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On intrinsic collision probability of subkilometer asteroids with the Earth

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Abstract.
The number of known Earth crossing asteroids (ECAs) approaching the Earth within a distance of 0.1 AU during a time period of 150 yr is used to derive the intrinsic collision probability P_i. P_i strongly depends on the asteroid absolute magnitude H in the studied interval between 17.0 and 20.0. However, it is shown that the dependence may be strongly reduced after a higher percentage of the total population of ECAs up to a given H is known.

Key words: Earth – asteroids – collision

1. Earth crossing asteroids population

The Earth is subjected to a steady state bombardment from surrounding asteroids of various sizes and moving on various orbits. Destroyed asteroids are continuously replaced by others and we suppose that the number of threatening objects down to a given size remains the same for several centuries. Their orbital elements change in the course of time especially after close encounters with the Earth (or other planets). The perihelion distances q of impactors on the Earth are about 1.0 AU or less - they have the Aten and Apollo type orbits, which further in the text will be called the Earth crossing asteroids (ECAs). There are also other definitions of this term in the literature with no relevant impact on the results of this paper.

Along with the orbital elements, the absolute magnitude H is also a basic parameter that characterizes asteroids. In the usual absence of a known albedo, we can infer possible size ranges of these bodies from H assuming a typical albedo, or albedo limits. Photometric, radar and even spacecraft data show that small asteroids are frequently of irregular shapes. Usually, by the size of the object we do not mean its maximum length, but rather the effective diameter of a spherical body with the same volume. For example, if H is about 17.8 we know that the typical ECA is of about 1 km in size (Stuart and Binzel, 2004)

Table 1. Discoveries of ECAs between 1998 and June 2004 in several ranges of the absolute magnitude H according to MPC database. The total number of known ECAs observed more than 30 days is in the last column.

H				year				all
	1998	1999	2000	2001	2002	2003	2004	
(0;14.0⟩	0	0	1	0	0	0	0	5
(14.0;14.5⟩	1	1	0	0	0	0	0	6
(14.5;15.0⟩	2	3	2	1	0	0	0	13
(15.0;15.5⟩	1	2	1	2	0	3	3	27
(15.5;16.0⟩	4	2	5	0	4	1	1	28
(16.0;16.5⟩	6	4	10	7	7	5	2	55
(16.5;17.0⟩	3	7	6	9	9	7	2	63

apart from the fact that some of them may still exceed even 2 km in size (dark) or just in their maximum length (elongated shape) and some of them may be below 1 km (bright and of a nearly spherical shape, or binaries). It should be mentioned that the computed orbital elements and H may be erroneously derived if the asteroid is observed on a short arc, at a large phase angle, and if some astrometrical and brightness data are not precise enough. To homogenize data, only asteroids observed for more than 30 days (this value was chosen empirically) are taken into consideration in the present analysis although this is done at the expense of their completeness. Some faint asteroids are hardly observed in a single apparition during such a long time and depending on the orbit it may take several years when the next observational window will be open again for them.

The power of sky survey telescopes in speeding up the process of ECAs discoveries is astonishing. The first survey telescope - LINEAR - has been working since 1998. Later on, its operation was improved and several new survey telescopes were constructed. The sky is covered much better in terms of the completeness up to a visual magnitude of 19 - 20 and of the total scanned area, including the southern hemisphere. The numbers of ECAs discoveries in a given year (from 1998 up to June 2004), as well as the total known population sorted to several ranges of H according to Minor Planet Centre (MPC) database, are summarized in Table 1. The brightest ECA since 1998 is 2000 UV_{13} with $H = 13.5$. It seems that we probably know all the ECAs up to $H = 15.0$. If projected to nighttime sky, such bright ECAs are easy targets for survey telescopes whatever is their orbit. No new discovery was reported in 2002 and 2003. Surprisingly, the second half of 2004 brought one such bright ECA - 2004 QY_2 with $H = 14.7$. This is missing in Table 1. On the other hand, Table 1 would contain only one more asteroid - 2004 QQ with $H = 16.7$ (from the last interval of H) - if the statistics for the whole year 2004 is included.

The numbers of ECAs with $H \geq 15.5$ diminished after 2002, in spite of improvements in observational technique. This may point to a fact that at present we are about to know nearly all the ECAs up to $H = 16.0$ or so. In other words, the discoveries of bright ECAs with $H \leq 16.0$ in 2004 could be among the latest. (Actually, the whole text below - down to equation 1 - is about proving this assumption.) A couple of stray ECAs with $H \leq 16.0$ could still be discovered later (as was the case of 2004 QY_2), for example, if their visual magnitude in the recent past dropped below 19 - 20 only when they were projected to star crowded fields near Milky Way, or when their angular distance from the Sun was very low, or when they were projected deep in the southern hemisphere only. But these are believed to comprise a tiny part of the total population. Our knowledge about the orbits of kilometer sized ECAs may be completed in a few years and we will gradually improve our knowledge about the orbits of ECAs down to subkilometer sizes. This achievement is planned with today and future sky survey telescopes, with the assistance of the follow-up observatories.

In our analysis the total population of ECAs up to a given H will henceforth be denoted as $N(< H)$. We can assume a steady increase of $log(N(< H))$ with an increasing H over a small interval of H. (Assuming that the albedo distribution does not depend on the size D, the relation is equivalent to the cumulative size distribution of the population in the form $(N(> D)) = KD^b$, where K is a constant and b is the cumulative size population index.) However, for an incomplete interval ($H > 16.0$) the uncertainty in estimates of $N(< H)$ grows with increasing H. The orbital elements and H values were taken from the continuously updated astorb.dat database (Bowell et al., 1994) that is also available on line (Bowell, 2004). They slightly differ from the values in MPC Orbit Database for unnumbered asteroids.

The population of known ECAs up to a given H observed longer than 30 days by June 30, 2004, vs. H is plotted in Fig. 1. The true population over the incomplete region ($H > 16.0$) can be assessed in the first approximation by the fit line constructed from the data over the complete region. However, the total numbers over $H < 16.0$ cannot be reliably described by a single equation. Much better agreement with the true population is achieved by using two different equations - the first one for the interval $\langle 13.5; 14.5\rangle$, the second one for the interval $\langle 15.0; 16.0\rangle$. Taking this into account, there would be just 3 points left from the second interval for the construction of the fit line to approximate $log(N(< H))$ over the incomplete region. Such estimate is not reliable over a large H interval.

However, independent estimates based on the analysis of the discoveries from sky survey telescopes and on models of their orbits also exist. According to Bottke et al. (2002) there are about 650 ± 100 ECAs having $H < 18$, while according to Stuart (2001) and assuming that ECAs comprise $68\% \pm 1\%$ of the so called Near-Earth asteroids (Bottke et al., 2002) there are about 850 ± 100 such bodies. The equations of the fit lines based on numbers of known ECAs from various H intervals also imply the numbers of ECAs having $H < 18$. Table 2 contains such equations and numbers. It is clear from it that except

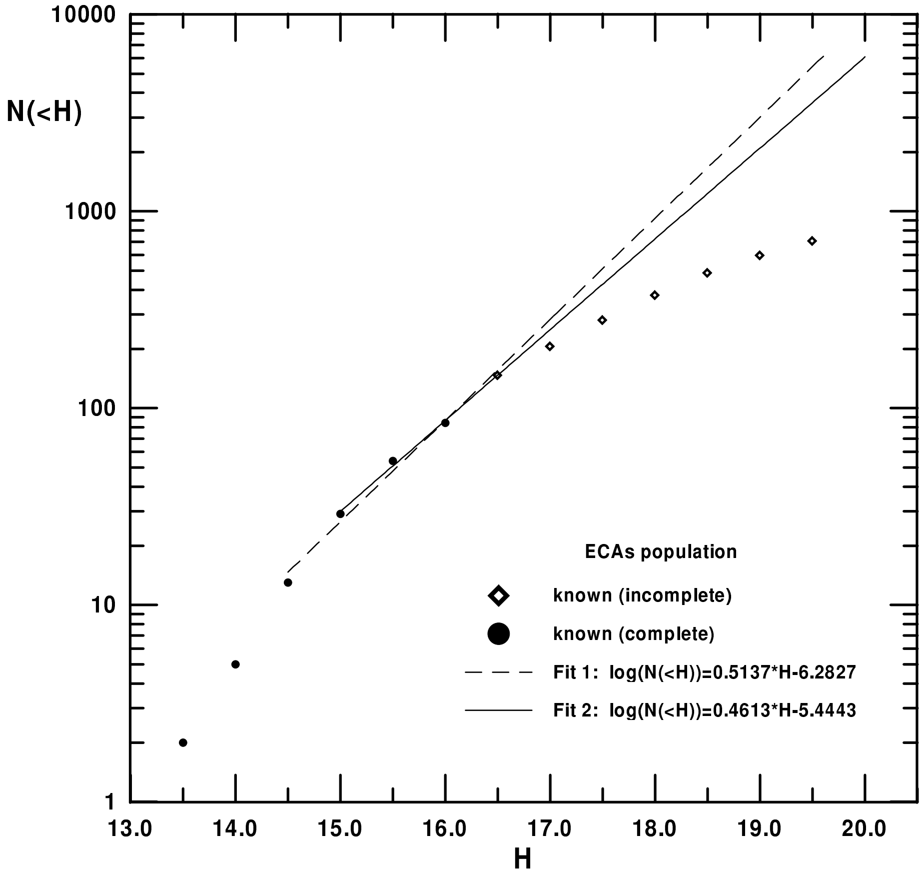

Figure 1. Cumulative distribution of Earth crossing asteroids observed more than 30 days by June, 30, 2004 (according to Bowell's database). Two fit lines are chosen to see their possible total number up to a given absolute magnitude H.

for the interval $\langle 15.0; 16.0\rangle$ there are just two intervals, namely $\langle 14.5; 16.5\rangle$ and $\langle 15.0; 16.5\rangle$, that could be used for the construction of the fit lines and at the same time are not in contrast with the expected ECAs numbers having $H < 18$. These fit lines are seen in Fig. 1 as Fit 1 and Fit 2, respectively. Both fit lines slightly steepen by the time the population up to $H = 16.5$ is complete (and observed more than 30 days). In this regard, the real ECAs population for an incomplete region seems to be slightly higher than is shown at least by Fit 2, which is considered to be a lower limit of $N(< H)$ here.

The average value from the two equations (Fit 1 and 2) is used here as a

Table 2. Equations of several fit lines constructed according to the numbers of ECAs known by June, 2004. The estimate of the total ECAs population up to the absolute magnitude $H = 18$ is in the last column.

H	$log(N(< H))$	$N(< 18)$
$\langle 13.5;14.5\rangle$	0.8129 H - 10.6761	9039
$\langle 13.5;15.0\rangle$	0.7798 H - 10.2183	6578
$\langle 13.5;15.5\rangle$	0.7252 H - 9.4541	3976
$\langle 13.5;16.0\rangle$	0.6609 H - 8.5421	2260
$\langle 13.5;16.5\rangle$	0.6191 H - 7.9442	1583
$\langle 14.0;15.0\rangle$	0.7634 H - 9.9779	5798
$\langle 14.0;15.5\rangle$	0.6897 H - 8.9218	3110
$\langle 14.0;16.0\rangle$	0.6138 H - 7.8208	1689
$\langle 14.0;16.5\rangle$	0.5739 H - 7.2350	1245
$\langle 14.5;15.5\rangle$	0.6184 H - 7.8405	1953
$\langle 14.5;16.0\rangle$	0.5402 H - 6.6798	1106
$\langle 14.5;16.5\rangle$	0.5137 H - 6.2827	920
$\langle 15.0;16.0\rangle$	0.4619 H - 5.4528	727
$\langle 15.0;16.5\rangle$	0.4613 H - 5.4443	723

working estimate for the total population of ECAs:

$$logN(< H) = 0.4875\ H - 5.8635. \tag{1}$$

By enhancing H interval we obtain a more reliable estimate for the true population of the ECAs over the uncomplete interval of H. It is also less dependent on the used database (for example one in MPC vs. that by Bowell). Interestingly, from the similarity of the equations of the fit lines at the bottom of Table 2 one can infer that we probably know nearly all ECAs up to $H = 16.5$.

To be rigorous, we should exclude from the ECAs those objects that are inside the Earth's orbit - with the aphelion distance smaller than 1.0 AU. We really know some members of this group but they are not included into the analysis here as none of them was observed for more than 30 days.

2. Impact frequency

Knowing $N(< H)$, one way how to assess the ECAs impact frequency f on the Earth is through the intrinsic collision probability P_i according to equation

$$f = N(< H)\ P_i\ R_C^2, \tag{2}$$

where R_C is the Earth's capture radius, which is computed from equation

$$R_C = R\,\sqrt{(1 + V^2/v^2)}, \tag{3}$$

where R is the Earth's effective radius, V is the escape velocity from the Earth ($\sim 11.2\,\mathrm{km\,s^{-1}}$), and v is the geocentric velocity of the asteroid (see, for example, Farinella and Davis, 1992; and Manley et al., 1998). This term includes Earth's gravitation - the slower the encounter velocity the larger the distance within which the Earth can capture the asteroid. The mean value of R_C depends on the mean v, that means on a typical ECA orbit, although the encounter velocities corrected for gravitation may differ nearly by one order of magnitude. The mean encounter velocity for the known ECAs (by June, 2004) with $H \leq 18.0$ is nearly $18.0\,\mathrm{km\,s^{-1}}$ for the period between 1900 and 2100 according Yeomans et al. (2004) from JPL. This was based on 803 approaches. As a working estimate we can use

$$R_C^2 = 1.4\,R^2 \tag{4}$$

corresponding to $v \sim 17.7\,\mathrm{km\,s^{-1}}$.

For a group of N known ECAs, P_i may be derived from the number of their approaches to the Earth, denoted as M, within a given distance r and during a given time period t, as

$$P_i = M/(N\;t\;r^2). \tag{5}$$

For the main belt asteroids and the Trojans P_i depends on H (Yoshikava and Nakamura, 1994; Dell'Oro et al., 1998; Galád and Gray, 2002). P_i increases with increasing H. We can study here if this is also the case for the Earth.

3. Intrinsic collision probability

Several authors aim to compute the orbital evolution to the past and to the future for all known possible impactors. Along with this they compute approaches to the Earth. However, short observational arcs of several asteroids prevent to do this job accurately for a long time interval (this is another reason why the ECAs with short arcs were excluded from the analysis here). We can use the results of Chesley and Milani (1999), which are available also online (Milani et al., 2004). They continuously compute approaches of individual asteroids to the Earth for the period of 150 yr between 1950 and 2099 (before 2000 approaches for the period of only 100 yr, namely between 1975 and 2074 were computed). Just the ECAs observed longer than 30 days and their approaches within 0.1 AU to the Earth were selected here for P_i computation. This process was repeated several times. Selection of a shorter time interval and a smaller range of approaches would weaken reliableness of the results, but within several years there is a chance to overcome even this drawback. Other two sources (Yeomans et al., 2004, from JPL and Williams, 2004, from MPC) were taken for comparison and check not to miss some data about approaches. They are independent. The P_i computation should thus be reliable.

In Table 3, eight different dates from the end of 1999 to 2004 were selected to see how P_i evolves with time and if it depends on H. Four groups of ECAs - those with $H \leq 17.0$, $H \leq 18.0$, $H \leq 19.0$ and $H \leq 20.0$, respectively - were studied. It seems that P_i really depends on H - it was always much lower for the ECAs up to 17.0 magnitude than for those up to 19.0 and up 20.0 magnitude. Only P_i values for the ECAs up to 18.0 magnitude in the recent dates were similar to those up to 17.0 magnitude.

It can be noticed that P_i generally decreased as a larger portion of the ECAs population became known for all groups. Only P_i values for the ECAs up to 17.0 and 18.0 magnitudes started to fluctuate around some value near $47 \times 10^{-18}\,\mathrm{yr}^{-1}\mathrm{km}^{-2}$. We can assess that this is probably very close to the true value of P_i for km sized ECAs. The general decrease of P_i can be explained as a consequence of an observational bias effect - asteroids on nearby orbits with a small semimajor axis a, eccentricity e and inclination i, frequently approach the Earth and could be discovered easier and sooner in comparison to asteroids on distant, elongated and more inclined orbits. Table 3 contains also the mean values of a, e, i and the general trend towards their larger values is seen, with only minor exceptions (from the second date it includes asteroid 1999 XS35 with $H = 17.25$, moving on extraordinary orbit $a \sim 17.91\,\mathrm{AU}$, $e \sim 0.95$, $i \sim 19°$).

Let's try to compensate for this bias. We can estimate what was the portion of known bodies N (the values are in Table 3) to the total population $N(< H)$ (using equation (1)) in the mentioned dates for all of our four groups. Interestingly, the comparison of P_i values for different H when normalized to the total population shows that P_i dependence on H is strongly reduced, as it can be seen in Fig. 2. The differences in P_i tend to be obliterated. Until we know a larger portion of the total population for two groups - up to 19 and up to 20 magnitudes - it is expected that P_i for them should decrease further, probably towards the values for the remaining studied groups.

The 100 yr interval is not large enough for P_i computation for the first dates - its values are higher than those based on 150 yr interval (this is also due to the observational effect). None of them were included in Fig. 2, though for the latest dates P_i values are comparable. Otherwise a nearly continuous downslope would not be followed for various H (just for individual groups of H).

Table 3. The intrinsic collision probability P_i derived from the ECAs approaches within 0.1 AU from the Earth during 150 years (the values during 100 years are in parenthesis). N is the number of known ECAs observed more than 30 days up to a given H, M is the number of their approaches during 150 years (the numbers during 100 years are not presented except for 1999 which are in parenthesis), a is the mean semimajor axis, e eccentricity and i inclination, respectively.

Time	H	M	N	P_i $[\times 10^{-18}\,\mathrm{yr}^{-1}\mathrm{km}^{-2}]$	a [AU]	e	i [°]
1999 Dec	≤ 17.0	(145)	103	(62.90)	1.656	0.554	22.003
	≤ 18.0	(222)	151	(65.69)	1.636	0.546	19.979
	≤ 19.0	(314)	196	(71.58)	1.619	0.535	18.683
	≤ 20.0	(384)	235	(73.01)	1.605	0.524	17.239
2000 Jun	≤ 17.0	203	118	51.25 (57.94)	1.639	0.559	22.661
	≤ 18.0	344	182	56.30 (61.62)	1.695	0.545	20.240
	≤ 19.0	519	245	63.10 (67.30)	1.657	0.533	18.869
	≤ 20.0	700	299	69.74 (72.63)	1.633	0.522	17.372
2000 Dec	≤ 17.0	218	133	48.83 (54.76)	1.691	0.574	23.137
	≤ 18.0	367	208	52.56 (56.28)	1.725	0.560	21.234
	≤ 19.0	571	288	59.06 (62.06)	1.677	0.542	19.748
	≤ 20.0	766	349	65.38 (67.73)	1.655	0.531	18.185
2001 Jun	≤ 17.0	228	148	45.89 (51.63)	1.706	0.579	23.363
	≤ 18.0	391	238	48.94 (52.00)	1.739	0.572	21.456
	≤ 19.0	611	330	55.15 (58.09)	1.683	0.548	20.110
	≤ 20.0	836	400	62.26 (65.01)	1.661	0.537	18.438
2001 Dec	≤ 17.0	251	163	45.87 (50.99)	1.794	0.578	23.806
	≤ 18.0	441	270	48.65 (50.97)	1.770	0.573	21.928
	≤ 19.0	693	381	54.18 (56.53)	1.696	0.550	20.362
	≤ 20.0	975	469	61.93 (64.31)	1.665	0.538	18.541
2002 Dec	≤ 17.0	299	185	48.14 (49.75)	1.725	0.580	24.577
	≤ 18.0	517	325	47.39 (48.12)	1.750	0.581	22.689
	≤ 19.0	904	482	55.87 (57.20)	1.676	0.556	20.410
	≤ 20.0	1308	609	63.98 (65.23)	1.653	0.545	18.687
2003 Dec	≤ 17.0	310	194	47.60 (48.37)	1.738	0.585	24.800
	≤ 18.0	570	352	48.24 (48.87)	1.775	0.587	22.924
	≤ 19.0	1008	545	55.09 (55.75)	1.695	0.560	20.680
	≤ 20.0	1514	712	63.34 (64.20)	1.669	0.548	18.819
2004 Jun	≤ 17.0	326	206	47.14 (47.72)	1.760	0.588	24.831
	≤ 18.0	601	376	47.61 (47.89)	1.780	0.587	22.934
	≤ 19.0	1073	597	53.54 (53.89)	1.709	0.562	20.783
	≤ 20.0	1647	791	62.02 (62.48)	1.672	0.549	18.952

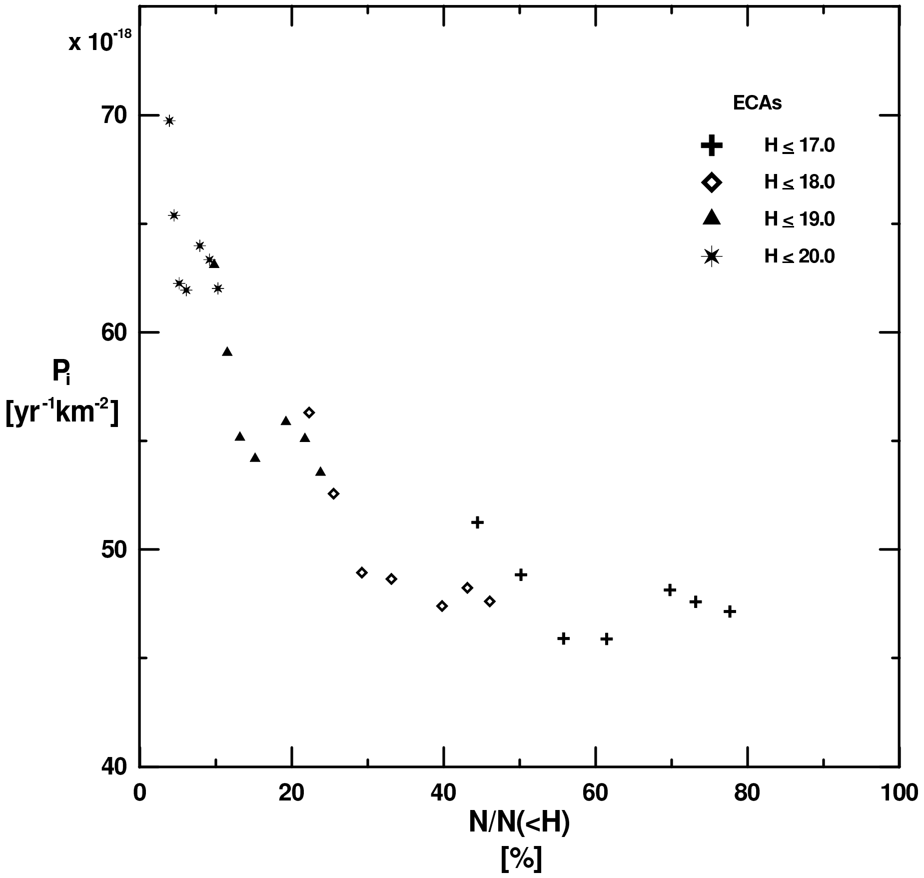

Figure 2. The intrinsic collision probability P_i related to the absolute magnitude H. The estimated portion of known ECAs from the total population up to a given H is on the x-axis. Seven different dates between 2000 and 2004 were chosen.

4. Discussion and conclusion

From our working formula of the ECAs population $N(< H)$ (equation (1)) an impact frequency of 50 m and larger sized bodies (approximately equivalent to $H < 24$) can be estimated. These objects are capable of causing a noticeable local catastrophe such as at the Tunguska event in 1908, or possible coastal floodings due to impact to the ocean. Bottke et al. (2000) pointed out that there is a steady increase of $log(N(< H))$ from 15 up to 22 magnitude. If our estimate is valid up to 24 magnitude and R_C and P_i remain the same as for kilometer sized bodies ($\sqrt{1.4}\ R$ and $47 \times 10^{-18}\,\mathrm{yr}^{-1}\mathrm{km}^{-2}$, respectively), we have nearly 7×10^5 bodies and every 5-6 centuries one of them hits the Earth. According to the estimate of $N(< H)$ based on Fit 1, hits would occur every 3-4 centuries, while according to the estimate based on Fit 2 every 8-9 centuries. Basically, none of the results is in contrast with the historical records of local catastrophes and the fit lines could be considered as lower and upper limits for $N(< H)$ in the interval under study in this respect. We should bear in mind, however, that P_i and even R_C could still be slightly higher for small ECAs if they have a different typical orbit. The impact frequency would rise accordingly, so Fit 1 could be slightly out of reality. On the other hand, we can't forget that the cumulative distribution has probably some kind of a wavy structure over such a long interval of H. Similarly, as in the interval of $H \leq 16.0$ the wavy structure can be observed down to the sizes studied in meteor science and the general slope of the fit line is expected to rise (e.g. Ceplecha, 1996; Morrison et al., 2002).

As we are interested in much larger sizes here - the region of H from 17.0 to 20.0 - the fit lines (in Fig. 1) neither differ so much from each other, nor (probably) from the real population. Both our fit lines assume a larger population of ECAs having $H < 18$ than follows from the analysis by Bottke et al. (2002). It seems that these authors slightly underestimated this population as our lower limit (according to Fit 2) is just within the uncertainty of their value. Our working formula indicates 816 ECAs up to $H = 18$. It is slightly less than follows from the nominal value of such ECAs obtained by Stuart (2001).

One can still expect that P_i does depend on H as some small subkilometer ECAs were discovered on nearby orbits. They are numerous on so-called Aten type orbits and their typical orbit may differ from a typical orbit of larger asteroids. They could more frequently pass by the Earth and hit it. But expected P_i dependence on H is probably not so crucial in the interval of H between 17 and 20, and it is nearly constant - equal to about $47 \times 10^{-18}\,\mathrm{yr}^{-1}\mathrm{km}^{-2}$.

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