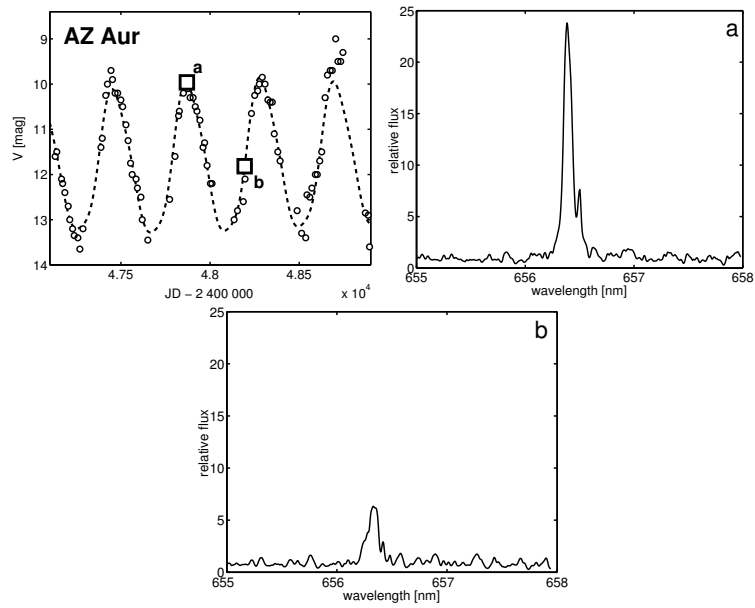


Figure 5. Light curve and H $\alpha$  emissions of AZ Aur.



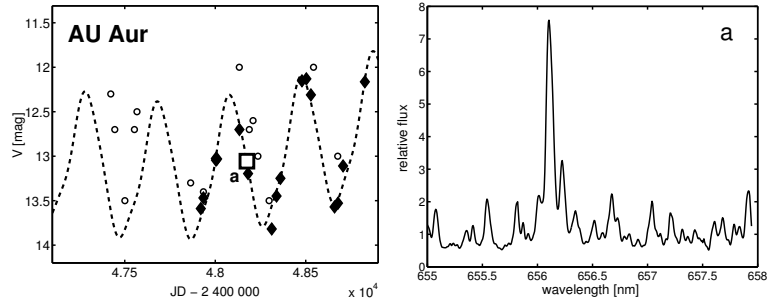


Figure 6. Light curve and H $\alpha$  emissions of AU Aur.

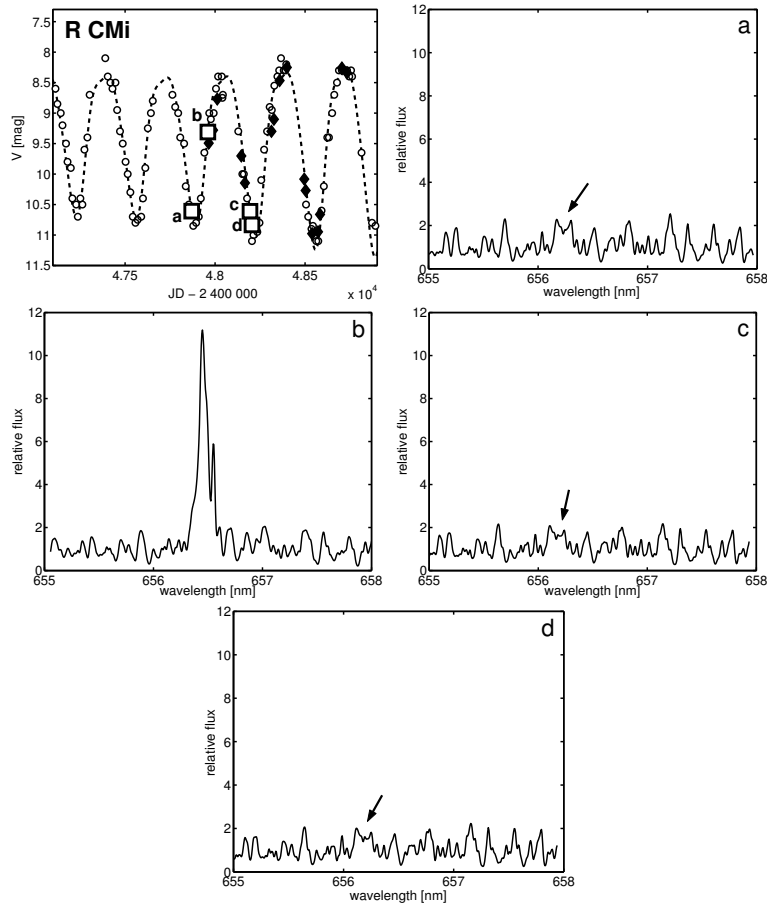


Figure 7. Light curve and H $\alpha$  emissions of R CMi.

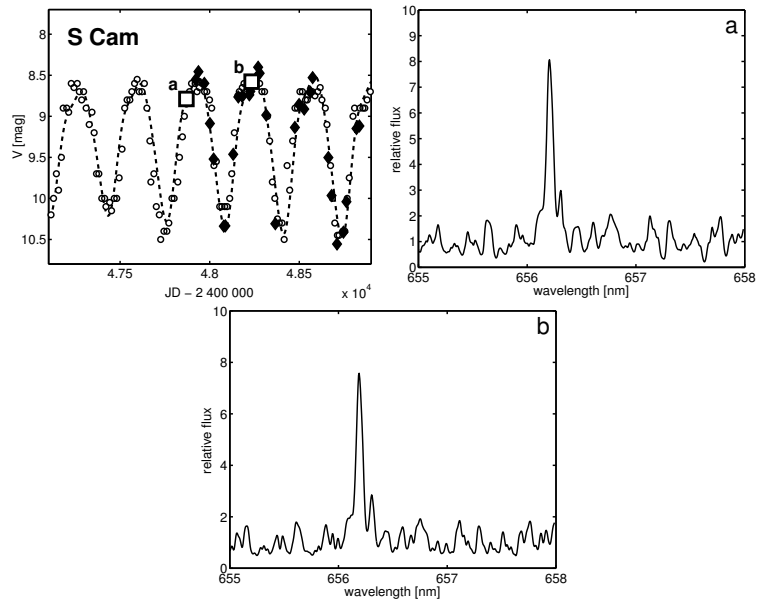


Figure 8. Light curve and H $\alpha$  emissions of S Cam.

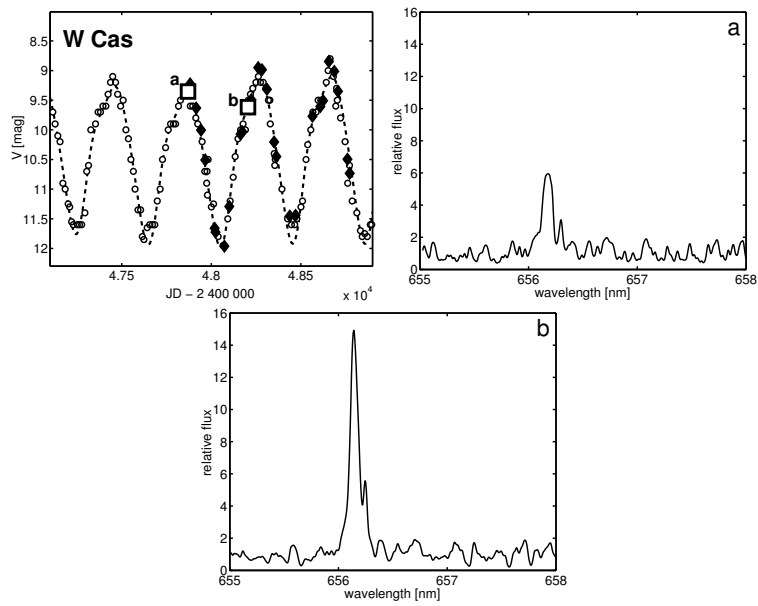


Figure 9. Light curve and H $\alpha$  emissions of W Cas.

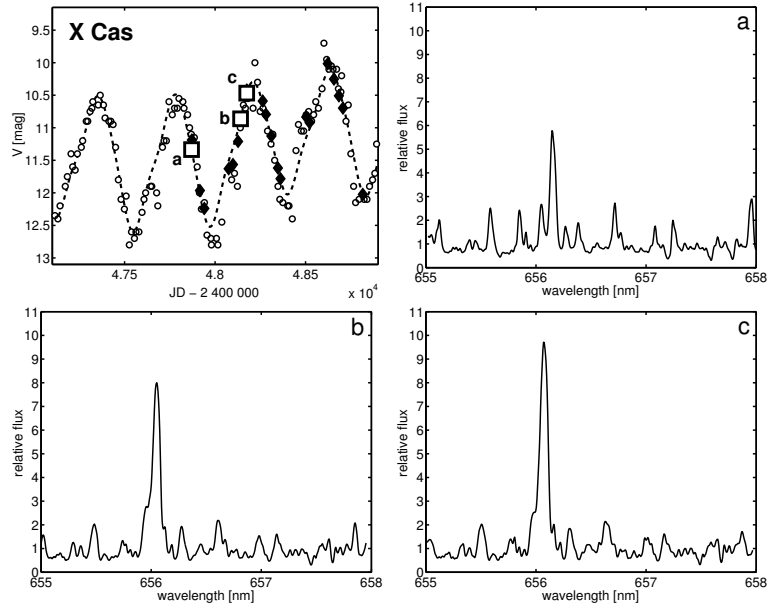


Figure 10. H $\alpha$  emissions and light curve of X Cas.

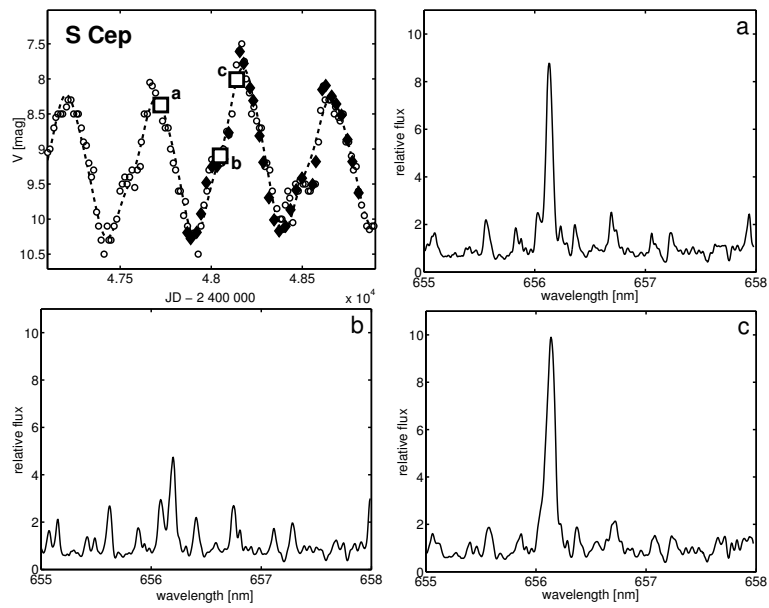
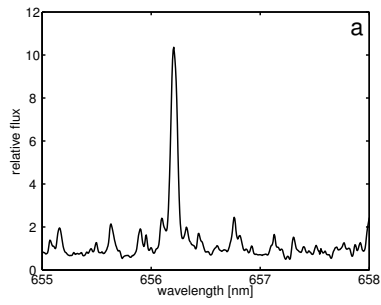
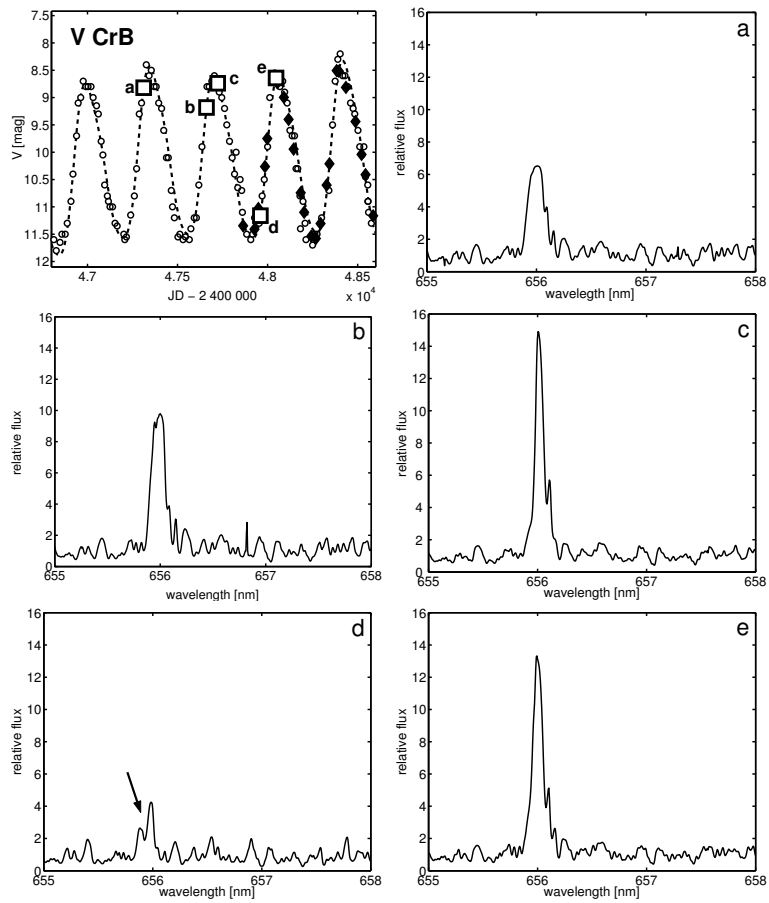


Figure 11. Light curve and H $\alpha$  emissions of S Cep.



**Figure 12.**  $H\alpha$  emission of HV Cas, corresponding photometric phase 0.5 (uncertain).



**Figure 13.** Light curve and  $H\alpha$  emissions of V CrB.

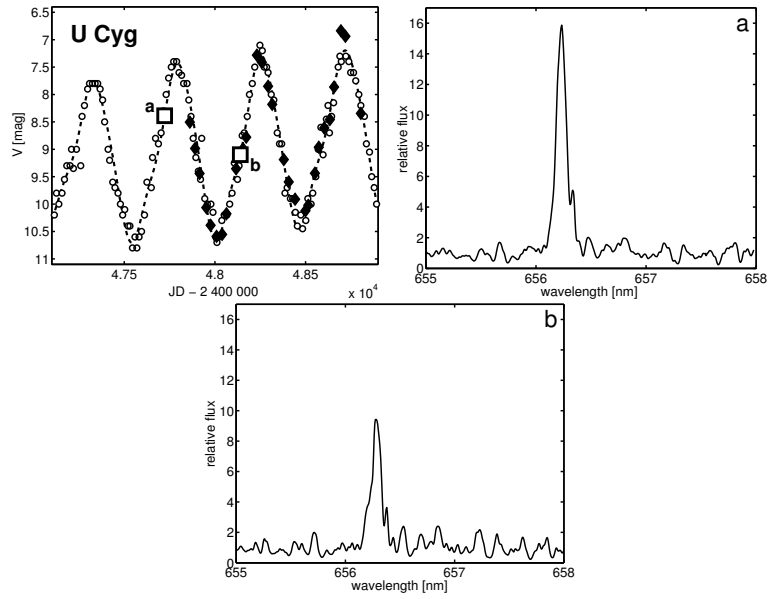


Figure 14. Light curve and H $\alpha$  emissions of U Cyg.

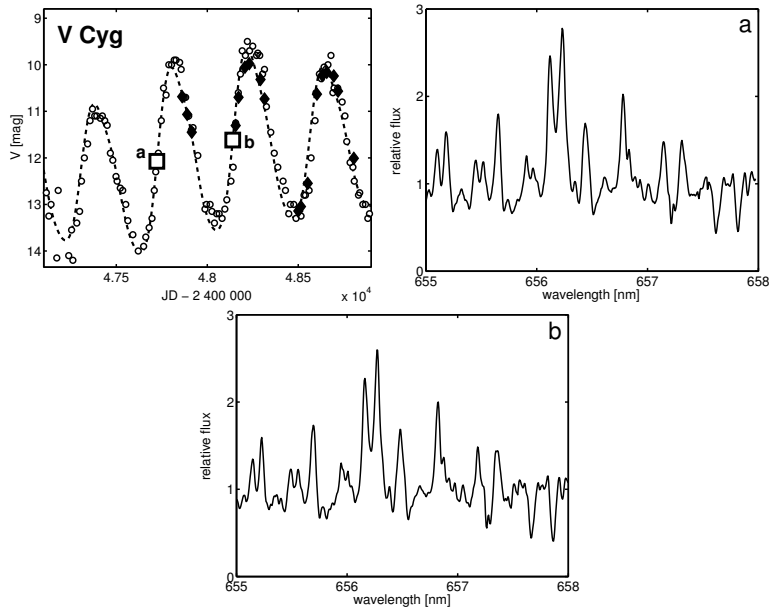


Figure 15. Light curve and H $\alpha$  emissions of V Cyg.

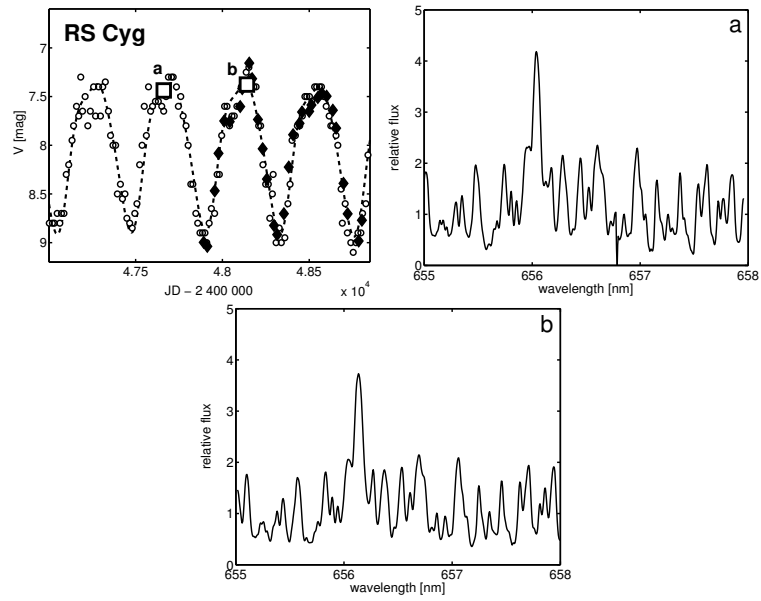


Figure 16. Light curve and H $\alpha$  emissions of RS Cyg.

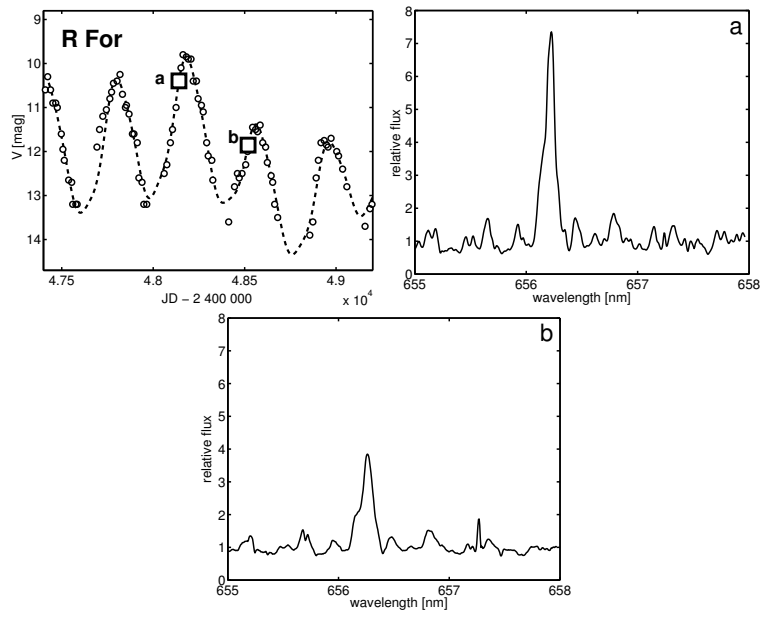


Figure 17. Light curve and H $\alpha$  emissions of R For.

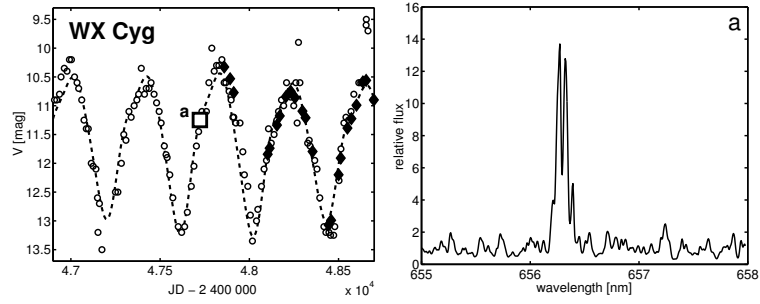


Figure 18. Light curve and H $\alpha$  emission of WX Cyg.

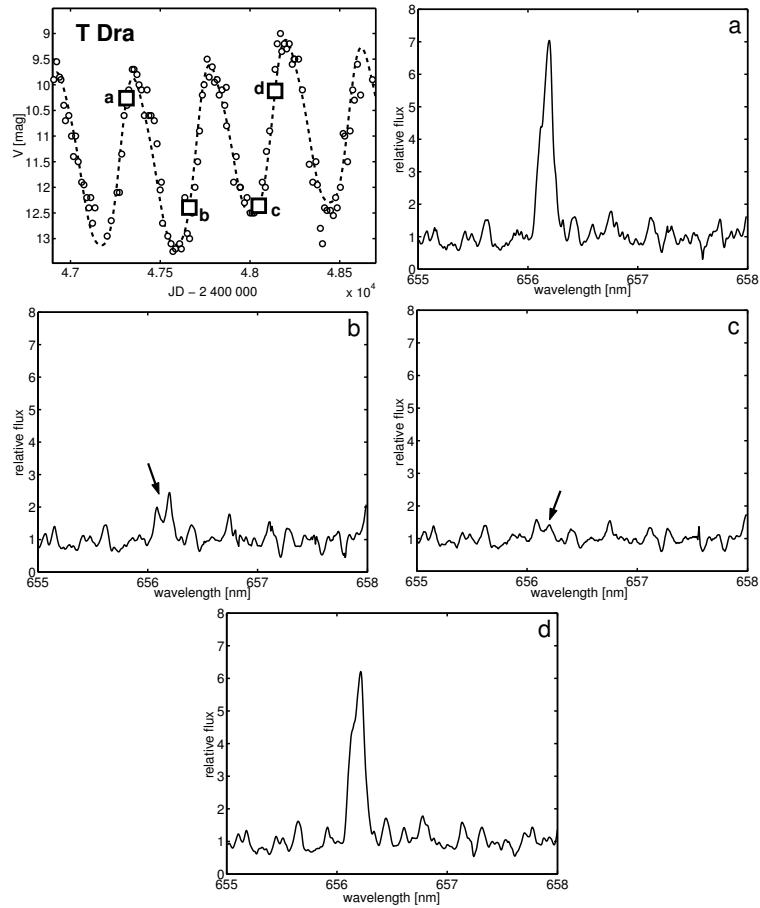
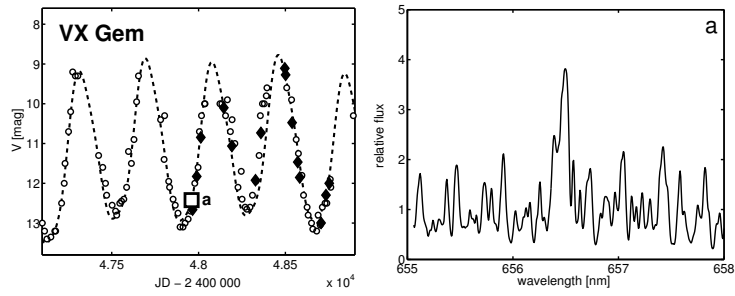
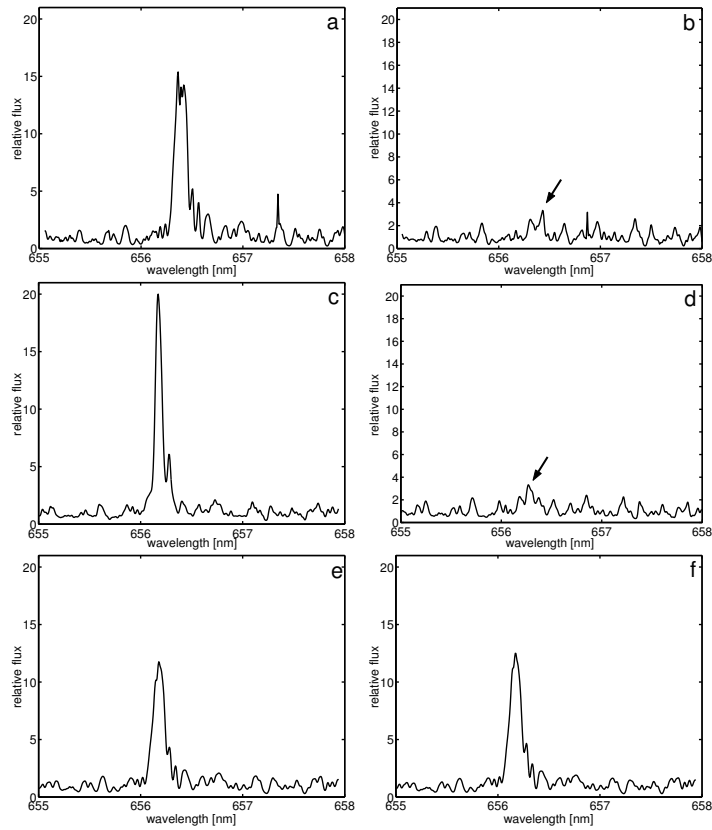


Figure 19. Light curve and H $\alpha$  emissions of T Dra.



**Figure 20.** Light curve and H $\alpha$  emission of VX Gem.



**Figure 21.** H $\alpha$  emissions of CZ Hya. The spectra (a), (b), (c), (d), (e) and (f) were exposed at photometric phases 0.72, 0.51, 0.98, 0.19, 0.78 and 0.80, respectively. The positions of weak emissions are labelled by arrows.



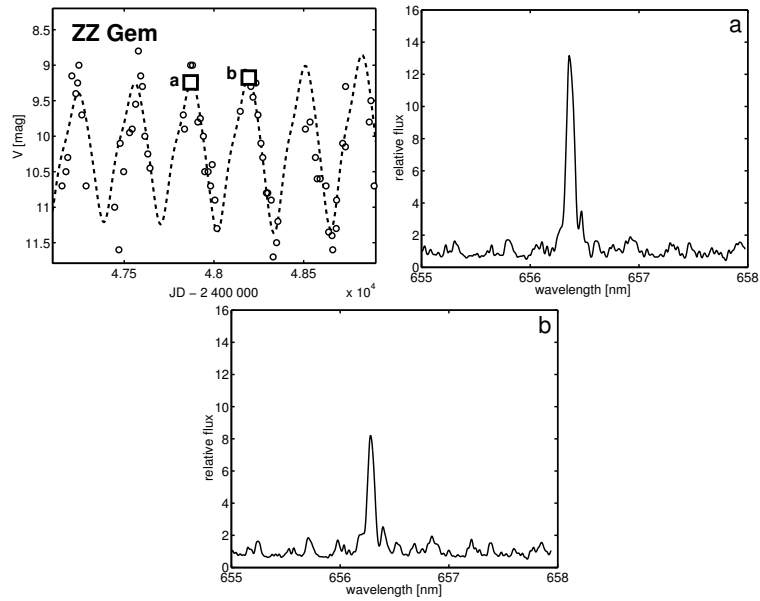


Figure 22. Light curve and H $\alpha$  emissions of ZZ Gem.

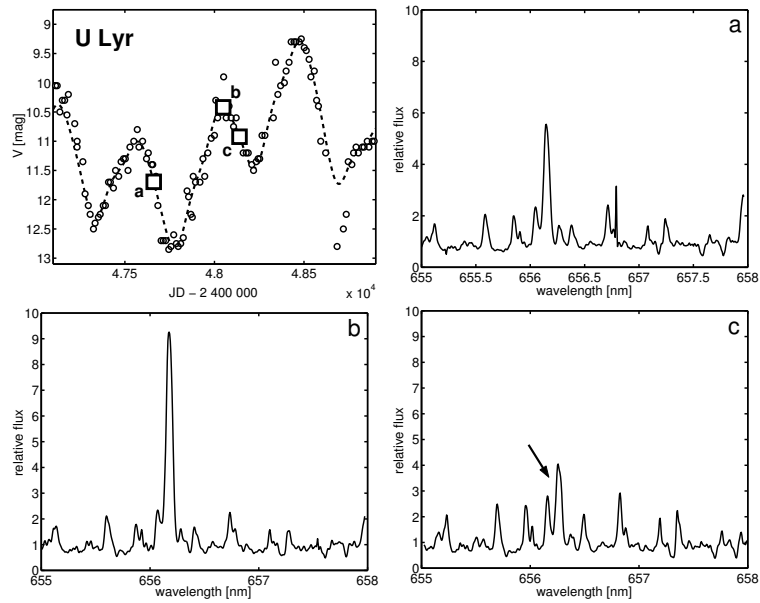
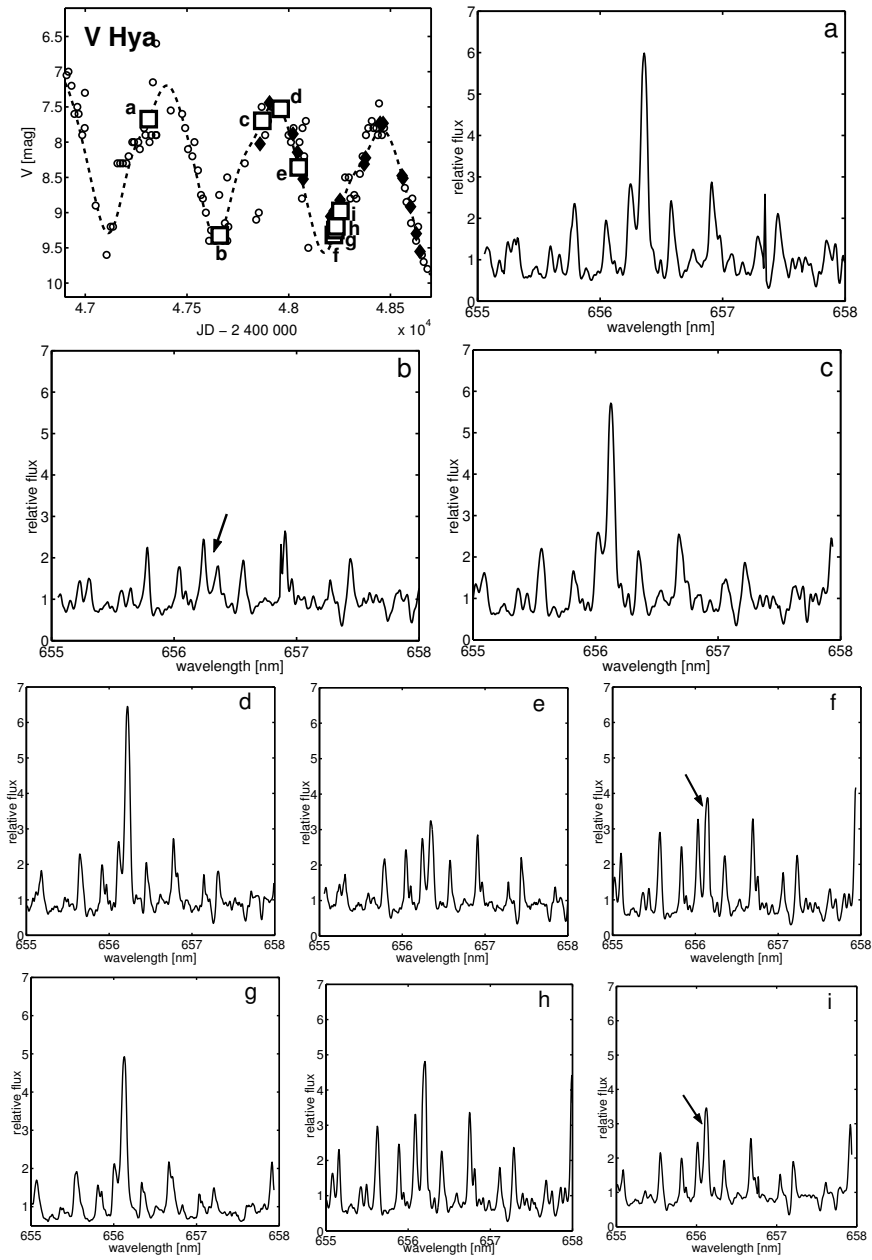


Figure 23. Light curve and H $\alpha$  emissions of U Lyr.



**Figure 24.** Light curve and H $\alpha$  emissions of V Hya.

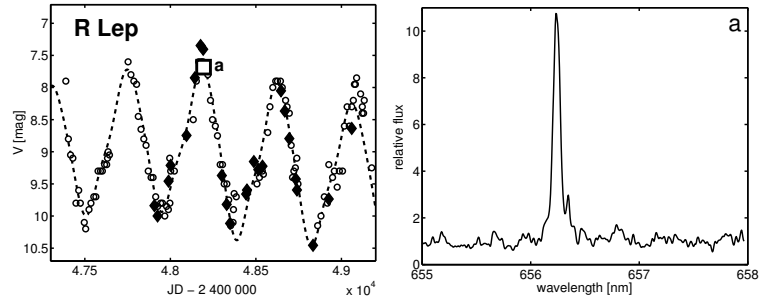


Figure 25. Light curve and H $\alpha$  emission of R Lep.

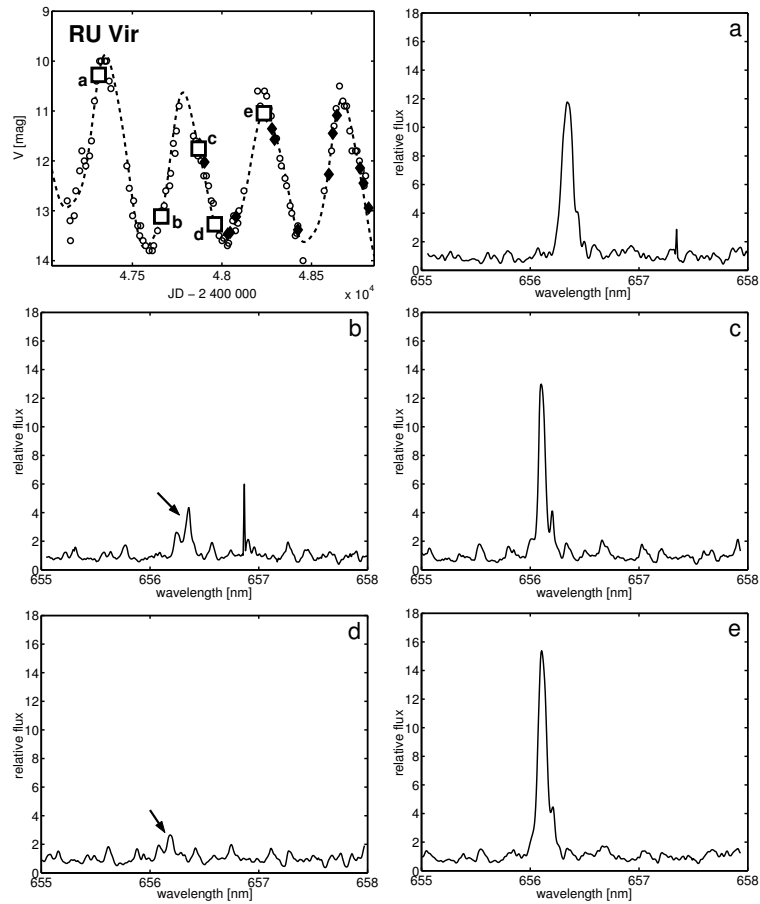
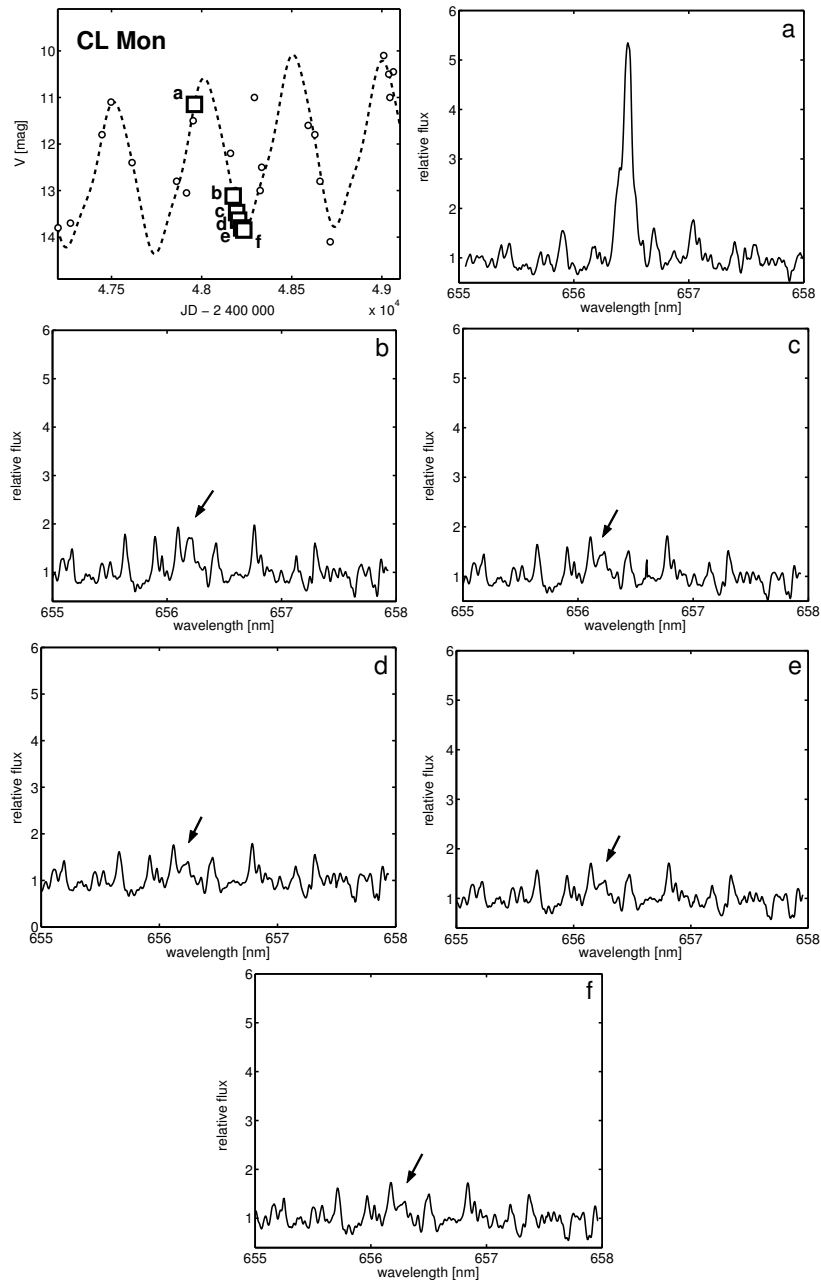


Figure 26. Light curve and H $\alpha$  emissions of RU Vir.



**Figure 27.** Light curve and H $\alpha$  emission of CL Mon. The positions of weak emissions are labelled by arrows.

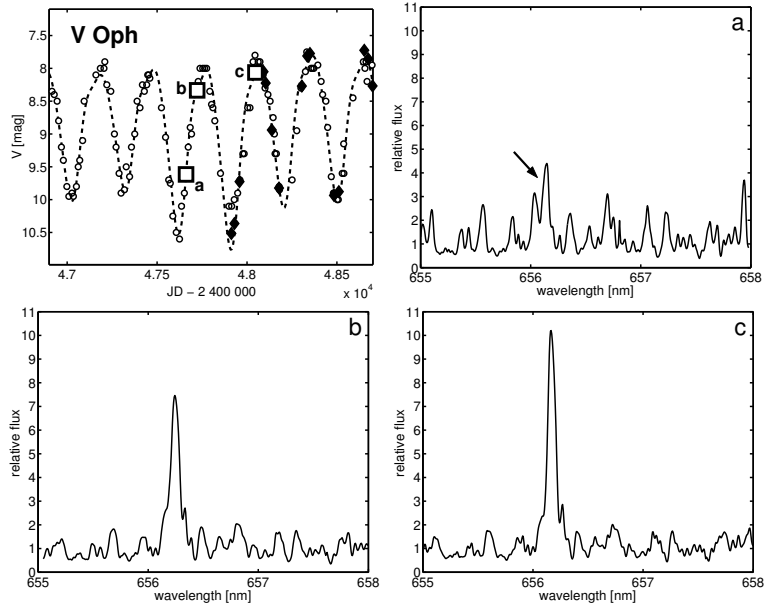


Figure 28. Light curve and H $\alpha$  emissions of V Oph.

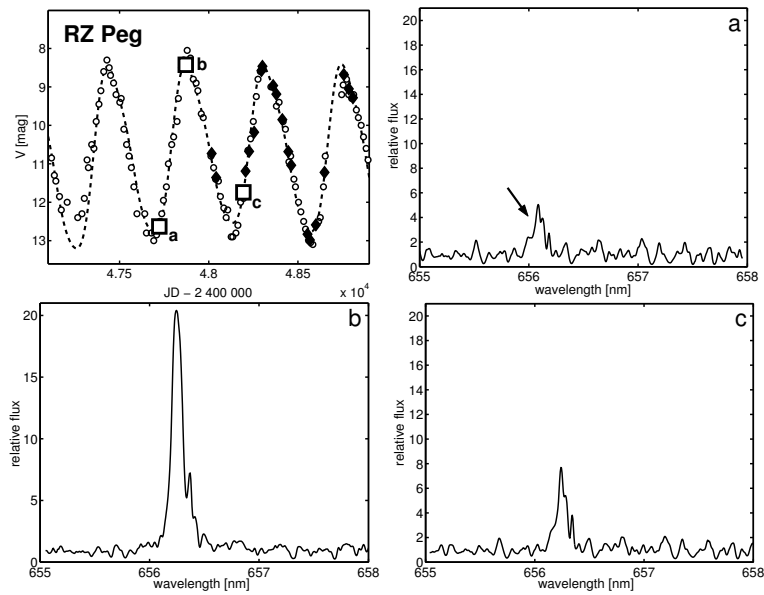


Figure 29. Light curve and H $\alpha$  emissions of RZ Peg.

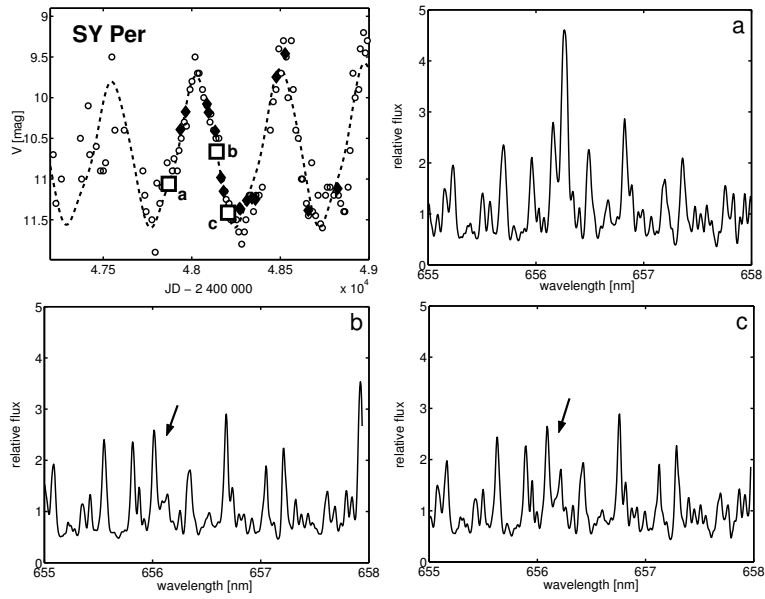


Figure 30. Light curve and H $\alpha$  emissions of SY Per.

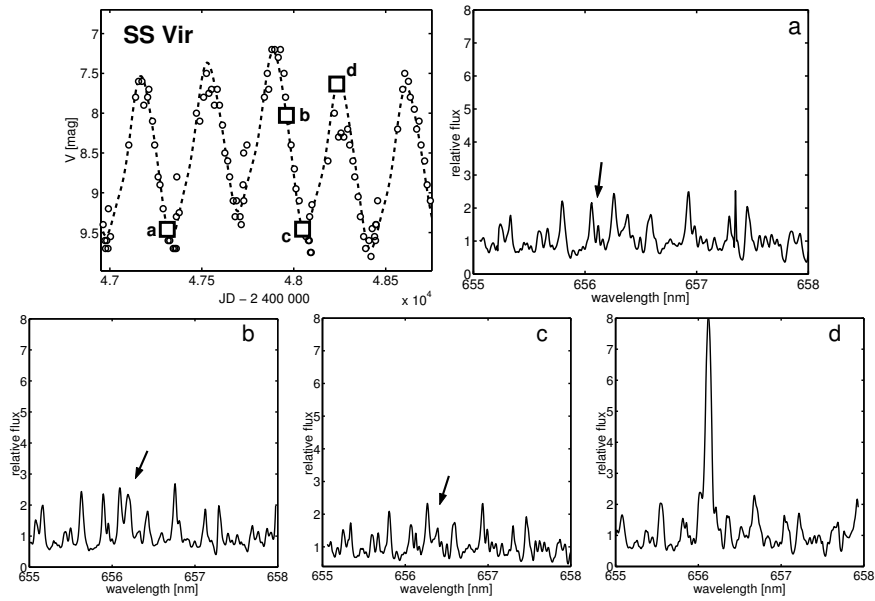
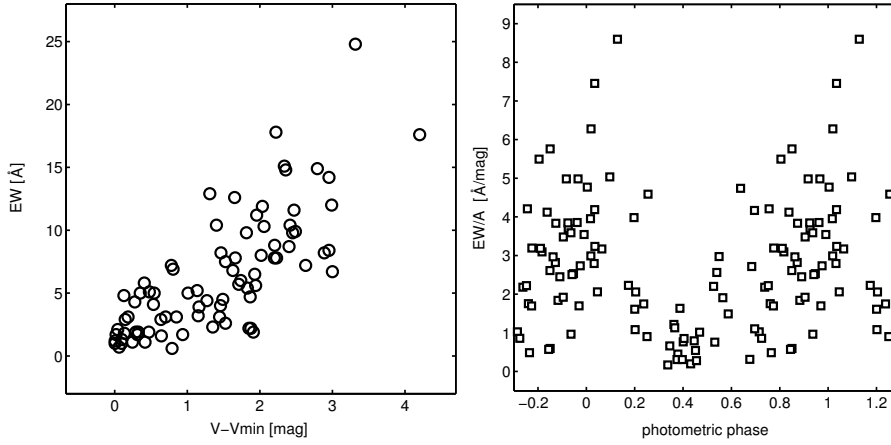


Figure 31. Light curve and H $\alpha$  emissions of SS Vir.



**Figure 32.** Left: Relation between the equivalent width of the H $\alpha$  emission and the magnitude difference given by the  $V$  magnitude predicted for the time of observations and the local minimum ( $V_{\min}$ ). Right: Phase diagram of the ratio of the equivalent width H $\alpha$  emission and the photometric amplitude in mags.

## 5. Conclusions

The above given atlas of  $V$  light curves and H $\alpha$  emission lines of 30 emission line carbon stars is based on the treatment of 89 spectrograms and 112177 photometric observations of the stars.

1. We have concluded that each of 30 stars, labelled as the object with H $\alpha$  emission contained in the Barnbaum (1994) list of 87 carbon stars, shows more or less periodic light variations with periods from 295 to 590 days and an amplitude in  $V$  light larger than 1.5 mag, what classify them as carbon Miras. Contrary, we can affirm that *all carbon Miras display H $\alpha$  emission*. This suggests a common basic mechanism to be in the effect for both the generation of hydrogen emission and  $V$  light variations in the continuum for carbon Miras.

2. The relationship between equivalent width of H $\alpha$  emission and its  $V$  magnitude is incontestable. If the star is near its light minimum the emission is practically invisible. The more the star vary from its light ground state, the brighter the H $\alpha$  emission is. This rule can be documented by the dependence of the equivalent width of the emission on the difference between the predicted  $V$  magnitude in the moment of the exposure and the expected minimal magnitude (see Fig. 32a).

3. The visual inspection of H $\alpha$  emission lines of the 28 carbon Miras variations with a known light curve shows that their equivalent width are dependent more on the photometrical phase than on its magnitude. This finding can be

demonstrated on the cases of S Cep, T Dra, or namely U Lyr depicted in Figs. 11, 19 and 23, respectively.

4. The phase graph of the ratio between the equivalent width and the photometric amplitude is plotted in Fig. 32b. All carbon Miras seem to form a relatively homogenous group of stars what justified hopefulness of our approach to treat variations of all carbon Miras as of it was only a single star (Gráf, 2003).

The detailed analysis of spectral and light variations of 30 carbon Miras will be published elsewhere.

**Acknowledgements.** The authors are extremely thankful to Dr. C. Barnbaum for providing them with her observational data. Dr. S. Kiyota is acknowledged for observational data from observers of VSOLJ (Variable Star Observers League in Japan). The work was carried out in the frame of the projects 205/04/1267 and 205/04/2063 of the Grant Agency of the Czech Republic. We acknowledge with thanks the variable star observations from the AAVSO International Database contributed by observers worldwide and used in this research. The authors thank Drs. J. Krtička and R. M. Komžík for their help with the preparation of the manuscript. We thank an unknown referee for a number of valuable remarks and suggestions, which helped to improve the manuscript considerably.

## References

- Barnbaum, C.: 1994, *Astrophys. J., Suppl. Ser.* **90**, 317  
Dušek, J., Mikulášek, Z.: 2003, *Contrib. Astron. Obs. Skalnaté Pleso* **33**, 13  
Gráf, T.: 2003, *Ph.D. Thesis, Masaryk University Brno*, in Czech  
Mikulášek, Z., Gráf, T.: 2005, *Astrophys. Space Sci.* **296**, 157  
Mikulášek, Z., Papoušek, J., Tremko, J., Žižňovský, J.: 2001, *Contrib. Astron. Obs. Skalnaté Pleso* **31**, 90  
Mikulášek, Z., Žižňovský, J., Zverko, J., Polosukhina, N. S.: 2003, *Contrib. Astron. Obs. Skalnaté Pleso* **33**, 29