

## Preliminary analysis of light curves of seven carbon stars

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**Abstract.** We present a preliminary analysis of the light curves of the following carbon stars: WZ Cas, VY UMa, Y CVn, RY Dra, T Lyr, HK Lyr and TT Cyg, constructed on the basis of our own *BV* photoelectric observations obtained at the Brno Observatory in 1979-94 and Hipparcos observations. The analysis suggests that "semiregular" light curves of all studied stars can faithfully be expressed by a superposition of long-term changes and a set of medium-term harmonic variations (possibly pulsations) with periods from 50 to 500 days.

**Key words:** AGB stars – carbon stars – individual stars – stellar pulsations – periods – *UBV* photometry – Hipparcos photometry

### 1. Introduction

Chemically peculiar late stars, so-called "carbon stars", are asymptotic giant branch (AGB) objects whose atmospheres are vigorously polluted by chemical elements originating namely from carbon in nuclear processes in their deep interiors. Theoreticians claim that every star with the initial mass from  $1.5 M_{\odot}$  to  $4 M_{\odot}$  should go through the stage of a carbon star lasting about  $3 \cdot 10^5$  years before it finally transforms into a white dwarf (Groenewegen et al., 1995).

An evolutionary relatively homogenous group of carbon stars (carbons) is very important for several reasons. First, the luminosity of carbons is extraordinarily high, the stars contribute considerably to the entire radiative output of galaxies, namely in their IR region. These stars can be used as important probes of galactic structure and dynamics. Further, carbons are plentiful sources of diffuse interstellar matter and, in addition, play an outstanding role in the chemical evolution of galaxies.

The interiors of carbons consist of two quite dissimilar parts: in the center resides a very dense, small ( $\sim 10^6$  m) and extremely hot ( $\sim 10^8$  K) active

**Table 1.** Basic information about the seven carbon stars investigated in this paper from the Hipparchos and Tycho catalogues (Perryman et al., 1997).

star	HIP	$\alpha$ (J1991.25)	$\delta$	spec.	$m(V)$ [mag]	$(B-V)$ [mag]
WZ Cas	99	00 <sup>h</sup> 01 <sup>m</sup> 15.8 <sup>s</sup>	+60° 21' 19.1''	C5p	7.04	2.84
VY UMa	52577	10 <sup>h</sup> 45 <sup>m</sup> 04.2 <sup>s</sup>	+67° 24' 41.0''	C5III	5.95	2.38
Y CVn	62223	12 <sup>h</sup> 45 <sup>m</sup> 07.8 <sup>s</sup>	+45° 26' 24.8''	C7Iab	5.42	2.99
RY Dra	63152	12 <sup>h</sup> 56 <sup>m</sup> 25.9 <sup>s</sup>	+65° 59' 39.9''	C7I	6.63	3.27
T Lyr	90883	18 <sup>h</sup> 32 <sup>m</sup> 20.1 <sup>s</sup>	+36° 59' 55.7''	C8	7.57	5.46
HK Lyr	91774	18 <sup>h</sup> 42 <sup>m</sup> 50.0 <sup>s</sup>	+36° 57' 30.9''	C5II	7.97	3.08
TT Cyg	96836	19 <sup>h</sup> 40 <sup>m</sup> 57.0 <sup>s</sup>	+32° 37' 05.8''	C6.4e	7.63	2.92

stellar core which is enwrapped by a very extended ( $\sim 10^{11}$  m) and relatively cool ( $\sim 10^4$  K) envelope. The transport of energy released in the core towards their outermost layers is ensured by convection, the convective zone sometimes penetrates the regions affected by nuclear processes. The elements transmuted during the previous nuclear evolution (including the heavy elements originated from *s*-processes) are transported to the surface by convection and then into the outer space by their tremendous stellar wind.

Carbon stars exhibit moderate light variations (typically 0.6 mag) on time scales ranging from tens to thousands of days. The stars are mostly classified as semiregular, possibly pulsating variable stars of late spectral types. In general, our knowledge of the nature of light changes is incredibly poor as only one carbon star (VY UMa) has been photoelectrically observed sufficiently frequently for more than ten years (Percy et al. 2001). Our contribution aims at filling a bit the above mentioned information gap by the study of the photometrical behavior of seven brightest carbon stars in the time interval of 15 years.

## 2. Observations

### 2.1. Observatory of Masaryk University

In the framework of a project of a comprehensive study of late-type stars carried out by Miroslav Vetešník and Jiří Papoušek there were systematically observed a number of carbon variables, namely: WZ Cas, VY UMa, Y CVn, RY Dra, T Lyr, HK Lyr and TT Cyg. Basic information about them is given in Table 1. The programme of the photoelectric monitoring of late type stars was terminated in 1994 for temporary technical problems and a planned switch to CCD technique. The whole project definitely ceased after both of its guarantees left Masaryk University.

All observations of carbons were done by 0.6m reflector of the Observatory of Masaryk University at Kraví hora in Brno from 1979 to 1994. The telescope

was equipped with the photomultiplier EMI 9656 and a set of  $U$ ,  $B$ ,  $V$  filters of Johnson's international photometric system. The data reduction method was the standard one. The mean error of one observation was always less than 0.04 mag, but the uncertainty of the majority of observations was smaller than 0.007 mag.

As carbon stars rank among to extremely red objects, they could not be observed in  $U$  colour, T Lyr with  $(B-V) = 5.5$  mag was observed only in  $V$ . The original data will be provided by the authors upon request. They are also accessible at the address <http://www.hvezdarna.cz/dusek/carbon/>.

The coverage of light curves of individual stars differs considerably. Several stars of the set (Y CVn, T Lyr, HK Lyr) were observed relatively frequently; on the contrary, some stars were monitored only sporadically (RY Dra). The resulting light curves are plotted in Figures 1–7.

The data used in our study represent the longest continuous photoelectric light curves of WZ Cas, VY UMa, Y CVn, RY Dra, T Lyr, HK Lyr and TT Cyg ever published.

## 2.2. Space observatory Hipparcos

The aim of the Hipparcos mission was to obtain very precise measurements of positions of bright stars with accuracy better than one angular millisecond. To optimize the astrometric signal, the brightness of stars were many times measured in the three broadband photometric colors, too. This by-product of astrometric measurements turned out to be an effective and reliable source of information about the behavior of plenty of variable stars. The authors of the Tycho and Hipparchos catalogue made a rough analysis of the measurements and showed that in many cases they are in disagreement with published periods and amplitudes of light changes.

The precision of individual measurements of the Hipparcos magnitudes varies from 0.003 to 0.05 mag. The extremely wide filter  $H_p$  had a maximum of its transmittance at the wavelength of 450 nm, but it transmitted the radiation well in the whole range from 340 to 890 nm. The precise description of its characteristics can be found in the preface to the Tycho or Hipparchos catalogue (Perryman et al., 1997). Besides this, the satellite Hipparcos also used another detectors which gained information about the stellar brightness in instrumental  $B_T$  and  $V_T$  systems. These measurements exhibited a large scatter and frequent rough errors, especially in the case of the  $B_T$  magnitudes. This was the reason why we used in our analysis  $H_p$  data only.

Observations in the photometric system  $H_p$  were obtained in a relatively short time from October 1989 until March 1993 via astrometric detectors of the Hipparcos mission. During these years, the brightness of each star was measured more than a hundred times. Measurements were conducted in about 35 sequences, each of them comprising several individual measurements done in the time interval shorter than 0.5 day. As we did not find any short-time variations,

**Table 2.** Global characteristics of the theoretical light curves of the variable stars.

	$\overline{m_v}$ [mag]	$\overline{(H_p - V)}$ [mag]	$\overline{(B - V)}$ [mag]	$P_{char}$ [d]	$A_{char}$ [mag]	$\left(\frac{\partial B}{\partial V}\right)$	remark
WZ Cas	7.299 $\pm 0.024$	-0.294 $\pm 0.029$	3.145 $\pm 0.032$	157	0.379	2.04 $\pm 0.26$	1.
VY UMa	6.056 $\pm 0.010$	-0.066 $\pm 0.012$	2.452 $\pm 0.014$	134	0.187	1.61 $\pm 0.18$	-
Y CVn	5.465 $\pm 0.007$	-0.215 $\pm 0.013$	3.019 $\pm 0.011$	192	0.328		2.
RY Dra	6.493 $\pm 0.050$	-0.352 $\pm 0.041$	3.222 $\pm 0.080$	351	0.950	2.19 $\pm 0.21$	-
HK Lyr	8.056 $\pm 0.011$	-0.178 $\pm 0.019$	3.302 $\pm 0.014$	218	0.382	1.88 $\pm 0.12$	-
T Lyr	8.399 $\pm 0.008$	-0.883 $\pm 0.013$	-	314	0.600	-	3.
TT Cyg	7.724 $\pm 0.010$	-0.188 $\pm 0.016$	2.794 $\pm 0.015$	126	0.253	1.43 $\pm 0.14$	-

we decided to work with median values characterizing the particular sequence instead of individual measurements.

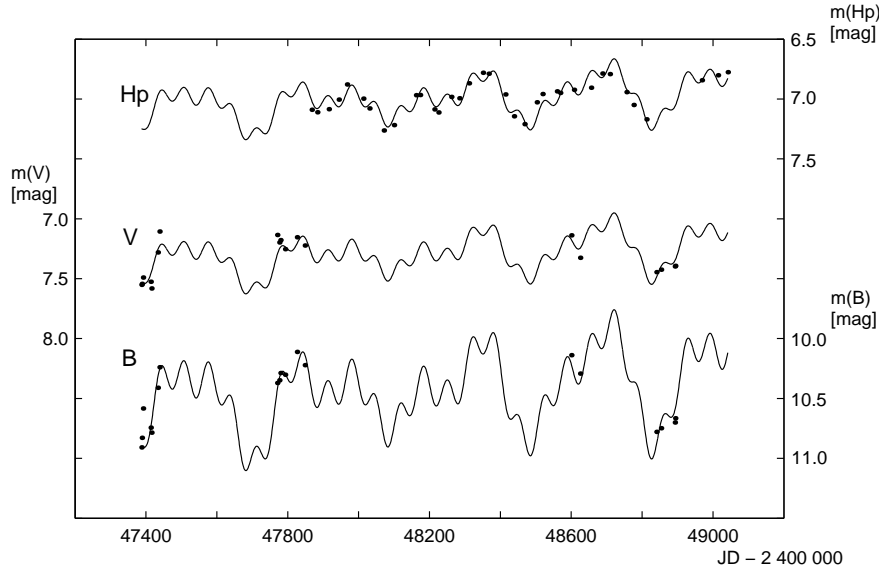
Measurements in the photometric system  $H_p$  turned out to be relatively reliable and sufficiently precise. Assuming constancy of the stellar light during the sequence we found that the standard deviation of the individual  $H_p$  magnitude for the observed group of carbons was usually 0.016 mag, and a mean uncertainty of the set of measurements during one sequence was 0.010 mag. These results are in agreement with those given in the Hipparchos catalogue (Perryman et al., 1997). The authors estimated the uncertainty of the median of brightness in all measurement files from 0.0004 to 0.0007 mag (for the stars brightness between 2–12 mag), the uncertainty of one individual measurement being 0.011 mag.

### 3. Light curves and their analysis

The observed light variations in all colors ( $B$ ,  $V$ ,  $H_p$ ) are continuous and sequentially monotonous in the range of several tens of days. We failed to find

any significant light variations in the range from a fraction of a day to a few days. Buchler et al. (2001) came to the same conclusion.

We found that the light changes in various colors positively correlate, but the correlation is not absolute. The light changes running in  $H_p$  and  $V$  seem to be almost parallel; the ratio of  $B$  and  $V$  amplitudes of light variations always exceeds one (typically 1.8) and is found to be almost constant for a particular star. So the slope of the correlation ( $\frac{\partial B}{\partial V}$ ) was chosen as one of the parameters of the description of observed light curves.



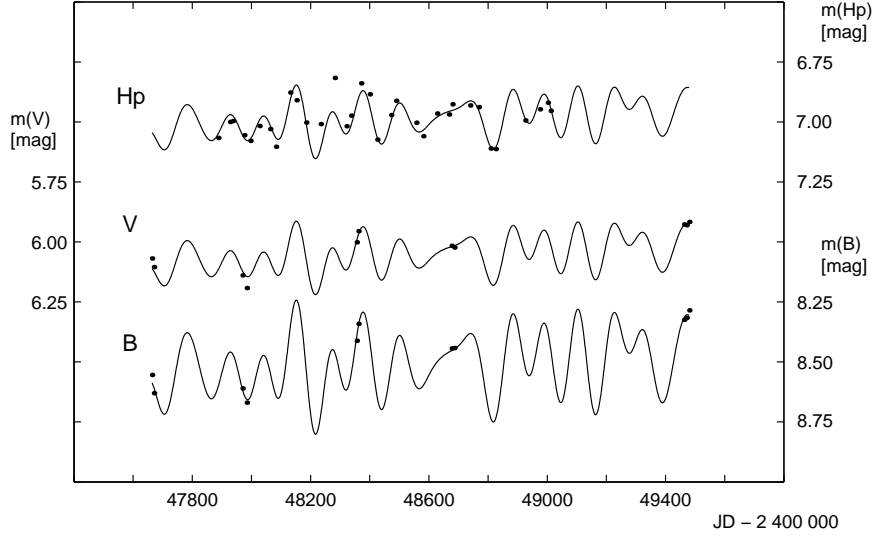
**Figure 1.** Light curves of WZ Cas.

After a long search for an appropriate representation of light variations we found that observed light curves of all carbon stars in all colors can satisfactorily be approximated by a superposition of (may be) aperiodic long-term variations and a sum of several middle-term harmonic variations with different amplitudes and periods. Long-term variations were fitted by a polynomial up to its fifth degree, the periods of sinusoidal medium-term variations were found to lie in the interval from 50 to 500 days. The expressions of the light curve in  $V(t)$ ,  $B(t)$  and  $H_p(t)$  were taken in the form:

$$V(t) \simeq \bar{V} + F(t), \quad (1)$$

$$B(t) \simeq \bar{V} + \overline{(B-V)} + \frac{\partial B}{\partial V} F(t), \quad (2)$$

$$H_p(t) \simeq \bar{V} + \overline{(H_p - V)} + F(t), \quad (3)$$



**Figure 2.** Light curves of VY UMa.

where the quantities:  $\overline{V}$ ,  $\overline{(B-V)}$ ,  $\overline{(H_p - V)}$  and  $\frac{\partial B}{\partial V}$  are free parameters connected to the individual colors. The light curve  $F(t)$  is a sum of long-term and medium-term parts,

$$F(t) = \sum_{i=1}^k a_i \left( \frac{t - T_m}{t_s} \right)^i + \sum_{j=1}^l A_j \sin \left( \frac{2\pi(t - M_{0j})}{P_j} \right), \quad (4)$$

which are characterized by  $k$  parameters  $\{a_i\}$  and  $3l$  parameters:  $\{P_j, M_{0j}, A_j\}$ .  $T_m$  is the mean value of the moments of observations;  $t_s$  are standard deviations of them. Periods  $P_j$  were found by means of a special interactive code (see Dušek 2002), where the selected periods had to fulfil several strict constraints.

It turned out to be useful to introduce two global quantities referring to the medium-term light variations – the so-called characteristic amplitude  $A_{char}$  and characteristic period  $P_{char}$  of the star.

$$A_{char} = \sqrt{\sum_{j=1}^l A_j^2}, \quad (5)$$

$$P_{char} = A_{char} \left( \sum_{j=1}^l \frac{A_j^2}{P_j^2} \right)^{-1/2}. \quad (6)$$

Such quantities optionally reflect a global shape of the light curve, being only slightly dependent on the details of the expression chosen the global characteristics of a light curve of variable stars:  $A_{char}$ ,  $P_{char}$ ,  $\overline{V}$ ,  $\overline{(B-V)}$ ,  $\overline{(H_p - V)}$  and

$\frac{\partial B}{\partial V}$  are displayed in Table 2. Other parameters necessary for a complete approximation of the light curves, namely the set describing medium-term harmonic variations and the set of parameters of the polynomial expression of long-term changes  $\{a_i\}$ , are given in Table 3, supplemented by a discussion referring to individual stars.

Light curves of individual stars in all colors are depicted in Figures 1–7, the dependence of the characteristic amplitude of light changes  $A_{char}$  on the logarithm of the characteristic period  $P_{char}$  is displayed in Fig 8.

#### Remarks on Table 2.

1. For WZ Cas only the simultaneous  $B$  and  $V$  observations were accepted.
2. A large sample of data on Y CVn allowed us to use a bit more complicated approach, where the quantity  $\frac{\partial B}{\partial V}$  for long-term and medium-term variations was separated:  $\left(\frac{\partial B}{\partial V}\right)_s = (2.48 \pm 0.09)$  and  $\left(\frac{\partial B}{\partial V}\right)_m = (1.71 \pm 0.08)$ .
3. T Lyr is the reddest star in our set:  $(B-V) = 5.5$  mag, so it was too faint in  $B$  to be measured at the Masaryk University Observatory.

#### Remarks on Table 3.

- **WZ Cas:** In the GCVS (Cholopov et al., 2000) it is given the period of 186 d, which is identical with our second period:  $(186.2 \pm 1.8)$  d. Kiss et al. (1999) came to the same conclusion by a thorough analysis of visual observations. He also revealed another period of variations: 373 d, which agrees well with ours:  $(379 \pm 5)$  d.
- **VY UMa:** Several periods given in our list are independently confirmed by other analyses. Particularly gratifying is the agreement in periods 124.7 d and 188 d, identified in an extensive and homogenous set of measurements done by the robotic photometric telescopes (Percy et al., 2001). Our period  $(119.7 \pm 0.4)$  d is identical with the period of 120 d derived from the visual estimates of the brightness by amateur observers (Ofek et al., 1995). We are convinced that these coincidences speak in favour of our approach.
- **Y CVn:** Our period  $(268.6 \pm 0.5)$  d confirms, in the range of its uncertainty, the existence of the period 267.8 d found by Dušek (1996). The GCVS (Cholopov et al., 2000) period 157 d has not been confirmed.
- **RY Dra:** The fit of the light curve is questionable and very probably not real, due to an uneven distribution of the  $BV$  observations in the course of time. The star is an interesting object for photometric observations.
- **HK Lyr:** GCVS (Cholopov et al., 2000) classified the star as a quite irregular variable. Nevertheless the analysis of Hipparcos photometry by Perryman et al. (1997) led to the period of 186 d, which is close to the period found by us  $(184.6 \pm 0.4)$  d.
- **T Lyr:** We have not found any reference on the periodicity of the star.
- **TT Cyg:** We found three periods of the star in the literature: GCVS (Cholopov et al., 2000) indicates the period of 118 d, the analysis of visual observations of amateur observers (Kiss et al., 1999) led to the periods of  $(390 \pm 10)$  d and  $(188 \pm 5)$  d. Two shorter periods are close to ours:  $(116.6 \pm 0.3)$  d and  $(187.1 \pm 0.5)$  d.

**Table 3.** Other parameters necessary for a complete entry of the model light curves.**WZ Cassiopeiae:**

$JD(T_m) = 2\,448\,170$ ,  $t_s = 527$  d

The degree of the polynomial: 1

$$a_1 = (-0.059 \pm 0.009) \text{ mag}$$

$P$ [d]	$A$ [mag]	$JD(M_0)$
$379.4 \pm 5.1$	$0.134 \pm 0.016$	$2\,448\,158.7 \pm 7.1$
$186.2 \pm 1.8$	$0.089 \pm 0.016$	$2\,448\,077.3 \pm 5.2$
$293.2 \pm 6.6$	$0.072 \pm 0.018$	$2\,448\,235 \pm 12$
$67.5 \pm 0.3$	$0.069 \pm 0.013$	$2\,448\,198.4 \pm 2.1$

**VY Ursae Maioris:**

$JD(T_m) = 2\,448\,463$ ,  $t_s = 504$  d

The degree of the polynomial: 1

$$a_1 = (-0.024 \pm 0.005) \text{ mag}$$

$P$ [d]	$A$ [mag]	$JD(M_0)$
$119.7 \pm 0.4$	$0.066 \pm 0.006$	$2\,448\,462.6 \pm 1.9$
$189.1 \pm 1.6$	$0.045 \pm 0.007$	$2\,448\,393.5 \pm 4.4$
$285.2 \pm 5.4$	$0.035 \pm 0.007$	$2\,448\,339.3 \pm 9.5$
$104.6 \pm 0.6$	$0.034 \pm 0.007$	$2\,448\,444.6 \pm 3.1$

**Y Canum Venaticorum:**

$JD(T_m) = 2\,446\,683$ ,  $t_s = 1699$  d

The degree of the polynomial: 5

$$a_1 = (0.132 \pm 0.014) \text{ mag}$$

$$a_2 = (-0.158 \pm 0.018) \text{ mag}$$

$$a_3 = (-0.019 \pm 0.019) \text{ mag}$$

$$a_4 = (0.152 \pm 0.010) \text{ mag}$$

$$a_5 = (-0.035 \pm 0.007) \text{ mag}$$

$P$ [d]	$A$ [mag]	$JD(M_0)$
$268.6 \pm 0.5$	$0.082 \pm 0.006$	$2\,446\,578.3 \pm 2.9$
$167.1 \pm 0.1$	$0.092 \pm 0.005$	$2\,446\,752.4 \pm 1.4$
$413.8 \pm 1.3$	$0.064 \pm 0.005$	$2\,446\,557.8 \pm 5.1$
$163.1 \pm 0.3$	$0.046 \pm 0.005$	$2\,446\,705.0 \pm 2.9$
$247.1 \pm 0.6$	$0.050 \pm 0.005$	$2\,446\,582.0 \pm 3.9$
$216.6 \pm 0.5$	$0.042 \pm 0.005$	$2\,446\,721.0 \pm 4.0$
$84.9 \pm 0.1$	$0.025 \pm 0.004$	$2\,446\,687.5 \pm 2.0$
$94.8 \pm 0.2$	$0.024 \pm 0.005$	$2\,446\,708.3 \pm 3.1$



**Table 3.** (continued)**RY Draconis:**JD( $T_m$ ) = 2 447 686,  $t_s = 1144$  d

The degree of the polynomial: 2

$$a_1 = (0.307 \pm 0.031) \text{ mag}$$

$$a_2 = (0.248 \pm 0.029) \text{ mag}$$

$P$ [d]	$A$ [mag]	JD( $M_0$ )
$394.6 \pm 2.3$	$0.285 \pm 0.031$	$2\ 447\ 659.4 \pm 6.8$
$344.3 \pm 2.5$	$0.219 \pm 0.033$	$2\ 447\ 830.9 \pm 8.5$
$274.4 \pm 0.9$	$0.233 \pm 0.021$	$2\ 447\ 681.4 \pm 3.9$
$480.3 \pm 4.1$	$0.205 \pm 0.026$	$2\ 447\ 886.3 \pm 9.7$

**HK Lyrae:**JD( $T_m$ ) = 2 447 898,  $t_s = 1037$  d

The degree of the polynomial: 3

$$a_1 = (0.021 \pm 0.013) \text{ mag}$$

$$a_2 = (-0.051 \pm 0.008) \text{ mag}$$

$$a_3 = (-0.019 \pm 0.006) \text{ mag}$$

$P$ [d]	$A$ [mag]	JD( $M_0$ )
$184.6 \pm 0.4$	$0.107 \pm 0.008$	$2\ 447\ 851.5 \pm 2.2$
$323.0 \pm 1.5$	$0.088 \pm 0.008$	$2\ 447\ 841.1 \pm 4.8$
$197.7 \pm 0.5$	$0.089 \pm 0.008$	$2\ 447\ 893.2 \pm 2.8$
$300.4 \pm 1.2$	$0.089 \pm 0.008$	$2\ 447\ 976.6 \pm 4.2$
$141.5 \pm 0.6$	$0.036 \pm 0.007$	$2\ 447\ 952.3 \pm 4.5$

**T Lyrae:**JD( $T_m$ ) = 2 448 007,  $t_s = 993$  d

The degree of the polynomial: 2

$$a_1 = (-0.003 \pm 0.006) \text{ mag}$$

$$a_2 = (-0.139 \pm 0.009) \text{ mag}$$

$P$ [d]	$A$ [mag]	JD( $M_0$ )
$392.5 \pm 1.0$	$0.210 \pm 0.009$	$2\ 447\ 998.4 \pm 3.8$
$529.8 \pm 3.6$	$0.116 \pm 0.009$	$2\ 448\ 007.3 \pm 3.4$
$271.4 \pm 1.4$	$0.098 \pm 0.012$	$2\ 448\ 033.9 \pm 3.3$
$467.9 \pm 3.8$	$0.085 \pm 0.009$	$2\ 447\ 935.3 \pm 6.7$
$280.8 \pm 1.9$	$0.076 \pm 0.012$	$2\ 447\ 926.9 \pm 8.2$
$194.2 \pm 0.7$	$0.070 \pm 0.008$	$2\ 448\ 030.0 \pm 5.4$
$172.6 \pm 0.6$	$0.060 \pm 0.008$	$2\ 448\ 200.7 \pm 7.0$
$110.8 \pm 0.4$	$0.034 \pm 0.008$	$2\ 448\ 156.8 \pm 2.6$

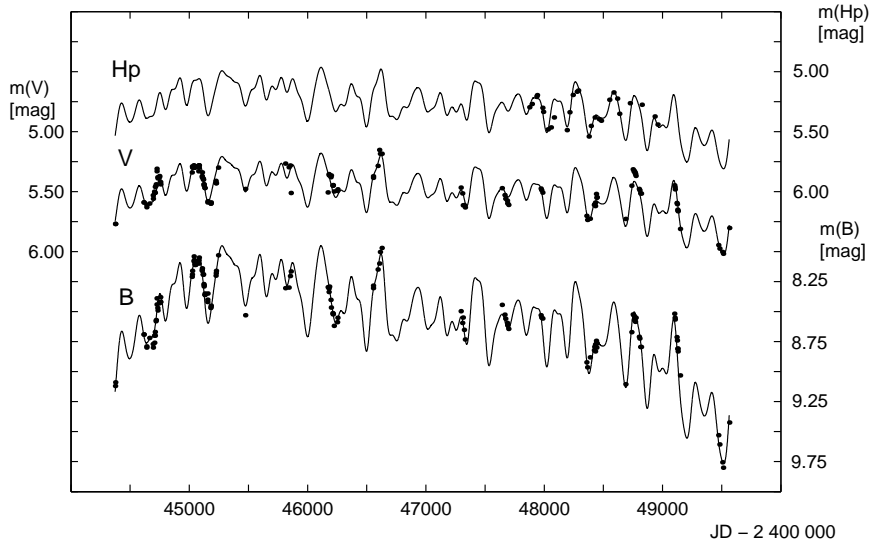
**Table 3.** (continued)**TT Cygni:**JD( $T_m$ ) = 2 448 059,  $t_s = 1069$  d

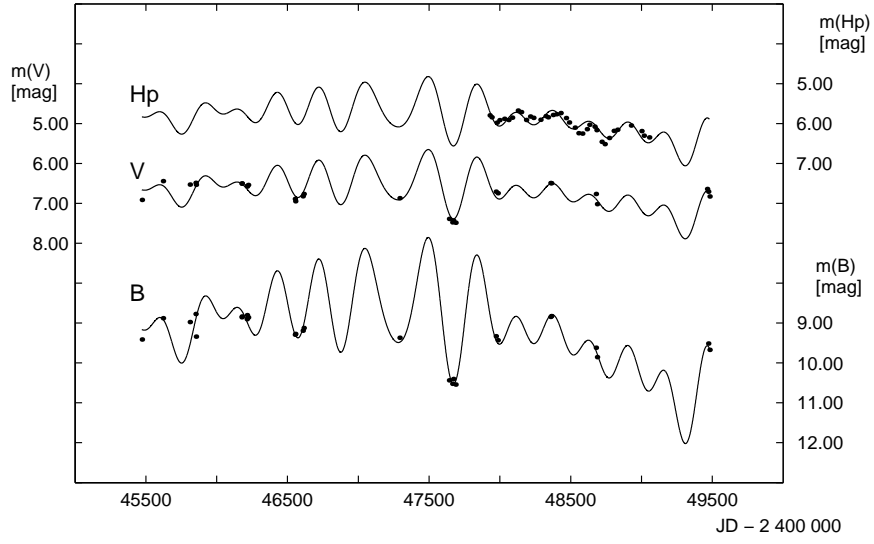
The degree of the polynomial: 0

$P$ [d]	$A$ [mag]	JD( $M_0$ )
$116.6 \pm 0.3$	$0.067 \pm 0.009$	$2\ 448\ 076.9 \pm 2.3$
$187.1 \pm 0.5$	$0.086 \pm 0.009$	$2\ 448\ 019.2 \pm 3.0$
$253.8 \pm 2.0$	$0.042 \pm 0.009$	$2\ 447\ 996.3 \pm 8.4$
$72.7 \pm 0.1$	$0.048 \pm 0.008$	$2\ 448\ 086.5 \pm 2.0$

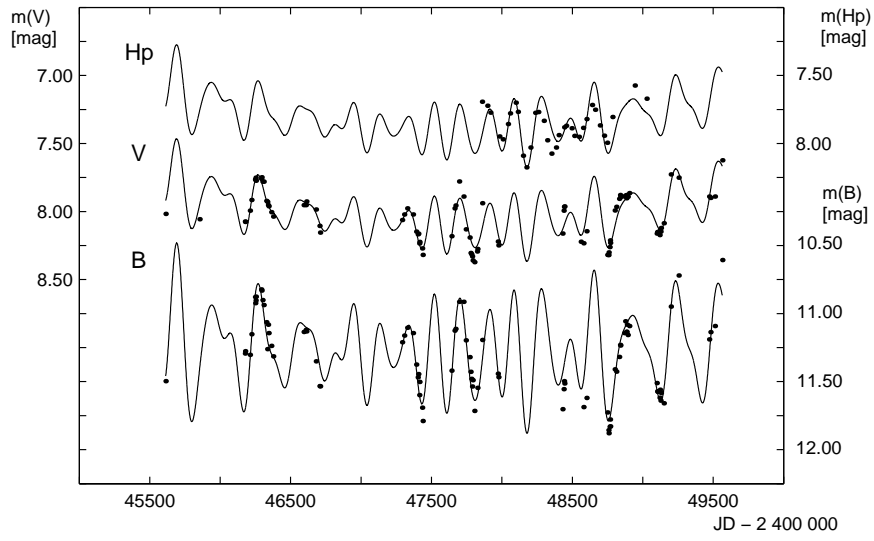
#### 4. Discussion of results and conclusions

The fact that carbon stars exhibit moderate light changes on the time scale of tens or hundreds of days has been known for more than a century. Carbon stars were classified mostly as irregular (type Lb) or semiregular (type Sb) variables, light changes of which were not strictly periodic. Nevertheless, we could occasionally find in the literature some values of periods of individual carbons, but the consistency of such information is very poor, as the majority of them were based on an insufficient, or unreliable, observational material.

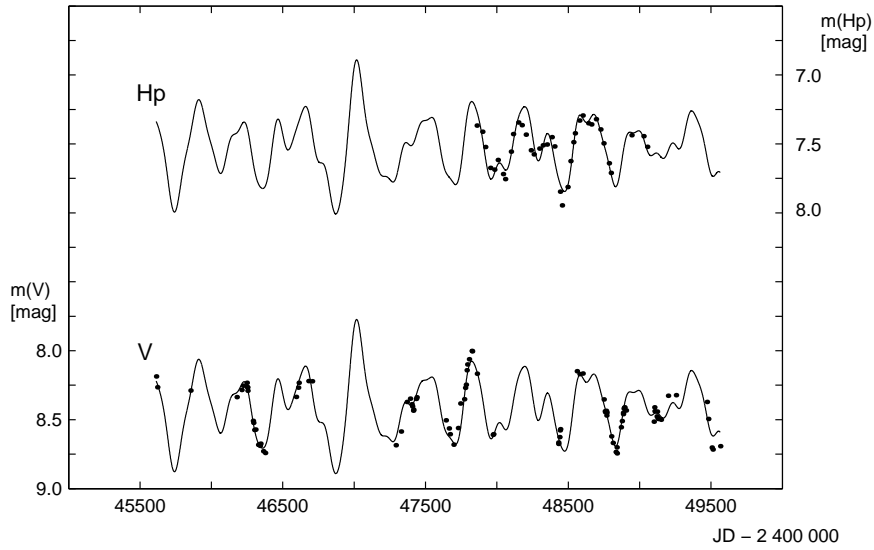
**Figure 3.** Light curves of Y CVn.



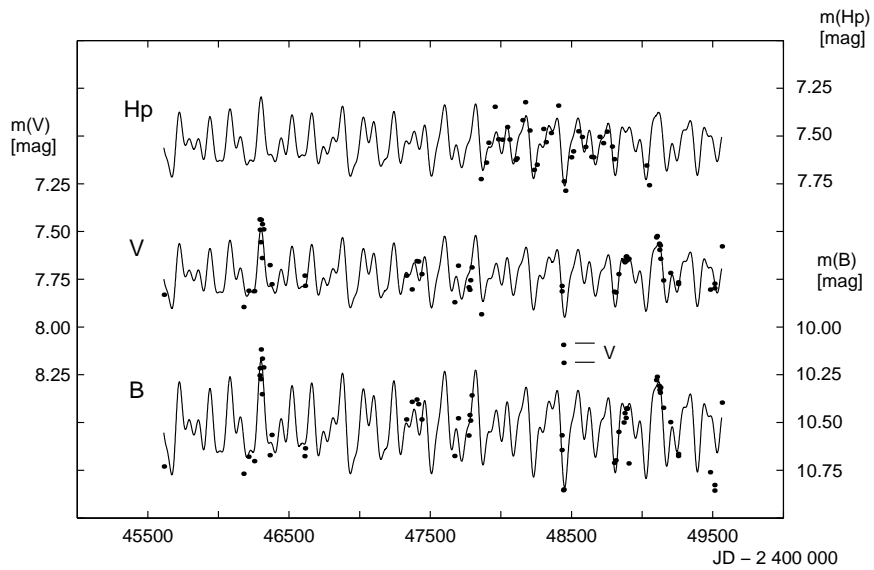
**Figure 4.** Light curves of RY Dra.



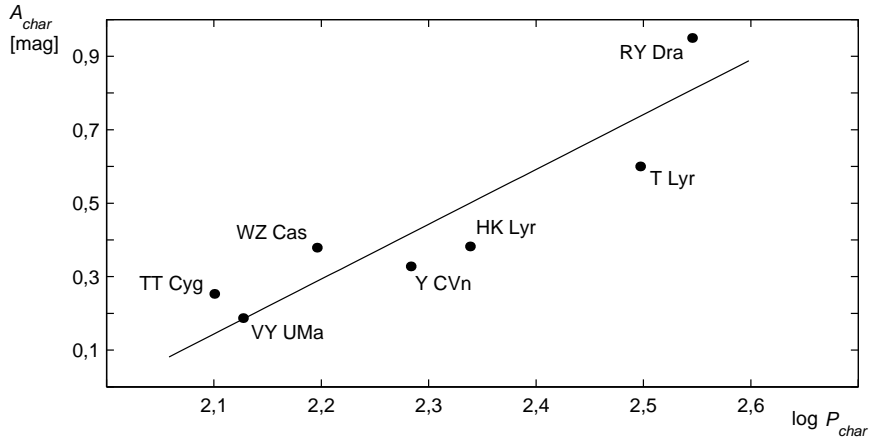
**Figure 5.** Light curves of HK Lyr.



**Figure 6.** Light curves of T Lyr. The star was too faint in *B* to be measured at the Masaryk University Observatory.



**Figure 7.** Light curves of TT Cyg.



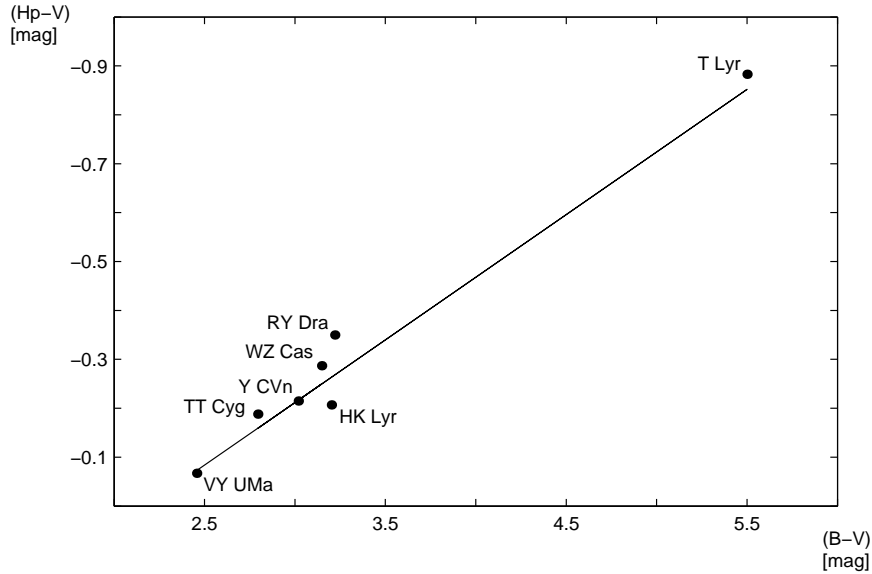
**Figure 8.** The correlation between characteristic amplitudes  $A_{char}$  and periods  $P_{char}$ .

Only several years ago there appeared statistically well-treated studies dealing with variability of carbons (Ofek et al., 1995; Perryman et al., 1997; Kiss et al., 1999; Percy et al., 2001), which were based on more trustworthy data. The authors of all papers have stated that light variations of carbon stars exhibit a certain periodicity and found for particular stars one or more periods.

The present analysis of the light curves of seven bright carbon stars, based on more than a thousand photoelectric measurements and covering the time interval of 15 years, favours our hypothesis that the light behavior of these large AGB stars has a multiperiodic nature. Credence of the reality of periods found by us is strengthened by the fact that practically all periods announced in the above-mentioned studies fit well into our period list. Hence, we believe that our approach to the study of carbon stars' variability is eligible. We also believe that our approximation of their light curves will be applicable to all carbons as they form, from the point of their inner structure, a relatively homogenous group of stars with the strictly defined initial mass and evolutionary status. This conclusion is supported by the apparent mutual similarity of light curves of all the seven carbon stars studied.

Nevertheless, a close inspection of the fit of light curves clearly shows that the approximation is not perfect in details. First, the assumption of a strict linear correlation of the middle-term variations in  $V$ ,  $B$  and  $H_p$  is not satisfied always – there are several sections where the light changes proceed in the opposite direction (see e.g. the  $B$  and  $V$  light curves of Y CVn).

Second, we do not get a complete agreement between the observed and fitted light curve, even by using more than 8 periods and a very high order of long-term variation polynomials. The deviations do not have the statistics of random deflections, the standard deviation of the fitting remains considerably larger



**Figure 9.** The correlation between  $\overline{(H_p - V)}$  and  $\overline{(B - V)}$ . The colour index  $\overline{(B - V)}$  of T Lyrae is taken from Perryman et al. (1997).

than the expected uncertainty of individual observations. It means that our approach is only a certain approximation to the reality.

After numerical removing of all other variations contained in the model, we can study forms of light variations related to particular periods. We have found that for all partial periodic light curves the sine approximation is fully adequate. This fact seems to be in accordance with the interpretation of observed medium-term light variations by higher mode pulsations of the stars.

Very important are conclusions based on the study of mutual correlations among global characteristics of the light curve, which are practically independent of the details of the approximation (the number of found periods, the order of the polynomial etc.). Using the data from Table 2, we have found a well-defined relation between  $\overline{(H_p - V)}$  and  $\overline{(B - V)}$  in the form (see Fig. 9):

$$\overline{(H_p - V)} = (-0.182 \pm 0.017) + (-0.246 \pm 0.039)[\overline{(B - V)} - 2.909]. \quad (7)$$

The relation can be useful for stellar astronomers who study red stars with  $(B - V)$  indices between 2.5 and 5.5 mags.

According to our expectation, we have found a positive correlation between the characteristic periods  $P_{char}$  and the mean color index  $\overline{(B - V)}$ . The correlation can easily be interpreted if we admit that medium term variations are caused by pulsation of various modes. It is known that the characteristic period

of pulsating stars increases with its linear dimension as well as the ruddiness of the AGB stars. The correlation is well defined (except for RY Dra), which means that the high value of  $\overline{(B-V)}$  of a carbon star is caused rather by the star itself than by expected circumstellar reddening.

Very impressive is an almost linear correlation between the characteristic amplitudes in mags and the characteristic periods in days (see Fig 8), which may indicate that the amplitude of an assumed pulsation increases with the radius of a star. The correlation can be expressed as:

$$A_{char} = (1.49 \pm 0.31) \log \left( \frac{P_{char}}{199} \right) + (0.44 \pm 0.05). \quad (8)$$

Although our observational data on carbon stars belong to the best material in the field, they are far from ideal. For a reliable evaluation of the details of carbons' light variability, it would be necessary to observe them regularly, without interruptions longer than 10 days, for about ten years. Such an observational task could be feasible by means of robotic photometric telescopes or by a (not yet operating) special photometric satellite.

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