The contribution to the inverse scattering problem in the Martian atmosphere

M. Kocifaj

Astronomical Institute of the Slovak Academy of Sciences, Interplanetary Matter Division, Dúbravská cesta 9, 842 28 Bratislava, The Slovak Republic

Received: February 17, 1999

Abstract. The accessible optical data for the Martian atmosphere is theoretically analyzed. Fast evaluation of radiative transfer in the atmosphere is based on Eddington's approximation, a two-stream method. The results show a feasible profile of radiative fluxes in the Martian environment and confirm the possibility of identification of large submicron particles using this approach. To get more precise results, the MRSM (multiple radiation scattering model) was used. The improvement of accelerated MRSM is included. The scattering problem formulated in terms of upward and downward radiance structure is applied to get information about the particle size distribution. Expressions for spectral radiances are simplified to the form of Fredholm integral equations of the first kind, which are solvable by special methods. The analysis of the observed spectral dependence of particle optical thickness in the visible spectrum brings a wide particle size distribution with a modal radius larger than 0.2 μm.

Key words: dust particles – Martian atmosphere – radiation scattering

1. Introduction

The description of radiative transfer in disperse media is a difficult problem due to factors such as environmental inhomogeneity, its large optical thickness (which supports the effects of multiple scattering) and the presence of particle systems of different population. However, the structure of the radiation field modified by such a medium contains fundamental information about its composition and properties. Light scattering in planetary atmospheres has been studied for a very long time. The main interest of the optical, polarimetric or photometric studies is, of course, to obtain a relevant knowledge about the planetary environment.

The atmosphere of Mars together with the Earth's atmosphere can be classified as optically thin media. This means that optical measurements yield information about both the surface soil and additional scatterers in the Martian atmosphere. A model of molecular atmosphere without aerosols has often been employed in past years to determine the nature of the Martian surface from polarimetric measurements. However the large values of the optical thicknesses

Contrib. Astron. Obs. Skalnaté Pleso **29**, (1999), 14 – 30.

of the Martian atmosphere τ_{Mars} found by Viking landers strongly suggest that the Martian aerosol is a very important optical component in the atmosphere. Shortwave radiation, which represents the maximum energy content in the solar spectrum, is modified mainly by dust particles. The influence of CO_2 has effect only in the far infrared region where it produces appreciable cooling of the Martian troposphere (Lindner, 1993). The Martian atmosphere therefore appears as a dusty atmosphere for shortwave radiation.

The dust particles in the atmosphere could be small grains of surface soil, small transparent white clouds crystals or very small mist aerosols. The IRIS experiment by the Mariner 9 spacecraft confirmed the silicate nature of mineral dust particles. The cameras mounted on Viking landers have measured the atmospheric extinction as a function of zenith angle. The angular distribution of sky brightness were available too. The measured optical thickness was surprisingly high, between 0.2 and 1.0. However, the large values of about 1 correspond to the dust storms. The standard background conditions are characterized by constant dust haze which describes a permanent loading of the Martian atmosphere by flying dust grains. The wavelength dependence of scattering found in form λ^{-4} may characterize a thin permanent haze consisting of small particles. The measurements from the MARS-5 orbiter indicate the residual particle loading with particle optical thickness τ_p no greater than 0.1 at 592 nm. There are submicron grains with refractive index $m = 1.55$, reminiscent of the terrestrial dust above continents. The PHOBOS-2 craft have measured dust over the Mars equator, where τ_p varies between 0.1-0.2 and the effective modal radius of the polydisperse particle system r_m is about 0.4 μm (Ebisawa and Dollfus, 1993). The gas scattering contribution is not important and $\tau_R \ll \tau_p$, where τ_R represents Rayleigh scattering by gas molecules and τ_p is produced by dust particles. The variations of gas pressure in the atmosphere may produce only a few percent of the total brightness of the sky (Moroz et al., 1993). The single scattering albedo $\overline{\omega}$ of dust particles in red was estimated during the Viking mission between 0.90 and 0.98 and the asymmetry factor $\overline{< cos >}$ between 0.55 and 0.65. All this data is consistent with permanent pollution. Moroz et. al (1993) considers a single scattering albedo to be 0.86 at 550 nm. Lindner (1991) uses a similar value. The optical properties of the dust grains fit that of a nonabsorbing material with a real part of the refractive index equal to 1.56 (Ebisawa and Dollfus, 1993). Haan (1987) uses a similar value of 1.58 which is produced by a mixture of basalt and montmorillonite 50-50%. The absorption by the particles can be ignored because the imaginary part of the complex refractive index of the Martian particles is about several thousandths (Petrova, 1993). This is slightly higher than for terrestrial basalt or andesite.

2. Radiative transfer in the Martian dust atmosphere

Martian particles corresponding to the constant haze are expected to be submicron sized. The particles of the Martian atmosphere are, of course, nonspherical and they are likely to be approximately disc-shaped (Murphy et al., 1990). Although the microsized particles were observed by Viking landers too, these irregular grains are representative of the largest particles in the Martian dust population. The optical thickness of such an environment may exceed 0.4 and its lifetime is expressed in weeks.

The light scattering by irregular particles can be calculated using Mie theory only if the particle size is smaller or comparable to the wavelength of the incident radiation. Nevertheless, it is known that the phase function of scattered radiation by such particles is approximated well by Mie theory.

The scattering matrix of dust particles generally varies with wavelengths. It was found that the asymmetry parameter of Martian dust particles changes with wavelength too. The scattering matrix for a Martian dust is assumed to have only three independent elements, owing to the constraints on the collection and type of particles (using rigorous Mie theory), and has the form

$$\begin{pmatrix} a_1(\theta) & b_1(\theta) & 0 & 0 \\ b_1(\theta) & a_1(\theta) & 0 & 0 \\ 0 & 0 & a_3(\theta) & 0 \\ 0 & 0 & 0 & a_3(\theta) \end{pmatrix} . \tag{1}$$

where θ is the scattering angle. Utilizing the Mie theory is a weak point in this approach, but it may be supposed that the errors introduced by this simplification do not have any significant effect on the conclusions.

The next simplification of the radiative transfer equations in Martian dust is based on scattering theory by nonabsorbing media, because the imaginary part of the particle refractive index found by Petrova (1993) is three magnitudes smaller than the real part. This fact significantly accelerates the solving of the inverse problems for the dust system. For example, it enables us to apply Van de Hulst's approximation for the efficiency factors for scattering using an anomalous diffraction approach (Van de Hulst, 1957).

Finally, a multiple scattering should be applied to solve the radiation transfer equation in disperse media at a high precision level. In such a case, both downward and upward radiation energy fluxes may be expressed as follows

$$\begin{aligned} F^{+}(h_1,\xi_0) \; = \; & \int_{\lambda=\lambda_1}^{\lambda_2} \int_{\xi=0}^{\pi/2} \int_{\alpha=0}^{2\pi} I_\lambda^{+}(h_1,\xi_0,\xi,\alpha) \; cos\xi \; sin\xi \; d\xi \; d\alpha \; d\lambda \; + \\ & + \int_{\lambda=\lambda_1}^{\lambda_2} F_{\lambda,dir}(h_1,\xi_0) \; cos\xi_0 d\lambda, \end{aligned} \tag{2}$$

and

$$F^{-}(h_1,\xi_0) = \int_{\lambda=\lambda_1}^{\lambda_2} \int_{\xi=0}^{\pi/2} \int_{\alpha=0}^{2\pi} I_\lambda^{-}(h_1,\xi_0,\xi,\alpha) \; cos\xi \; sin\xi \; d\xi \; d\alpha \; d\lambda, \tag{3}$$

where λ_1 and λ_2 define a studied spectral band, ξ is the zenith angle of the sky element, α the difference between the solar azimuth and the azimuth of the sky element, ξ_0 the zenith angle of the Sun, h_1 the altitude in the atmosphere and $F_{\lambda,dir}(h_1,\xi_0)$ the monochromatic flux density of direct solar radiation at altitude h_1. The total radiances for downward and upward directions are calculated as a sum over all scattering orders,

$$I_\lambda^+(h_1,\xi_0,\xi,\alpha) = \sum_{n=1}^{N\to\infty} I_{n,\lambda}^+(h_1,\xi_0,\xi,\alpha) \tag{4}$$

$$I_\lambda^-(h_1,\xi_0,\xi,\alpha) = \sum_{n=1}^{N\to\infty} I_{n,\lambda}^-(h_1,\xi_0,\xi,\alpha), \tag{5}$$

where each scattering order depends on particle characteristics, such as particle size distribution (PSD), complex refractive index, vertical stratification, and, of course, on other characteristics of the atmosphere and the Martian surface, respectively. The measurements of degree of linear polarization in backscattered light (Ebisawa and Dollfus, 1993) independently confirm the small value of τ_p in the Martian atmosphere. The decreasing of particle concentration with altitude in the Martian atmosphere is smaller than in the Earth's atmosphere. This fact limits the effect of multiple scattering. The vertical stratification of the concentration of background dust is approximated by a Gaussian profile of the form

$$exp(-z^2/650),$$

where z is altitude in kilometres (Lindner, 1991). However, the calculation model must include high scattering orders, because the value of $\tau_p = 0.2$ prohibits the complete exception of multiple scattering.

Perhaps the most difficult part of studying light scattering in planetary atmospheres is accounting for a groundsurface having realistic reflection properties. Many types of surfaces occur for the Earth. For Mars the situation is somewhat better because no vegetation is present. In such case a model of Lambertian surface is a good approximation for the reflexivity of the Martian surface. The considered average Martian albedo A_{Mars} is about 0.1-0.2 (Moroz et al., 1993).

To check an elementary scattering as a starting point of the calculation of radiation characteristics, we use the well-known Eddington's approximation for the optically thin Martian atmosphere. This method is a so-called two-stream method, which is applicable to horizontally homogeneous environments. The Earth's atmosphere under cloudless conditions was systematically studied using such methods. Eddington's approximation applied to the Martian atmosphere brings a fast solution of the radiation transfer equation by obtaining reliable data. The calculation setup of numerical modelling is summarized in table 1.

The particle size distribution (PSD) fits a model of haze-L (Mc Cartney, 1977)

$$f(r) = Ar^a e^{-br^\gamma}, \tag{6}$$

Table 1. Calculation setup for radiative transfer modeling in Martian atmosphere

particle optical thickness	$\tau_p = 0.2$
effective particle refractive index	$m = 1.56$
Gaussian altitude profile	$exp(-z^2/650)$
single scattering albedo	$\overline{\omega} = 0.95$
asymmetry factor	$\overline{< cos >} = 0.6$
Martian surface albedo	$A_{Mars} = 0.2.$

where r is a particle radius, and parameters specified were chosen according to the Haan (1987), i.e. $a = 2$, $b = 6.32\ \mu m^{-1/2}$, and $\gamma = 0.5$. The applicability of this function will be discussed in the next chapter. The results of calculation for different solar zenith angles ξ_0 are presented in the following figures: 1, where cos ξ_0=0.2 (ξ_0=78.5^o), 2, where cos ξ_0=0.4 (ξ_0=66.4^o), and 3, where cos ξ_0=1.0 (ξ_0=0.0^o).

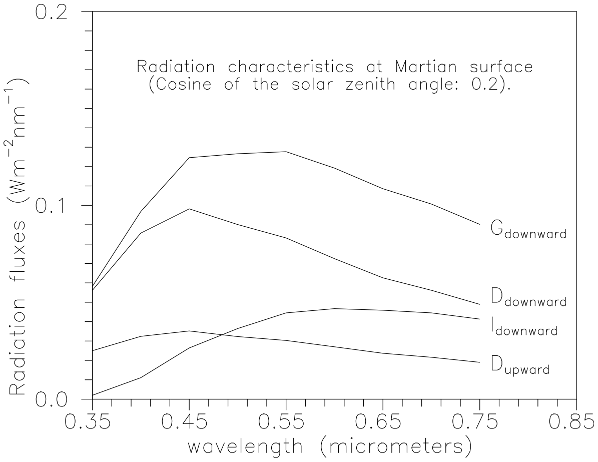

Figure 1. Radiation fluxes in Martian atmosphere calculated using Eddington's approximation. Zenith angle of the Sun equals to 78.5^0

The downward diffuse $D_{downward}$ and global $G_{downward}$ spectral radiation fluxes, and direct solar radiation flux $I_{downward}$ are calculated at the Martian surface, the upward spectral diffuse radiation fluxes D_{upward} are calculated at the top of the atmosphere. The maximum of the function $G_{downward}(\lambda)$ lies close to the maximum energy distribution in the solar spectrum, but its position is a little bit shifted to the ultraviolet spectrum due to radiation attenuation and scattering by aerosol particles. This effect is observable at all elevations of the Sun, because it is mainly a result of the particle optical thickness. The downward radiation fluxes at large solar zenith angles are comparable or greater than the

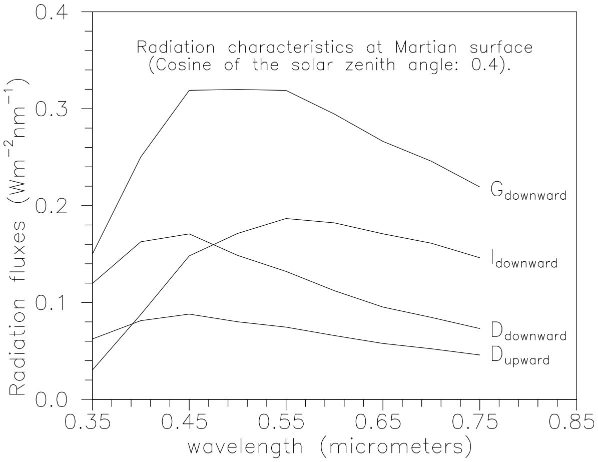

Figure 2. Radiation fluxes in Martian atmosphere calculated using Eddington's approximation. Zenith angle of the Sun equals to 66.4^0

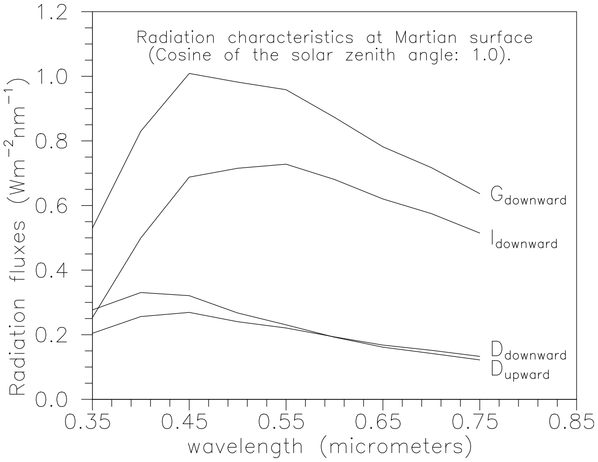

Figure 3. Radiation fluxes in Martian atmosphere calculated using Eddington's approximation. Zenith angle of the Sun equals to 0.0^0

direct radiation flux. A weak decreasing of $I^{IR}_{downward}(\lambda)$ in comparison with the profile of intensity distribution in solar spectrum indicates the presence of Mie's particles, i.e. large submicron particles, for which the spectral profile of $\tau_p(\lambda)$ can be approximated by a power function very close to λ^{-1}. A similar result is also known for terrestrial aerosols (Burki et al., 1995). The modal radius of such particles in the Earth's atmosphere is about 0.1 μm. Analysing the theoretically calculated radiation fluxes, one can see that the existence of large submicron particles in the Martian atmosphere may be identified when processing the measurements of downward diffuse radiation fluxes. High values of $D_{downward}$ in the infrared spectrum (IR) correspond to the high scattering efficiency in this spectral region.

The next analysis uses basic knowledge of Mie's theory. The efficiency factor for scattering $Q_{sca}(x)$ in anomalous diffraction approximation is a function only of parameter x, which depends on a ratio of particle radius and wavelength of incident radiation. The first maximum of $Q_{sca}(x)$ is most important in scattering processes in planetary atmospheres. This mode usually correlates well with the mode of the wavelength dependence of radiation characteristics in the atmospheric environment. A shift in the maximum scattering efficiency of the disperse medium to the longer wavelengths implies the increasing of the mean effective radius of scatterers in such environment, in order to have a constant value of x. Similar, but not identical facts are valid for upward radiation fluxes at the top of the atmosphere, although the contribution of reflected radiation by the planetary surface plays an important role in optically thin atmospheres. Taking into account a complex of scattered, reflected, and attenuated radiation fluxes, the approximately neutral course of the function $D_{upward}(\lambda)$ is feasible. The presented results of the calculations confirm this fact by all previewed cases. Concerning the results presented in Figs. 1, and 2, the best conditions to measure a scattering properties of the Martian atmosphere are at large zenith distances of the Sun. Unfortunately, the diffuse radiation is modified by the effect of multiple scattering in this case, and the inversion techniques tend to suffer from this defects.

The methods of high-level precision such as line-by-line should be applied to get the detailed results for a radiation field structure. Spectral radiances for a selected scattering order can be calculated on the basis of a multiple scattering radiation model (MRSM) (Kocifaj and Lukáč, 1998). The first scattering order for the downward radiance contains direct solar radiation scattered to the Martian surface. The upward radiance profile for the elementary scattering is generated by the radiation scattered to the top of the atmosphere and direct downward radiation reflected by the Martian surface. Global radiation is a sum of downward diffuse radiation integrated over whole halfspace and direct solar radiation corrected according to a cosine law. Each next scattering order is computed using recurrent formulae on the basis of the previous order. A feasible computation time is achieved by this way. Dramatic acceleration is reached by applying the optimized calculation scheme, which enables us to preserve the precision of computed results while strikingly decreasing the CPU time. It has been shown that no all scattering orders must be precalculated to gain a reliable result. It is possible to cut the series at a certain term indexed by *NT'*. The rest is approximated by geometrical series with a good accuracy. Expressed mathematically, the total value of the selected radiation characteristic *Rc* will be calculated as follows

$$Rc_{total} = \sum_{i=1}^{NT'} Rc_i + Rc_{NT'} \frac{\overline{q}}{1-\overline{q}}. \tag{7}$$

The smaller the optical thickness of the studied medium, the smaller the number of terms (NT'). Two scattering orders can be saved by this way in the Earth's atmosphere for clear sky if $\tau_{Earth} = 0.2$ (Kocifaj and Lukáč, 1998). The error is under ten percent.

A spatial structure of the spectral radiance (in $Wm^{-2}nm^{-1}sr^{-1}$) of the Martian sky calculated using MRSM is presented in Figs 4, and 5. Displayed numeric values describing the individual isolines are enlarged in two magnitudes. The centre of the picture corresponds to the zenith. Four azimuth angles are marked along the horizon. The profile of the radiance field is similar for the whole visible spectrum, therefore only the results for a reference wavelength of 550 nm of an incident radiation are presented here. Weak changes of isoline structure of the radiances is caused by open (wide) particle size distribution with vague mode position. Isolines of the spectral sky radiances observable from the Martian surface are previewed in Fig 4. At the first glance the radial structure of isolines is evident. Horizontally homogeneous profile of 'total' radiance is typical for hazy or turbid atmospheres. But this is not our case, because the discussed isolines belong to the 'tail' of a scattering phase function for large submicron or micron particles, and the corresponding intensity of scattered light is two magnitudes less than in the solar aureola. It is an effect of the purely dust atmosphere for the selected wavelength. Finally, the radial structure is constituted by high scattering orders as is shown in Fig. 6. However, the maximum contribution to the total downward diffuse flux is probably caused by the solar aureola. Here, a radiance profile is unambiguously constructed by a first scattering order. This fact enables to use aureola data to retrieve the PSD (Box and Deepak, 1979; Nakajima et al., 1983; Box and Box, 1990; Tonna et al., 1995). The largest particles of the Martian dust population may be gained by this way, because projection of scattering angle interval ($\theta < 5 - 10^o$) to relevant particle radii interval in Hilbert space lies in the micron range. On the other hand, the 'aureola' around the reflex of the Sun in upward radiance structure is based on a second scattering order (Fig. 5). It is due to convention explained above (the diffuse downward radiation reflected by the Martian surface is included for the first time in the second term of the series for upward radiation, i.e. it belongs to the second scattering order). First and second scattering orders for upward radiation are therefore obvious comparable at large scattering angles (Fig. 7). An influence of multiple scattering expressed by scattering orders higher than 2 strongly falls down. The intensity changes in backscattered radiation (observed outside of Mars) are not so critical comparing them with the radiance distribution at the Martian sky (observed from the planetary surface). Differences found in figures 4 and 5 are at a level of one magnitude. Displayed isoline structure for a sky brightness in first scattering order approximation (Fig. 6) correlates well with theoretical results for optically thin atmospheres (Minin, 1988).

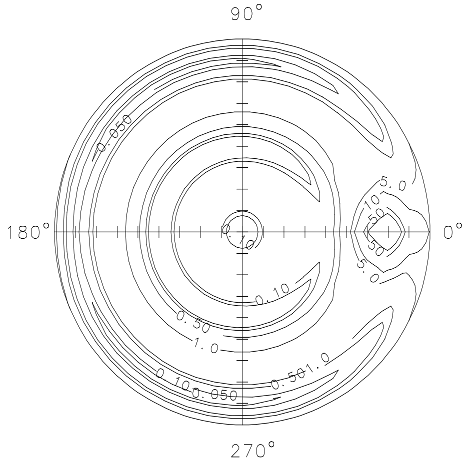

Figure 4. A spatial structure of the spectral downward radiance (in $Wm^{-2}nm^{-1}sr^{-1}$) at Martian sky calculated using MRSM. The displayed values are enlarged in two magnitudes. Wavelength of an incident radiation is 550 nm.

Comparing the radiation fluxes calculated using Eddington's approximation with the results based on MRSM theory, the fluxes are underestimated in several tens of percent when a two-stream method is used.

3. Inversion of radiation data

Retrieving the particle characteristics using the inversion methods is one of the major problems of atmospheric optics. While such inversions are required to be unique, signatures may exist in the scattered light which correspond to specific physical properties of the particles. Schuerman et al. (1981) shows that particle shape irregularity may produce a radiation field different from a field produced by spheres. Interpreting such measurements via Mie theory is impossible especially for the degree of polarization. However, the systematic studies of light scattering confirm the applicability of Mie theory to calculate a total scattered intensity (i.e. first component $a_1 = S_{11}$ of the scattering matrix) by irregular particles. The wavelength of an incident radiation cannot be too small in comparison with particle size and a statistically large amount of randomly oriented particles must be available. The scattering phase function in the forward direction correlates well with the phase function for spheres of an identical volume to the really shaped particles. Then, using rigorous Mie theory the inverse problems in atmospheric remote sensing can be formulated in terms of the Fredholm integral equation of the first kind. Spectral radiance in the first scattering order approximation is directly convertable to the given integral equation form.

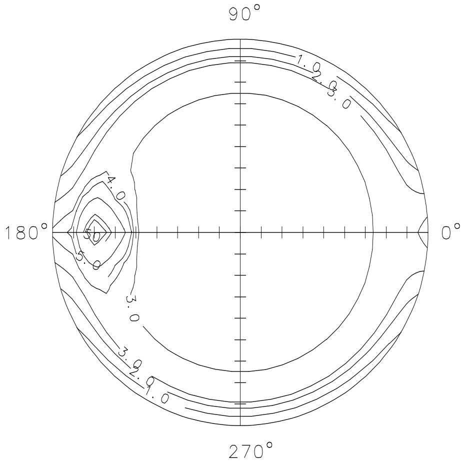

Figure 5. A spatial structure of the spectral upward radiance (in $Wm^{-2}nm^{-1}sr^{-1}$) at top of the Martian atmosphere calculated using MRSM. The displayed values are enlarged in two magnitudes. Wavelength of an incident radiation is 550 nm.

The inversion of the first kind Fredholm integral equations is an example of an ill-posed problem which is notoriously difficult to solve. Such problems fail to fulfill at least existence of a solution, uniqueness of the solution, and continuity of the solution on the data function. Several methods were developed to solve such inverse tasks, although no general rules may be formulated. The singular-function theory or eigenfunction theory (Box et al, 1992, Box and Box, 1985) are applicable to such tasks.

Accelerated MRSM based on the first scattering order can be successfully applied to calculate the spatial brightness distribution of the downward radiation at the Martian surface. Aerosol scattering produce too small values in backward directions in comparison with forward scattering, therefore a second scattering order should be calculated for upward radiation if the planetary surface is assumed to be totally absorbing. Calculation of upward radiation characteristics could be based on first scattering order approximation when the planetary surface albedo is greater than 0.1. Wang and Gordon (1994) have presented such a model of calculation for radiances at the top of the Earth's atmosphere with an error smaller than several percent. This method may fail when aerosol is strongly absorbing. Even so the success of an inversion of radiance data at the top of the atmosphere strongly depends on precision of surface albedo determination. Taking into account the average columnar size distribution function of the aerosol particles, the upward radiance measured at selected almucantar is a function of scattering angle θ^-

$$I_1^-(\theta^-) = K_1 \; P_p(\theta^-) + K_2, \tag{8}$$

where P_p is a scattering phase function for particles, and functions K_1 and K_2 in

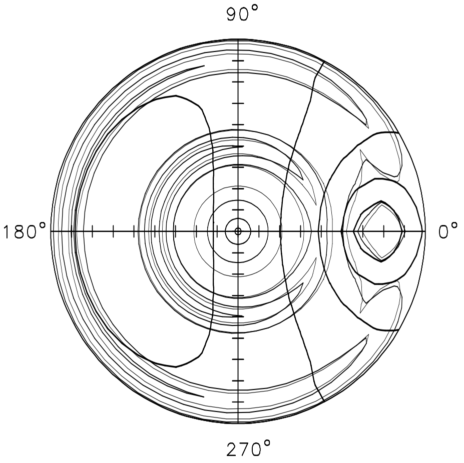

Figure 6. The contribution of individual scattering orders to the total sky radiance at Martian surface. Wavelength of an incident radiation is 550 nm. (Scattering order/pen width): (1/2), (2/1), (3/0.5)

generally depends on the vertical profile of particle concentration in the Martian dust atmosphere, surface albedo, wavelength, and zenith angles of the Sun ξ_0 and of selected almucantar ξ, respectively. The scattering angle θ^- is given by the equation

$$cos(\theta^-) = -cos(\xi_0)\ cos(\xi) + sin(\xi_0)\ sin(\xi)\ cos(\alpha), \tag{9}$$

where α represents a difference of solar azimuth and azimuth of a selected sky element. Using measured almucantar data, the functions K_1 and K_2 become constant values for our study. The inversion of the equation for $P_p(\theta^-)$ leads to solve a Fredholm integral equation of the first kind, because

$$P_p(\theta^-) = K_3 \int_{r_1}^{r_2} f(r) \frac{i_1 + i_2}{2}\, dr, \tag{10}$$

where K_3 is a constant value during inversion procedure (but depends on particle optical thickness), i_1 and i_2 are well-known normalized Mie scattering functions, and r_1 and r_2 characterize transformants of scattering angle interval $< \theta_1^-, \theta_2^- >$, for which the radiance data is available. Inversion of such an integral equation is based around the Mellin transform of the kernel and was described by Shifrin (1971). His method is applicable for anomalous diffraction approximation, when the kernel of an integral equation belongs to so-called product-type kernels $K(x, y) = K(xy)$ (Schmeidler, 1955)

Average value of q (eq. 7) for the Earth's atmosphere under low turbidity conditions is about 0.33, but in comparison to the Martian atmosphere this value is logically underestimated. While aerosol optical thickness in the Earth's

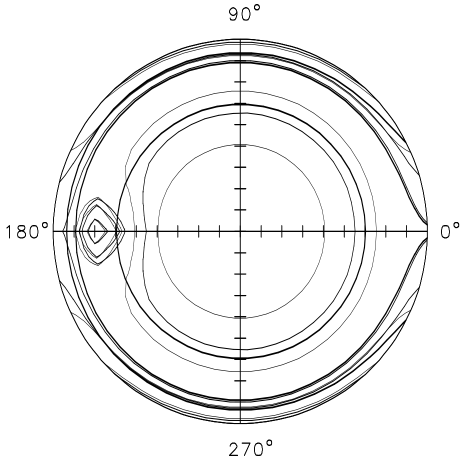

Figure 7. The contribution of individual scattering orders to the total sky radiance at top of the Martian atmosphere. Wavelength of an incident radiation is 550 nm. (Scattering order/pen width): (1/2), (2/1), (3/0.5)

atmosphere should be only about 0.05 to save the value of total atmospheric optical thickness 0.2 (the molecular component contribution is about 0.15), the Martian atmospheric optical thickness is equal to the particle optical thickness. The martian atmosphere is more dusty than the atmosphere of the Earth from this point of view. Multiple scattering effect makes complications in the dust atmosphere, because the size of particles is comparable to the incident wavelength and the refractive index is far from environmental refractive index (i.e. from the optical properties of air molecules). Besides, the extinction coefficient of the aerosol particles varies slowly the wavelength in the visible spectrum. The spectral optical thickness in Rayleigh atmosphere is described by the power law λ^{-4}, while aerosol optical thickness depends on wavelength usually as λ^{-1}. Summarizing all these statements, we have found average value of $\overline{q} = 0.54$ for the Martian atmosphere.

We have improved the accelerated MRSM to calculate high scattering orders for selected radiation characteristics Rc as follows

$$Rc_{total} = \sum_{n=1}^{n=N-1} Rc_n + Rc_N \sum_{n=1}^{\infty} \overline{Q}^{n-1} \left[n + A_{Mars} \frac{n[(n-2)^2 + n + 4] - 6}{6} \right] . \tag{11}$$

The second series should start with a scattering order N, which brings a maximum contribution to Rc_{total}. In optically thin atmosphere it is often the first term, and then

$$Rc_{total} = Rc_1 \sum_{n=1}^{\infty} \overline{Q}^{n-1} \left[n + A_{Mars} \frac{n[(n-2)^2 + n + 4] - 6}{6} \right] . \tag{12}$$

Assuming absolutely absorbing planetary surfaces ($A_{Planet} = 0$) gives the result

$$Rc_{total} = \frac{Rc_1}{(1-\overline{Q})^2}. \tag{13}$$

The dependence of $\overline{Q}$ on solar zenith angle can be approximated for Martian atmosphere by the equation

$$\overline{Q} = \frac{0.45}{1+cos^2(\xi_0)}. \tag{14}$$

The results presented in Figs. 6, 7 correlate with the average value of $\overline{Q} = 0.39$. This value of $\overline{Q}$ equivalent to the Earth's atmosphere is about 0.2, which corresponds to the $\overline{q} = 0.36$.

With the exception of the uncertainties in absolute values, the total radiance profile is well-good fitted by improved accelerated MRSM. Using of this approach the inverse problem for PSD may be solved using a complete calculation only for the first scattering order. Obtained relative values of PSD can be then calibrated using a measured value of particle optical thickness $\tau_p(\lambda)$ at selected wavelength

$$\tau_p(\lambda) = \pi \int_0^\infty Q_{sca}(r,\lambda,m)\ r^2\ f(r)\ dr, \tag{15}$$

where efficiency factor for scattering Q_{sca} for nonabsorbing homogeneous spheres expressed in terms of anomalous diffraction theory is

$$Q_{sca}(\rho) = \frac{2\overline{m}-1}{\overline{m}}\left[2 - \frac{4}{\rho}sin\rho + \frac{4}{\rho^2}(1-cos\rho)\right], \tag{16}$$

where $\rho = 4\pi(\overline{m}-1)r/\lambda$, and $\overline{m}$ is the effective complex refractive index of the particles. Formula (16) is usuable also for optically hard particles, with refractive index less than 1.6.

Data of spectral optical thickness for the dust atmosphere could be also directly used to retrieve the PSD, whereas equation 15 is a Fredholm integral equation of the first kind. The size interval in which PSD can be found depends on the spectral range of measured values of τ_p (Kocifaj, 1994). Generally, the stability of the inverse method is improved when the measured profile of $\tau_p(\lambda)$ contains a mode of this function. The relationship of wavelength interval and particle size interval represents the transformation in Hilbert space, where PSD is required to be quadratically integrable and a continuous function. Results of theoretical calculations show that the greater particle modal radius the greater the wavelength belonging to the position of the mode for the function $\tau_p(\lambda)$. The measurements of optical thickness of the Martian atmosphere give a relatively constant value of τ_p in the visible spectrum (Lindner, 1991). A first estimation based on the previous discussion suggests that particles larger than tenths of microns could be found in the atmosphere. Particle size distribution in hazy atmosphere is often fitted by gamma distribution (Cartney, 1977). Using equation 15, we have modelled the profile of the particle optical thickness in such an environment as the Martian atmosphere. The τ_p at a reference wavelength 500 nm is 0.2 according to the measurements. The results presented in Fig. 8 show the individual profiles of τ_p for different particle modal radii. The figure indicate that the particle size should be larger than 0.2 μm to match condition of small variability of τ_p in the visible spectrum. The presence of particles larger than one micron in PSD guarantee the continuity of a relatively constant profile of $\tau_p(\lambda)$ in the far infrared, but no data is available to check the reality of such a course. In this case we may assume that particles of the size between 0.2 and 1 μm have to be the basis of the Martian dust population. Our conclusion coincides with results published by Ebisawa and Dollfus (1993), where the average particle size was estimated about 0.35-0.40 μm. Theoretical analysis and numerical processing of the measurements of $\tau_p(\lambda)$ profiles in the Earth's atmosphere under clear sky conditions and in cirrus clouds (Kompanovskij, 1972) are also weight to predictions of the existence of large particles in the Martian atmosphere.

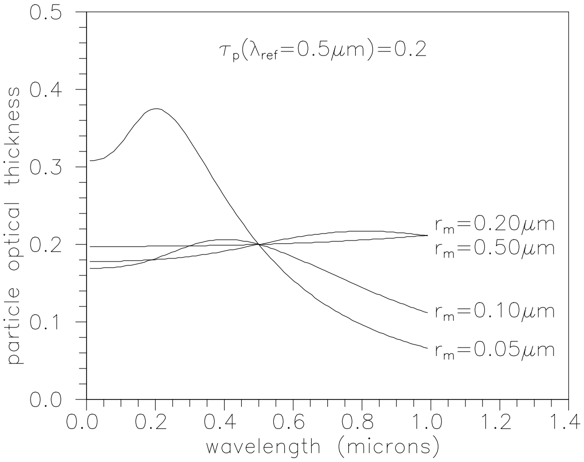

Figure 8. Particle optical thickness profile calculated for modified gamma distributions of different particle modal radii.

Starting with the obtained result that particle modal radius lies in submicron range, we will analyse PSD for its profile in the next step. The value of 0.2 for particle optical thickness is an additional condition, which must be fulfilled in the visible spectrum. Fig. 9. shows the profiles of $\tau_p(\lambda)$ for several particle size distributions with modal radius $r_m = 0.35$ μm. Fig. 10 brings a preview of modified gamma distribution functions, which were used in the calculation procedure. PSD indexed by 1 fulfill the condition of constant value of τ_p in the visible spectrum, while PSD indexed by 4 violates it. PSD_1 corresponds to 'open' size distribution consisting of populations of small and large particles, while PSD_4 characterizes the considerably narrower distribution around the modal radius. However, the amount of microsized particles decreases more rapidly than the amount of very small particles. These results are in good agreement with PSD proposed by Haan (1987). Ebisawa and Dollfus (1993) recommend using Junge distribution with power parameter n = 3.7. The Junge's particle size distribution corresponding to condition $\tau_p(\lambda=500\text{ nm}) = 0.2$ is shown in Fig. 10. The decline of the large particles with their size is rendered by this function although the amount of small particles is evidently overloaded. Knowledge of systematic studies in the Earth's atmosphere incline to approximate PSD by Junge function only for 'clear' air i.e. for small particle concentration (e.g. in mountains). The PSDs with modal radius larger than 0.2-0.3 microns usually characterize hazy states and they are almost routinely expressed in terms of gamma function. Therefore we prefer the gamma function if the mathematical function is to be used for theoretical study. Lindner's resulting PSD (1993) is formulated in terms of the known haze-L model (based on gamma distribution).

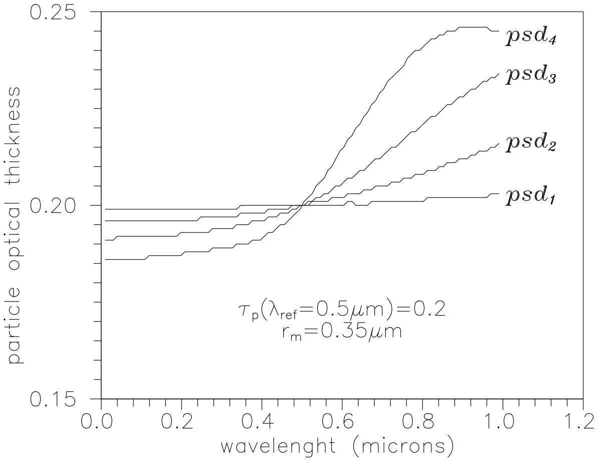

Figure 9. Particle optical thickness profile calculated for set of modified gamma functions.

Summarizing the previous analysis we may conclude that PSD in the Martian atmosphere should be wide with a modal radius situated in the submicron range. PSD will be probably similar to the gamma distribution. We have completed the final revision for PSD profile by numerical solution of the inverse problem for $\tau_p(\lambda)$. Accepting a relatively constant value of $\tau_p(\lambda)$ in the visible spectrum, the regularization of Tikhonov's functional was the way to retrieve the PSD in the submicron range (Kabanov et al., 1988, Kocifaj and Držík, 1997). The profile of PSD obteined supports the model of the gamma function (Fig. 10).

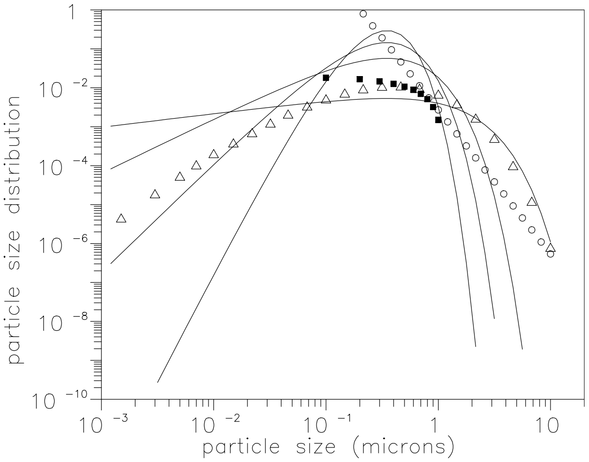

Figure 10. Particle size distributions. 1-4: model of modified gamma functions, triangles: gamma function according to Haan (1987), circle: Junge distribution according to Ebisawa and Dollfus (1993), full square: solution obtained by Tikhonov's regularization.

4. Conclusion

Theoretical analysis of the optical measurements by probes in Martian atmosphere shows that spectral radiance data can successfully be processed to retrieve a particle size distribution. First preliminary calculations using a fast two-stream Eddington's approximation may reflect the presence of large submicron particles in constant dust haze. These particles could be larger than equivalent background aerosols in the Earth's atmosphere. The improvement of accelerated MRSM presented in this paper enables to calculate the spectral radiative characteristics in the Martian atmosphere in terms of the first scattering order. We have then formulated the problem of usage of the upward radiance structure in inversion procedures for particle characteristics. Applying the anomalous diffraction theory the modified expressions for upward radiance lead to Fredholm integral equation of the first kind, which are so-called ill-posed problems. Their solution could be obtained by regularisation of Tikhonov's functional or using a Mellin's transform because the kernel of integral equation is of product-type. Calculation of downward radiance structure at Martian sky brings the intensity of solar aureola two magnitudes higher than the contribution of other sky elements. Whereas the large micron particles mainly influence the forward scattered radiation they are detectable using aureola data. The calculated results presented for individual scattering orders confirm that the aureole radiance is generated by an elementary scattering. The existence of large submicron particles in the Martian atmosphere was also predicted after theoretical analysis of the practically constant value of particle optical thickness in the visible spectrum. Inversion of a such profile gives a wide distribution with a modal particle radius between 0.2 and 1.0 μm. It is in good agreement with the independent measurements by landers, which give the modal radius about 0.35-0.40 μm and confirm the existence of large nonspherical particles in dust hazes.

Acknowledgements. This work has been supported by a Grant No. 4174/97 of the Slovak Academy of Sciences.

References

Box, M.A., Box, G.V.: 1990, *J. Opt. Soc. Am. A.* **7**, 1015
Box, G.V., Box, M.A.: 1985, *Appl. Opt.* **24**, 4525
Box, M.A., Deepak, A.: 1979, *Appl. Opt.* **18**, 1376
Box, G.P., Sealey, K.M., Box, M.A.: 1992, *J. Atmos. Sci.* **49**, 2074
Burki, G., Rufener, F., Burnet, M., Richard, C., Blecha, A., Bratschi, P.: 1995, *Astron. Astrophys.* **112**, 383
McCartney, E.J.: 1977, *Optics of the atmosphere*, John Wiley & Sons, New York
Ebisawa, S., Dollfus, A.: 1993, *Astron. Astrophys.* **272**, 671
Haan, J.F.: 1987, *Effects of aerosol on the brightness and polarization of cloudless planetary atmospheres*, Vrije Univ. Amsterdam, Amsterdam
Van de Hulst, H.C.: 1957, *Light scattering by small particles*, John Wiley & Sons, New York
Kabanov, M.V., Panchenko, M.V., Pkhalagov, Ju.A., Veretennikov, V.V., Uzhegov, V.N., Fadeev, V.Ja.: 1988, *Optical properties of maritime atmospheric hazes*, Nauka, Novosibirsk
Kocifaj, M.: 1994, *Studia Geoph. Geod.* **38**, 399
Kocifaj, M., Držík, M.: 1997, *J. Aeros. Sci.* **28**, 797
Kocifaj, M., Lukáč, J.: 1998, *J. Quant. Spectrosc. Radiat. Transfer* **60**, 933
Kompanovskij, V.I.: 1972, *In: Optika okeana i atmosfery* , 210
Lindner, B.L.: 1993, *Planet. Space Sci.* **41**, 743
Lindner, B.L.: 1991, *Icarus* **93**, 354
Minin, I.N.: 1988, *Theory of radiative transfer in planetary atmospheres*, Nauka, Moscow
Moroz, V.I., Petrova, E.V., Ksanfomality, L.V.: 1993, *Planet. Space Sci.* **41**, 569
Murphy, J.R., Toon, O.B., Haberle, R.M., Pollack, J.B.: 1990, *J. Geophys. Res.* **95**, 14629
Nakajima, T., Tanaka, M., Yamauchi, T.: 1983, *Appl. Opt.* **22**, 2951
Petrova, E.V.: 1993, *Planet. Space Sci.* **41**, 587
Schmeidler, W.: 1955, *Integralgleichungen mit Anwendungen in Physik und Technik*, AV Geest & Portig K.-G., Leipzig
Schuerman, D.W., Wang, R.T. Gustafson, B.A.S., Schaefer, R.W.: 1981, *Appl. Opt.* **20**, 4039
Shifrin, K.S.: 1971, *In: Theoretical problems of light scattering and their applications*, Nauka i Technika, Minsk
Tonna, G., Nakajima, T., Rao, R.: 1995, *Appl. Opt.* **34**, 4486
Wang, M, Gordon, H.R.: 1994, *Appl. Opt.* **33**, 7088