Analytical solution of the extended single-body problem and its applications

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Abstract. This paper deals with the solution of well-known single body problem for small meteoroids moving through Earth's atmosphere. The main result is an analytical solution which may significantly accelerate numerical modeling and bring a good view of the specifics of the resulting expressions. A luminous equation takes into account the deceleration process, and extends the basic physical model with the dynamical effect of mass dissipation at the particle surface. Both the particle mass and velocity can then be easily calculated at an arbitrary altitude in isothermal Earth's atmosphere. The analytical expressions offer an unambiguous mapping $[v_{avg}, M, h_{max}] \leftrightarrow [v_\infty, m_\infty, \delta]$. It means the information on observable parameters (average particle velocity in Earth's atmosphere v_{avg}, the maximum magnitude M and height of maximum luminosity h_{max}) is a sufficient basis to retrieve an incident velocity v_∞, mass m_∞ of the particle and its mean effective density δ.

Key words: meteoric physics – ablation – brightness – inverse problems

1. Introduction

Although the interaction of small extraterrestrial bodies with planetary atmospheres is a well-known problem, its extensions are still applicable to investigating both the atmospheric environment and properties of dust grains. The physical principles of this interaction (formulated as basic equations of motion) were presented by Pecina and Ceplecha (1983) as a so-called single-body problem. While large massive bodies are scattered in few substantial fragments, the motion of small cosmic dust grains in Earth's atmosphere is often well described by the simple fragmentation (or non-fragmentation) model (Borovička, et al., 1998; Borovička and Spurný, 1996).

Extensive research of a single-body problem results in many imporant conclusions consisting of radiation effects (Bellot Rubio, 1995; Nemchinov et al., 1995), meteoric ablation (Revelle, 1979; Bronshten, 1983; Hawkes and Jones, 1975), heights of meteors (Baggaley and Webb, 1980; Verbeeck, 1993), etc. Comparing theoretical methods and observational data (trajectories, lightcurves, spectra...), the meteoroid mass, composition, and structure can be estimated.

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The retrieval method is a well-defined inverse problem. Basic equations of the meteor phenomenon in the case of a single nonfragmenting body (Ceplecha, 1975) enable us to express the change of the particle mass m with velocity v as follows

$$m = m_\infty \exp\left\{\frac{\sigma}{2}(v^2 - v_\infty^2)\right\} \quad , \tag{1}$$

where m_∞ and v_∞ are the mass and velocity of the particle outside the Earth's atmosphere and σ is an ablation coefficient. This coefficient varies over a wide range, on average from 1.4×10^{-8} to 2.1×10^{-7} $s^2 m^{-2}$ (Ceplecha et al, 1998). However, the dependence of particle mass (or velocity) on altitude h in the atmosphere is still calculated by numerical methods. This paper brings an analytical solution for an isothermal model of Earth's atmosphere. The solution to this problem is described in the next section. The analytical expressions make it relatively simple to calculate the light intensity at each point of particle trajectory. The light intensity in classic single-body approximation is proportional to v^5,

$$I = -\tau(v)\frac{v^2}{2}\frac{dm}{dt} \quad , \tag{2}$$

because dm/dt is proportional to v^3. However, the luminous efficiency factor τ depends in general on velocity, too.

The main aim of this paper is to present an analytical solution of particle motion in the atmosphere, where particle mass and velocity are functions of altitude. The solution extends the luminous equation with considerable deceleration, and takes into account the dynamical effect of mass dissipation at the particle surface. Conclusions are related to the frequent questions of inverse problems. The information content of the input data necessary to get reliable results about characteristics of the cosmic dust particles (incident mass or velocity) is specified. The path-length of a meteor lightcurve, maximum magnitude, lifetime and other characteristics are investigated. The influence of the mass dissipation rate at the particle surface, scale height, drag coefficient, and ablation parameter on both particle motion and radiation effects is examined.

2. An extended single-body approximation

The luminosity of falling bodies is produced not only by ablation but also by deceleration, although this is only significant for meteoroids with speeds lower than 16 $km\ s^{-1}$ (Ceplecha et al, 1998). In such a case the general ballistic power lost by the meteorite must be applied (Celnikier, 1995), where deceleration is expressed by the second term of the following equation

$$I = -\tau(v)\left(\frac{v^2}{2}\frac{dm}{dt} + mv\frac{dv}{dt}\right) \quad . \tag{3}$$

Usually the empirical relations are applied to express the dependency of the meteor luminous efficiency τ on the velocity. Babadzhanov (1987) uses several model equations for $\tau(v)$ based on maximum luminosity of particle. While the model for bright meteors ($M \leq 0$) gives $\tau \sim v$, meteors with $M > 0$ can be characterized by τ, which depends on particle velocity: (1) τ is proportional to v if: 10 $km\ s^{-1} < v < 17\ km\ s^{-1}$, (2) $\tau \sim 1/v$ if $v \geq 17\ km\ s^{-1}$. Unfortunately, this classification requires information about the observed magnitude to select a corresponding equation. However, there are more appropriate formulae for numerical simulations based on velocity only. The expressions published by Pecina and Ceplecha (1983) are applied in this paper, where

$$\log \tau(v) = -12.834 - 10.307 \log v + 22.522 (\log v)^2 - 16.125 (\log v)^3 + 3.922 (\log v)^4$$

$$3 \leq v\ [km\ s^{-1}] < 25.372 \qquad (4a)$$

and

$$\tau(v) = 2 \times 10^{-14}\ v, \quad v \geq 25.372\ km\ s^{-1} \quad . \qquad (4b)$$

Equations (4a) and (4b) require v to be in $[km\ s^{-1}]$. Although the luminous efficiency is still not a well known quantity, the selection of either relation/value doesn't affect the analytical solution presented here. The luminous efficiency generally depends on many parameters (spectral region of observations, chemical composition of the meteoroid and the atmosphere, on meteoroid velocity and probably also on the mass) (Ceplecha et al, 1998). The value of $\log \tau = -11.4$ were for example retrieved for Perseids (in panchromatic).

Mass loss of a spherical body in the isothermal atmosphere with exponential decrease of the air density can be expressed (using Eqs. 2 and 4 in Ceplecha, 1975) as follows:

$$\frac{dm}{dh} = \omega\ m^{2/3} v^2\ e^{-h/H} \quad , \qquad (5)$$

where the atmospheric scale height H depends on the atmospheric layers which are taken into the calculation model. The validity of exponential law for the atmospheric parameters is seriously limited. Equation (5) is then applicable for not too thick atmospheric layers. The scale height H can reach values about $\approx 6 - 8$ km. Parameter ω depends on both particle density δ and entry zenith angle Z,

$$\omega = \pi \frac{\sigma \rho_0 \gamma}{\cos Z} \left(\frac{3}{4\pi\delta} \right)^{\frac{2}{3}} \quad . \qquad (6)$$

The ablation parameter σ is a function of the drag coefficient γ, heat transfer coefficient, and energy necessary for the ablation of a unit mass of the meteoroid. Parameter ρ_0 represents the atmospheric density at the Earth's surface. The Earth's atmosphere is not isothermal; nevertheless, it is useful to characterize the atmosphere over a certain altitude range by an exponential law of the type

$$\rho = \rho_0(h_0) \exp\{-(h - h_0)/H\} \quad , \qquad (7)$$

in which case ρ_0 and H are simple best-fit parameters to a measured (or modeled) density variation (Celnikier, 1995). The characteristic value of γ is usually within the range $\langle 0.5, 1.0 \rangle$.

On the other hand the mass dissipation rate at the particle surface causes changes in particle velocity, because meteoric ablation produces a net force on the parent particle. In general, this effect has an anisotropic character due to a limited active part of the particle surface. The physical formulation of this effect follows from well-known mechanics of bodies of variable mass (Meshcherskij, 1949) and was described by Katasev (1966). The simplification of his model results in the equation

$$\frac{1}{m}\frac{dm}{dh} \doteq \frac{v}{kv_t v + \frac{1}{\sigma}}\frac{dv}{dh} \quad . \tag{8}$$

The solution assumes that difference in particle velocity and the velocity of the molecules vaporised at its surface can be characterized by a constant value equal to kv_t. The reactive force is then proportional only to mass loss with time. The approximate value of proportionality coefficient k is 0.44, while the average thermal velocity of evaporated molecules v_t is about 1 $km\ s^{-1}$. For an isothermal atmosphere, classical evaporation theory, which gives the rate of evaporation of meteoric atoms, was presented long ago (Jones and Kaiser, 1966)

The mass-velocity dependence obtained by integrating equation (8) has the form

$$m = m_\infty \left(\frac{1+bv_\infty}{1+bv}\right)^{\sigma b^{-2}} \exp\left\{-\frac{\sigma(v_\infty - v)}{b}\right\} \quad , \tag{9}$$

where $b = \sigma k v_t$. Comparing equations (5) and (8) with reference to relation (9) , and integrating them over dv (from v to v_∞) and dh (from h to ∞), it can be found that

$$h = av_\infty H + H \ln\left(\frac{\omega H}{F(v, v_\infty, a, b) m_\infty^{1/3}}\right) \quad , \tag{10}$$

where $a = 1/(3kv_t)$, and

$$\begin{aligned} F(v, v_\infty, a, b) = 3ab\,(1+av_\infty)\,[Ei(av_\infty) - Ei(av)] + 3a^2(1+bv_\infty)\times \\ \left[\frac{e^{av_\infty}}{1+bv_\infty} - \frac{e^{av}}{1+bv}\right] - 3a\left\{av_\infty(a+b) + b\left[1+\left(\frac{a}{b}\right)^2\right]\right\}\times \\ \exp\left\{-\frac{a}{b}\right\}\left[Ei\left(av_\infty + \frac{a}{b}\right) - Ei\left(av + \frac{a}{b}\right)\right] \quad . \end{aligned} \tag{11}$$

Abramowitz and Stegun (1965) give five pages with function simply related to exponential integral Ei (the so-called E function). The height in the atmosphere h is then an analytic function of particle velocity v. Starting with $v = v_\infty$ and finishing with $v = 0$, the corresponding values of both, h and m (eq. 9) can then be calculated. Produced light intensity represented by magnitude M has the well known form

$$M = -2.5 \log I \quad . \tag{12}$$

Comparison of this theory with the Babadzhanov's model is presented on Fig. 1, where model configuration values of incident mass m_∞, and velocity v_∞ of the particle as well as an entry zenith angle were adopted according to Babadzhanov (1987). The resulting profile of the magnitude obtained using our model is apparently different from published curves. While Babadzhanov's model gives values of M smaller than observed data, our calculation leads to essentially large magnitudes in the first phase of the particle burning. However, the height of the maximum brightness is a better fit using formulae presented in this paper.

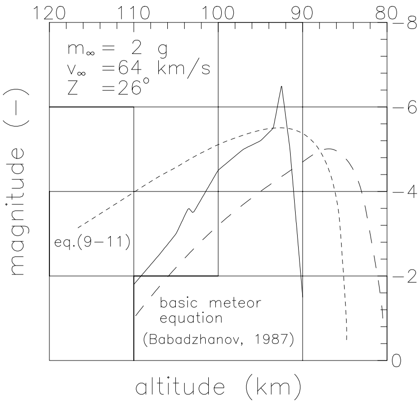

Figure 1. Theoretical and observational data for meteor magnitude at different altitudes in the Earth's atmosphere. The solid curve represents the measurement, the fine dashed curve corresponds to our theoretical model, the rough dashed curve represents Babadzhanov's model.

The best results can be obtained for very small so–called TV meteors (Koten, 1998). Comparison of the theoretical computation with the measured data (Koten 2000) is presented on Fig. 2 ($v_\infty = 23.36$ $km\ s^{-1}$, $m_\infty = 0.091$ g) and Fig. 3 ($v_\infty = 34.81$ $km\ s^{-1}$, $m_\infty = 0.0044$ g).

Excepting the incident mass and velocity of the particle, its fall could depend on several other parameters. The correlation of characteristics describing the particle radiation effects (trail length L, length of the extinction part of lightcurve B, lifetime t of the phenomenon, height h_{max} of maximum luminosity, and maximum magnitude M) and selected environmental/particle parameters (atmospheric density stratification, drag coefficient γ, ablation coefficient σ, and the average thermal velocity of evaporated molecules v_t) is documented in Tabs. 1-4.

The resulting dependencies illustrate the sensitivity of measurable characteristics to model settings. Starting parameters were configured as follows: $v_\infty = 40$ [$km\ s^{-1}$], $m_\infty = 20$ [g], $Z = 30$ [°], $\delta = 3.5$ [$g\ cm^{-3}$] (stone particle). The scale height H was chosen to vary over a relatively wide range (Tab. 1). The obtained results could then in general represent a measure of the interdependence between

Table 1. Dependence of selected characteristics (meteor trail length L, its extinction part B, lifetime t, height h_{max} of maximum luminosity, and maximum magnitude M) on atmospheric scale height H. Configuration settings: m_∞=20 [g], v_∞=40 [$km\ s^{-1}$], Z=30°, material=stone.

	H [km]				
	12.5	10.0	8.3	7.1	6.2
L [km]	68.60	54.88	45.74	39.20	34.31
t [s]	1.75	1.40	1.17	1.00	0.87
h_{max} [km]	125.78	98.39	80.47	67.88	58.56
B [%]	0.20	0.20	0.20	0.20	0.20
M [-]	−5.37	−5.62	−5.81	−5.98	−6.13

Table 2. Dependence of selected characteristics (meteor trail length L, its extinction part B, lifetime t, height h_{max} of maximum luminosity, and maximum magnitude M) on thermal velocity of evaporated molecules v_t. Configuration settings: m_∞=20 [g], v_∞=40 [$km\ s^{-1}$], Z=30°, material=stone.

	v_t [$km\ s^{-1}$				
	1.0	2.0	4.0	8.0	16.0
L [km]	43.90	49.50	58.35	71.26	79.25
t [s]	1.12	1.29	1.74	2.74	3.46
h_{max} [km]	76.93	77.06	77.85	81.21	80.66
B [%]	0.20	0.24	0.29	0.39	0.38
M [-]	−5.86	−5.83	−5.77	−5.60	−5.49

Table 3. Dependence of selected characteristics (meteor trail length L, its extinction part B, lifetime t, height h_{max} of maximum luminosity, and maximum magnitude M) on ablation coefficient. Configuration settings: m_∞=20 [g], v_∞=40 [$km\ s^{-1}$], Z=30°, material=stone.

	$\sigma \times 10^{-8}$ [s^2/m^2]				
	3.0	5.0	7.0	9.0	11.0
L [km]	45.91	43.90	43.08	42.20	39.48
t [s]	1.17	1.12	1.10	1.07	1.01
h_{max} [km]	73.87	76.93	79.95	81.28	82.30
B [%]	0.17	0.20	0.24	0.24	0.25
M [-]	−5.79	−5.86	−5.88	−5.89	−5.91

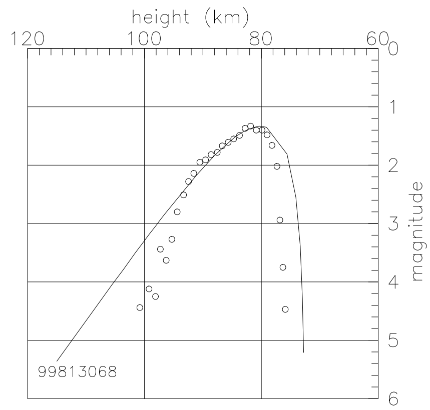

Figure 2. Measured (circles) and computed (solid curve) magnitude profile for TV meteor 99813068 (Koten, 2000). Incident parameters: $v_\infty = 23.36\ km\ s^{-1}$, $m_\infty = 0.091$ g.

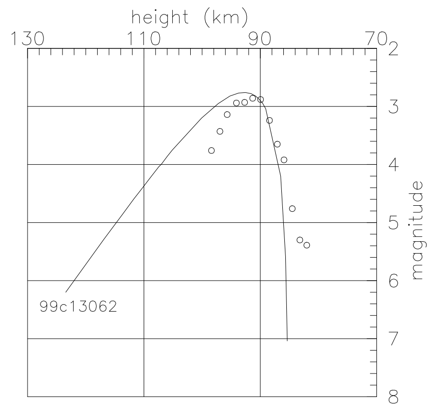

Figure 3. Measured (circles) and computed (solid curve) magnitude profile for TV meteor 99c13062 (Koten, 2000). Incident parameters: $v_\infty = 34.81\ km\ s^{-1}$, $m_\infty = 0.0044$ g.

the examined radiation effects and the planetary atmospheric environment. The smaller the value of H, the shorter both the trail-length of meteor lightcurve L and phenomenon lifetime t are. If decreases H, two times, the height h_{max} decreases approximately two times (from 125 km to 59 km), and the maximum magnitude increases two times (Tab. 1). If v_t increases the burning of meteoroid body decreases, and particle velocity decelerates. This is just a contrary effect to the decreasing of H (which could be understood as increasing the particle incident velocity – i.e. it correlates to fast burning). This fact is also represented by the moving of h_{max} from low atmospheric layers to high levels. The discussed

Table 4. Dependence of selected characteristics (meteor trail length L, its extinction part B, lifetime t, height h_{max} of maximum luminosity, and maximum magnitude M) on the drag coefficient γ. Configuration settings: m_∞=20 [g], v_∞=40 [$km\ s^{-1}$], Z=30°, material=stone.

	γ [-]				
	0.7	0.9	1.1	1.3	1.5
L [km]	43.90	43.90	43.90	43.90	43.90
t [s]	1.12	1.12	1.12	1.12	1.12
h_{max} [km]	76.38	78.39	79.99	81.33	82.47
B [%]	0.20	0.20	0.20	0.20	0.20
M [-]	-5.86	-5.86	-5.86	-5.86	-5.86

changes of v_t cause the deceleration of a particle burning in the last phase (in the so-called extinction part of the light curve) (Tab. 2). It corresponds to the brightness loss. The high values of ablation coefficient σ induce a decline of the apparent meteor trail length together with its lifetime. However, these changes are not so important (Tab. 3). While decreasing H by 2× is followed by L decreasing to $\approx L/2$, the increasing of σ by approximately one magnitude causes L to decrease by only about 1.2×. On the other hand, parameter B depends on the current value of σ. Both the B and the maximum brightness of the meteor increase with σ. It means that the profile of the light-curve will be modified too. The analysis of light-curve records could then be utilized to successfully evaluate the ablation coefficient. Particle motion in the atmosphere depends significantly on parameter ω, which is a product of the drag coefficient γ. However, it can be shown by the derivation of the presented equations that variability of γ has no effect on M, B, t, and L (Tab. 4). The modification of γ can only affect the resulting value of h_{max}. If γ increases by two times, h_{max} will be shifted from the lower to higher atmospheric layers, a difference of about 6 km.

3. Inverse problems of meteoric physics

Standard observational data (photographic records) offers only limited information on the meteor phenomenon. However, its content is sufficient to get reliable knowledge about particle characteristics, such as incident velocity v_∞, incident mass m_∞, and in specific cases, the material density δ, too. The average particle velocity in Earth's atmosphere can simply be determined from the actual trail length and lifetime of the meteor phenomenon. The actual trail length is recalculated using both the entry zenith angle and the length of the apparent trail length. Many simulations using the formuale presented in the previous sections confirm that v_∞ depends on the average velocity v_{avg} unambiguously. The average velocity can be obtained by multiplying the incident velocity v_∞ by the efficiency factor (Fig. 4). If v_{avg} is determined by observational techniques,

v_∞ is easily retrievable. It can easily be demonstrated that the information on maximum magnitude is not sufficient to retrieve particle characteristics. Two particles with different mass and velocity, that enter Earth's atmosphere from the same radiant can produce the same maximum luminous effect. This fact is documented by Fig. 5, where particles significantly different in mass were taken into account. The ratio of their masses reached the value of 40. The influence of the second term in equation (3) is documented by dashed curves, for which the deceleration term was omitted. The more massive the particle, the larger the differences between resulting magnitude profiles (Fig. 5, Fig. 6). However, this distinction (about 0.2-0.3^M) is at the level of precision of meteor photometry (0.4^M). Including this term into the theoretical model can at any rate improve the precision of the obtained results.

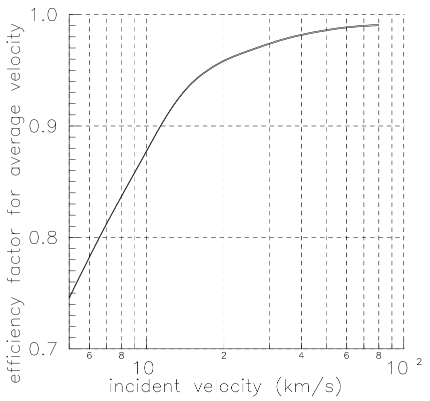

Figure 4. Efficiency factor reflecting the relationship between the incident velocity v_∞ of the particle in the interplanetary space and its average velocity inside the Earth's atmosphere.

The value of m_∞ can easily be determined by using figures 7 or 8 when both input parameters, v_∞ and maximum magnitude M, are known. The isolines represent the incident particle mass.

By including the next observable parameter into the calculation (e.g., height h_{max} of maximum luminosity), particles of different composition may be identified. The composition is represented by particle density in our model. Collecting all results presented above, the mapping $[v_{avg}, M, h_{max}] \leftrightarrow [v_\infty, m_\infty, \delta]$ is realisable when assuming the theory presented in the previous chapter is applicable for small particles. Particles equal in mass should flare at different altitudes in the Earth's atmosphere when they are different in composition (density). This fact is presented in Fig. 9, where results for stone and iron particles are compared. While the density of iron is approximately two times greater than the density of stone, the stone particle flares in atmospheric layers situated around 4 km above burning iron layers. In addition, this distance is practically independent of particle mass. The greater the particle mass, the lower the height of the max-

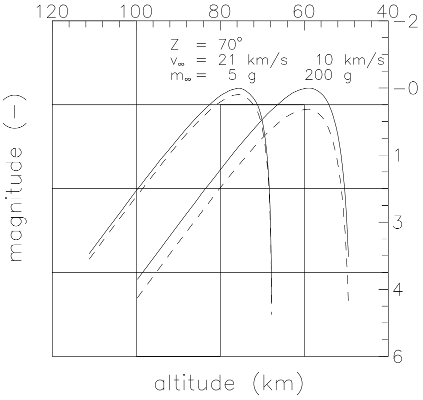

Figure 5. The height dependence of meteor magnitude in the Earth's atmosphere for particles different in mass and velocity. The solid curve represents a precise calculation using equation (3). The second (deceleration) term of the equation (3) was omitted when drawing the dashed curve.

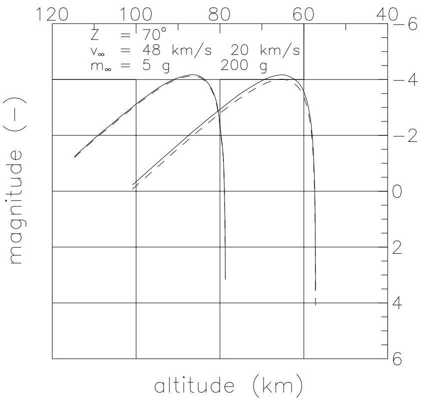

Figure 6. The height dependence of meteor magnitude in the Earth's atmosphere for particles different in mass and velocity. The solid curve represents a precise calculation using equation (3). The second (deceleration) term of the equation (3) was omitted when drawing the dashed curve.

imum meteor luminosity. Uncovering this effect, Baggley (1978) states that the mean height of sporadic meteors of magnitude -2 is about 82 km. The density of such a particle can be evaluated using the results of the calculation presented on Fig. 10 (scale height H=8 km). One can see that the average density δ could be approximately 1.4 $g\ cm^{-3}$ when the entry zenit h angle equals 30°, but the value of δ may be significantly larger (i.e. 5.5 $g\ cm^{-3}$) if the entry zenit h angle is about 70°. Taking into account mapping $[v_{avg}, M, h_{max}] \leftrightarrow [v_\infty, m_\infty, \delta]$, a

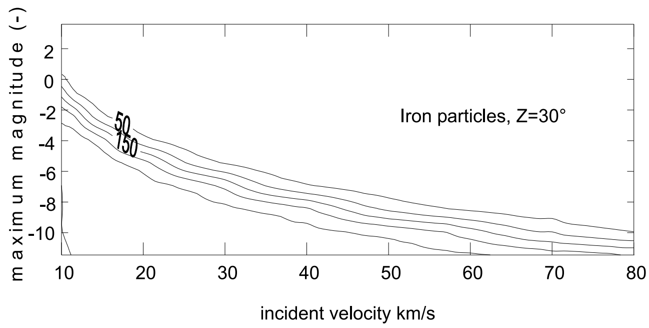

Figure 7. Isolines represent the particle mass m_∞ as a function of both, v_∞, and M. The entry zenit angle is 30^o.

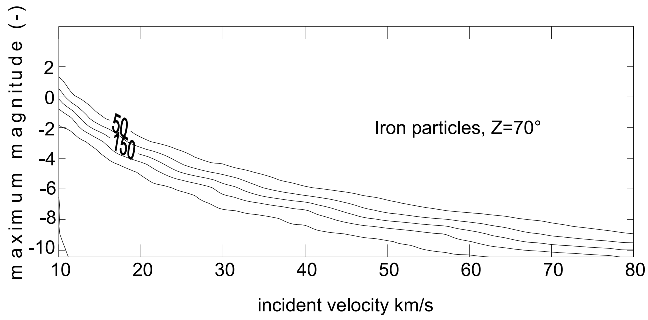

Figure 8. Isolines represent the particle mass m_∞ as a function of both, v_∞, and M. The entry zenit angle is 70^o.

minimal mass of the detectable particles can be specified assuming that only particles lighter than M_{obs} can be registered by the camera system. For example, the minimum mass of detectable iron particles coming into Earth's atmosphere under entry zenit h angle 30° and lighter than a defined value of M can be determined from Fig. 7. Accepting that the visual perception effect of the majority of observed meteors actually have magnitudes about +3, particles heavier than 0.1 g are detectable when their velocity is no less than 10 $km\ s^{-1}$. By collecting observed data on $[v_{avg}, M, h_{max}]$ and processing them the mass distribution function for a selected meteor shower could be found using our mapping method. On the other hand, the magnitude distribution can be simulated for a particle set with a known mass distribution. The approximate mass distribution of the meteor shower may also be evaluated by using the empiric formula that follows from our calculations (e.g. Figs 7. and 8.)

$$\frac{dm_\infty}{dM} = \frac{1}{p}\ \exp\left\{\frac{M+q}{p}\right\} \quad . \tag{13}$$

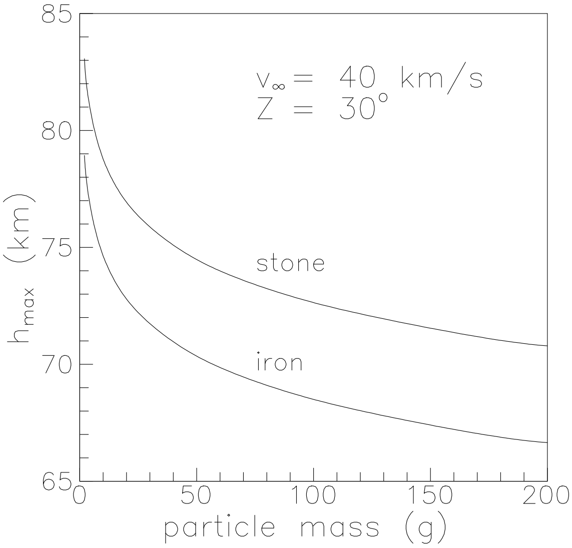

Figure 9. The height of maximum meteor luminosity as a function of particle mass. Two different material types (iron: 7.6 [$g\ cm^{-3}$], stone 3.5 [$g\ cm^{-3}$]) were taken into account.

The optimized values of parameters p and q were found to be approximately p=-1.09, and q=0.58. Then it is valid

$$\frac{dN}{dm_\infty} = -1.85\ e^{0.92M}\frac{dN}{dM} \quad , \tag{14}$$

where N represents the number of particles, the mass of which is between $\langle m, m+dm\rangle$, as well as the maximum magnitude that lies inside the range $\langle M, M+dM\rangle$. A known expression for the cumulative flux of sporadic meteors, the magnitude of which is less than M is

$$N = N_0\ e^{-const.\ M} \quad , \tag{15}$$

(Baggley, 1978), which implies that

$$N \sim m^{-(s-1)} \quad , \tag{16}$$

when using relation (14). Equation (16) fits a well-known model for mass distribution, which is applied for interplanetary dust particles (Porubčan et al., 1997; Hajduk, 1991), where $s > 1$ ($s \approx 1.7$ - 2.0).

4. Conclusion

The motion of meteoroid bodies in planetary atmospheres is the physical process that enables us to study both the atmospheric environment as well as the

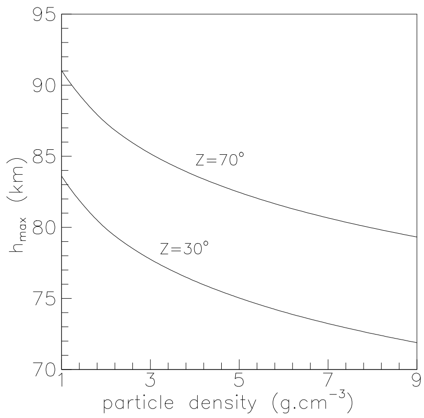

Figure 10. Dependence of burning height on particle density for different entry zenit angles.

characteristics of the particle. A single-body approximation may be applicable for a simple evaluation of the interaction between small (micron-size) particles and the atmosphere. However, there are limitations that must be taken into account. The extended single-body approximation presented in this paper brings an analytical solution of the particle motion in the Earth's atmosphere. For such a theory, the accelerated calculation model is available, because no numerical integration is necessary to retrieve the particle velocity (or particle mass) at an arbitrary altitude. The specific relations known mainly from numerical modelling can then be traced using the theoretical expressions. It was shown that omitting the deceleration term in the equation of motion can cause a variation of measured meteor luminosity of about 0.2-0.3^M (for large particles, the mass of which equals 200 g). The analysis documents that information consisting of the particle incident velocity v_∞ and the maximum magnitude M is enough to evaluate the incident mass of the particle, because the function $m_\infty(M, v_\infty)$ is strictly monotonous over the wide plane $M \times v_\infty$. Taking into account this fact together with expressions 8-10 one can see that mapping $[v_{avg}, M, h_{max}] \leftrightarrow [v_\infty, M_\infty, \delta]$ is possible for small particles. It means that information on the average particle velocity v_{avg} in the Earth's atmosphere, the maximum magnitude M, and height of maximum luminosity h_{max} is a sufficient basis to retrieve the incident velocity v_∞ and mass m_∞ of the particle and its mean effective density δ. By collecting observed data on $[v_{avg}, M, h_{max}]$ and processing them, the mass distribution function for a selected meteor shower can be determined using our mapping method. On the other hand, the magnitude distribution can be simulated for particles of known mass distribution.

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