The inverse problem of interaction of shortwave direct solar radiation with small dust particles in the high atmosphere

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Abstract. The method of inverse procedure of satellite measurements of the spectral direct solar radiation flux density is described in this paper. These data are the basis for obtaining the vertical profile of particle concentration and distribution in the middle and high atmosphere. Double inverse problem must be solved for integral equations of the first kind. The characteristics of meteoric particles of submicron dimension can be obtained from this spectral data in the shortwave spectrum.

Key words: shortwave solar radiation – small particles – Earth's atmosphere

1. Introduction

The optical properties of the middle and high atmosphere in the shortwave spectrum are determined by physico-chemical characteristics of gaseous and solid components. The most interesting gaseous component is ozone. The origin of solid microparticles to a large degree extraterrestrial. The intensity of the ascending flow and air density in the parts of the atmosphere being discussed are small enough, so that the heavier and larger particles ($> 1 - 10 \mu m$) spend a very short time there and, due to gravity, fall to lower lying layers (Link 1956, 1961). The lifetime of particles of a certain size and density in the middle and high atmosphere is presented in Fig.1. Long-term measurements enable the 'phon' turbidity to be determined (Zuev and Krekov 1986; Zuev and Kabanov 1987). Significant changes of the particle structure can be observed on a short-term basis in case of interaction incidents between the Earth's atmosphere and meteor streams, when the optical properties of the middle and high atmosphere are very variable (Lebedinec 1981). The vertical redistribution of particles takes place during the drift of meteoric microparticles. Heavy and large particles fall more quickly than small particles and consequently first influence the optical properties of the lower layers of the middle atmosphere. Small particles mainly modify the optical properties of the high atmosphere. The atmosphere's vertical transparency in this process can be relatively stable, but the horizontal

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transparency is subject to large changes (Link 1956, 1961). This enables the horizontal transparency function to be used to characterize the considered microparticles (e.g. of meteoric origin) in the middle and high atmosphere.

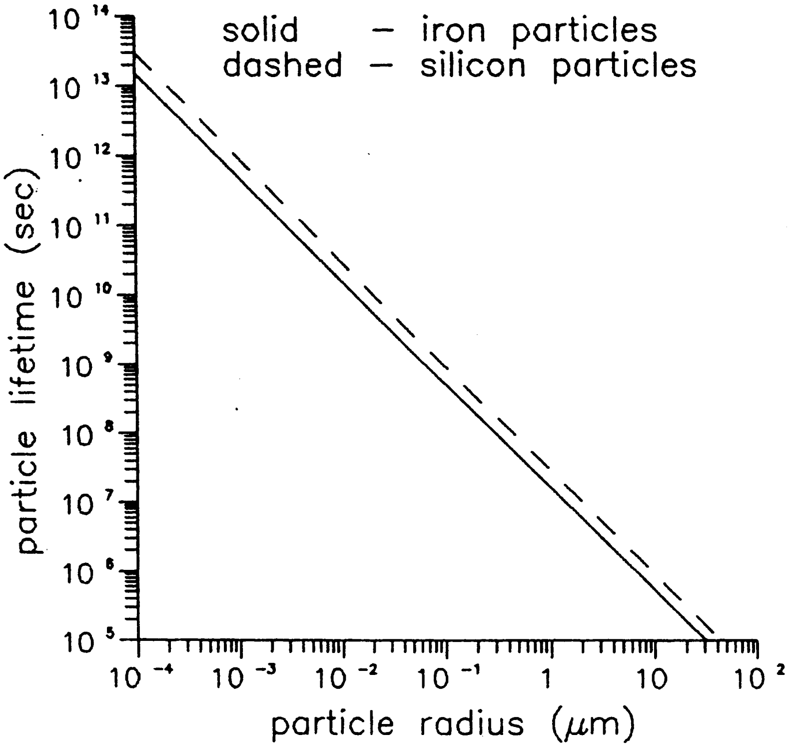

Figure 1. Dependence of particle lifetime in the Earth's atmosphere on particle radius. After Link (1962).

Lunar eclipse phenomena (Kocifaj) or dusk phenomena (Lebedinets 1981; Mikirov and Smerkalov 1981) can be applied to this research. The vertical profile of submicron particle concentration and distribution can be obtained using satellite data of the spectral flux density of direct solar radiation in the shortwave spectrum. The inverse method is described in this paper.

2. Attenuation of direct solar radiation in a spherical atmosphere

Solar radiation penetrating the Earth's atmosphere changes intensity and direction due to extinction and refraction. Sunbeams "flowing round" the Earth at different altitudes are deflected from the original direction. The deflection is the larger, the smaller the minimum sunbeam altitude h_0 (Fig.2). Vertical refraction is many times smaller than horizontal refraction (a horizontal homogeneous atmosphere is assumed) and in this case the incident angle α_0 at the top of the atmosphere (in altitude H) is equal to output angle α_v. In the standard atmosphere it is possible to the express the vertical profile of the atmosphere's refractive index as

$$n_\lambda(h) = 1 + c_\lambda \rho(h) \tag{1}$$

where λ is the wavelength, h the altitude, c_λ the light dispersion coefficient and ρ the air density. Some values of coefficient c_λ for the shortwave spectrum are given in (Link 1956). The values of horizontal refraction in the real atmosphere show that above 60-100 kilometres the refraction is almost equal to zero, and the sunbeams propagate in the original direction without change of intensity (Link and Neužil 1965). Data of the spectral horizontal transparency at different altitudes h_0 are required to determine the vertical particle concentration profile and particle distribution. Satellites with orbits above 100 kilometres can be used for this purpose. In this case the horizontal refraction is the function of the minimum altitude h_0 only. Coordinates η and H determine the satellite's position (Fig.2). The orbital period of the satellite should be sufficiently small for the vertical distribution of particles to vary ??? little during the measurement. The data of the spectral flux density of direct solar radiation must be measured along the orbit with boundary coordinates (η_1, H) and (η_2, H), when the minimum sunbeam altitudes h_0 are equal to 0 and H, respectively. The section of measurement is approximately one quarter of the orbit and for the given satellites it corresponds to a time interval of $\approx$ 1 hour. Since the velocity of fall of the submicron particles is very small, the particle distribution does not undergo larger variations during measurements.

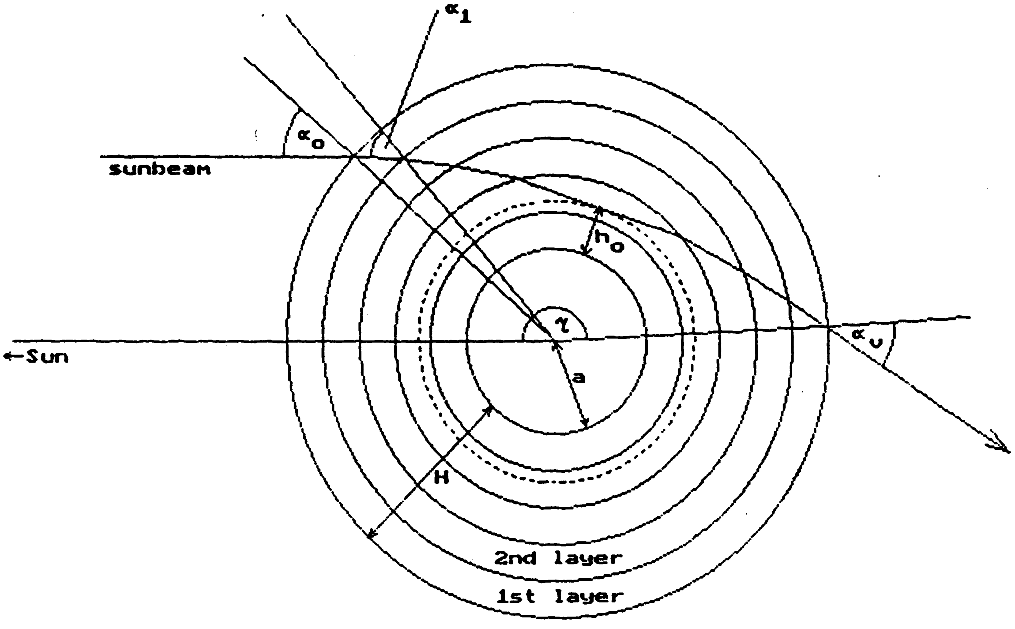

Figure 2. Scheme of sunbeam refraction. Description in text.

The direct solar radiation penetrating through the atmosphere is attenuated by refraction and extinction. The spectral flux density of direct solar radiation can be expressed as

$$I_\lambda(\eta, H) = I_{0\lambda} R_\lambda(\eta, H).exp\left\{-2\int_{h_0(\eta,H)}^{H} \beta_{ext,\lambda}(h) sec\ \xi_\lambda(h,\eta,H)dh\right\} \tag{2}$$

where $I_{0\lambda}$ is the spectral flux density of solar radiation at the top of the atmosphere, R_λ the refraction attenuation coefficient, $\beta_{ext,\lambda}$ the volume extinction coefficient, and ξ_λ the apparent solar zenith angle at altitude h. The refraction properties of the atmosphere in comparison with atmospheric extinction are very stable. Function R_λ may be assumed known and dependent on coordinates η and H only. In this case all functions $I_{0\lambda}, R_\lambda, I_\lambda$ and function

$$F_\lambda(\eta, H) = \ln\left[\frac{I_{0\lambda}.R_\lambda(\eta, H)}{I_\lambda(\eta, H)}\right]$$

are known. The problem of inverting the measured data leads to the solution of an integral equation of the first kind

$$F_\lambda(\eta, H) = 2\int_{h_0(\eta,H)}^{H} \beta_{ext,\lambda}(h) sec\ \xi_\lambda(h,\eta,H)dh \tag{3}$$

If the incident angle at the top of the atmosphere is α_0, angle η can be obtained from equation

$$\eta = \alpha_0 + 2\int_{h_0(H,\alpha_0)}^{H} \frac{(a+H)\sin\alpha_0}{(a+h)\sqrt{(a+h)^2 n_\lambda^2(h) - (a+H)^2\sin^2\alpha_0}}dh \tag{4}$$

where a is the Earth's radius. The apparent solar zenith angle ξ_λ at any altitude h is given by

$$\sin\ \xi_\lambda(h, \alpha_0, H) = \frac{(a+H)}{(a+h)}\frac{\sin\alpha_0}{n_\lambda(h)} \tag{5}$$

At the top of the atmosphere $\xi_\lambda(H, H, \alpha_0) = \xi(H, H, \alpha_0) = \alpha_0$, because $n_\lambda(h \geq H) = 1$. There exists an unambiguous transformation between the base coordinate systems (H, α_0) and (H, η), which is determined by eq.(4). Any function of η and H can then be expressed also in terms of α_0 and H (e.g. equation 5).

Ratio x/x" determines the refraction attenuation of radiation (Fig.3). In the process of radiation transfer through the atmosphere the energy concentrated in linear dimension x is spread to linear dimension x". It can be show that

$$R_\lambda(\eta, H) = \frac{x}{x"} = \frac{d\alpha_0}{d\eta} \tag{6}$$

Inverse differentiation is substantially simpler, therefore,

$$R_\lambda(\eta, H) = \left(\frac{d\eta}{d\alpha_0}\right)^{-1} \tag{7}$$

By differentiating the integrated function and lower boundary of the integral we obtain

$$R_\lambda(\alpha_0, H) = \left\{ 1 - \frac{2(a+H)\sin\alpha_0}{(a+h_0)B_\lambda(H, \alpha_0, h_0)} \frac{dh_0(\alpha_0, H)}{d\alpha_0} + \right.$$

$$\left. +2\int_{h_0(\alpha_0,H)}^{H} \frac{(a+H)}{(a+h)} \cos\alpha_0 . \left[\frac{1}{B_\lambda(H, \alpha_0, h)} + \frac{(a+H)^2 \sin^2\alpha_0}{B_\lambda^3(H, \alpha_0, h)} \right] dh \right\}^{-1} \qquad (8)$$

where $h_0 = h_0(\alpha_0, H)$ or $h_0 = h_0(\eta, H)$ and

$$B_\lambda(H, \alpha_0, h) = \sqrt{(a+h)^2 n_\lambda^2(h) - (a+H)^2 \sin^2\alpha_0}$$

Function $h_0(\alpha_0, H)$ is expressed in the equation

$$(a+H)\sin\alpha_0 = (a+h_0)n_\lambda(h_0) \qquad (9)$$

Refractive index n_λ in the whole atmosphere is only a little different from 1, but it must be take into account in relation (9). Inrelation (10)

$$\frac{dh_0(\alpha_0, H)}{d\alpha_0} = \frac{(a+H)\cos\alpha_0}{1 + N_\lambda(h_0)} \qquad (10)$$

coefficient N_λ is comparable with 1. Coefficient N_λ depends on the refractive index in the form $N_\lambda = c_\lambda \rho(h_0)(1 - a\gamma)$, and $N_\lambda \approx 0.25$ for average values of the vertical air density gradient $\gamma = 1.25.10^{-4}\, m^{-1}$, Earth's radius $a = 6378000\, m, c_\lambda \approx 3.10^{-4}\, m^3.kg^{-1}, \rho(0) \approx 1\, kg.m^{-3}$. The slope $dh_0/d\alpha_0$ can be obtained numerically. It is possible to tabulate this function for the standard atmosphere.

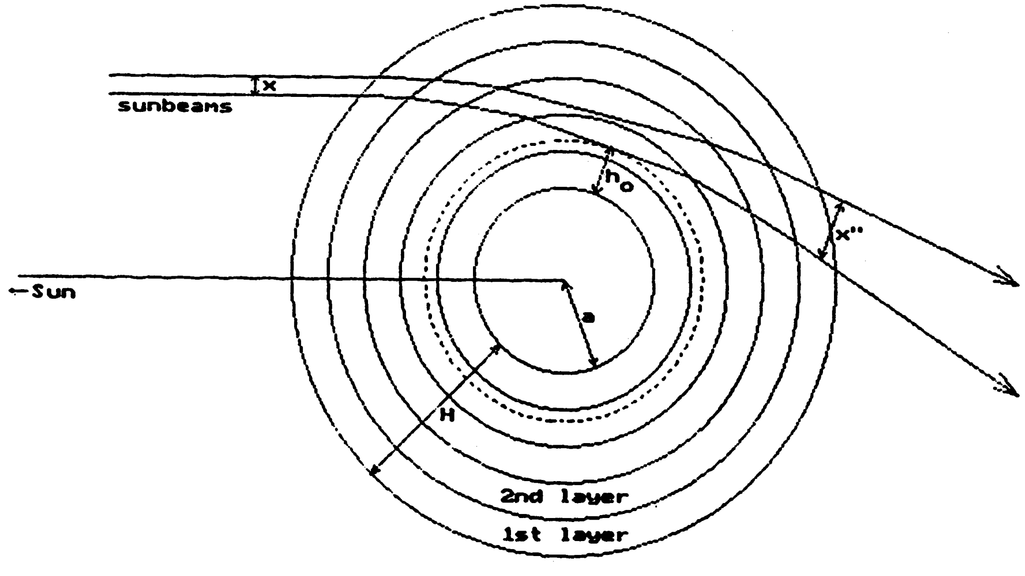

Figure 3. Scheme of refraction attenuation of solar radiation. Description in text.

For a layered atmosphere the integral equation

$$F_\lambda(\eta) = 2\int_{h_0(\eta)}^{H} \beta_{ext,\lambda}(h) sec\, \xi_\lambda(h,\eta) dh \tag{11}$$

can be simply solved numerically for different wavelengths. The concentration and distribution of particles in the lowest i-th layer can be obtained from the measured values $F_\lambda(\eta_i)$. The optical properties of the higher layers are determined from values $F_\lambda(\eta_1) \rightarrow F_\lambda(\eta_{i-1})$. In the first layer it holds approximately that

$$F_\lambda(\eta_1) = 2\beta_{ext,\lambda}(h_0 \rightarrow H).\sqrt{2(a+H)\Delta h_1} \tag{12}$$

where $\Delta h_1 = H - h_0$. Function $\beta_{ext,\lambda}$ in the high atmosphere is obtained with a certain error, because the optical thickness of the high atmosphere is very small and $F_\lambda \approx 0$. The iteration method can be used to solve integral equation (11), if the solution obtained in the previous iteration cycle is used in the actual calculation of the new values of function $\beta_{ext,\lambda}$. The spectral dependence of function $\beta_{ext,\lambda}$ in the layer under study is used to obtain the particle distribution function $f(r)$ by solving integral equation

$$\beta_{ext,par,\lambda}(h) = \int_0^\infty \pi r^2 f(r,h) Q_{ext}(r,\lambda,m) dr \tag{13}$$

In relation (13) $f(r,h)$ is the distribution function and r the particle radius. As regards $\beta_{ext,par,\lambda}(h)$,

$$\beta_{ext,par,\lambda}(h) = \beta_{ext,\lambda}(h) - \beta_{ext,mol,\lambda}(h) - \beta_{ext,ozone,r}(h) - \beta_{ext,tc,\lambda}(h) \tag{14}$$

Functions $\beta_{ext,par,\lambda}(h), \beta_{ext,mol,\lambda}(h), \beta_{ext,ozone,\lambda}(h), \beta_{ext,tc,\lambda}(h)$ are the volume extinction coefficients for microparticles, the clean atmosphere, ozone and trace components, respectively, Q_{ext} is the efficiency extinction factor. Since the scattering and absorption coefficients for nonspherical and inhomogeneous particle depend on the shape, orientation and distribution of these particles in a diffusive medium in a complicated way (Pollack and Cuzzi 1980; Yeh 1964; Erma 1968 a, 1968 b, 1969) it is practically impossible to solve this integral equation for arbitrary particles. At present Mie's theory is the only possible way of solving this inverse problem. For the fast numerical application of Mie's theory effective procedures of calculating Mie's function have been developed (e.g. Cachorro 1991; Zege and Kokhanovsky 1992). For submicron particles of known optical properties some approximations can be used in this theory, and the resulting function $f(r)$ will be obtained rather simply (Shifrin and Perelman 1963, 1964; Kocifaj and Lukáč). Well-known are the regularization methods (Goncharsky and Cherepashuk and Yagoda 1978; Imanaliev 1981) and the Kolmogorov-Smirnov test (Press, Flannery, Teukolsky and Vetterling 1988). After obtaining the distribution function, the concentration of particles can be calculated from $N(h) = \int_0^\infty f(r,h)dh$.

3. Conclusion

The similar method of determination of vertical profile of particle extinction coefficient $\beta_{ext,par,\lambda}(h)$ from the optical satellite measurements of the spectral solar radiation flux density is presented. The measurements of horizontal transparency in visible spectrum are used to determine the size distribution and vertical profile of concentration of submicron particles. To calculate the particle characteristics in double inverse procedure in concrete case the vertical profiles of molecule, ozone and trace components concentration must be known. The Mie's theory of radiation scattering for particle set of spherical shape is applied. The accuracy of resulting function $\beta_{ext,\lambda}(h)$ decrease with altitude due to the decrease of optical thickness ($F_\lambda \approx 0$). Using the measured input data of spectral horizontal transparency in the integral equations (3) and (13) the particle characteristics will be obtained.

4. Appendix - List of symbols

λ - wavelength
h - altitude
$n_\lambda(h)$ - air refractive index
c_λ - light dispersion coefficient
$I_\lambda(\eta, H)$ - spectral flux density of direct solar radiation for given coordinates η, H
$I_{0\lambda}$ - spectral flux density of solar radiation on the top of the atmosphere
$R_\lambda(\eta, H)$ - spectral refraction attenuation coefficient for given coordinates η, H
$h_0(\eta, H)$ - minimal altitude of sun beam for given coordinates η, H
$\beta_{ext,\lambda}(h)$ - spectral volume extinction coefficient on the altitude H
$\beta_{ext,par,\lambda}(h)$ - spectral volume particle extinction coefficient on the altitude H
$\beta_{ext,mol,\lambda}(h)$ - spectral volume extinction coefficient for clean atmosphere on the altitude H
$\beta_{ext,ozone,\lambda}(h)$ - spectral volume ozone extinction coefficient on the altitude H
$\beta_{ext,tc,\lambda}(h)$ - spectral volume extinction coefficient for trace components on the altitude H
$\xi_\lambda(h, \eta, H)$ - apparent solar zenith angle at the altitude H for given coordinates η, H
a - Earth's radius
α_0 - incident angle of sun beam at the top of the atmosphere
$f(r, h)$ - particle distribution function on altitude h
r - particle radius
$Q_{ext}(r, \lambda, m)$ - Mie's efficiency factor for extinction
m - particle refractive index
$N(h)$ - particle concentration on altitude h

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