Applicability of meteor radiant determination methods depending on orbit type

I. High-eccentric orbits

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Abstract. It is evident that there is no uniform method of calculating meteor radiants which would yield reliable results for all types of cometary orbits. In the present paper an analysis of this problem is presented, together with recommended methods for various types of orbits. Some additional methods resulting from mathematical modelling are presented and discussed together with Porter's, Steel-Baggaley's and Hasegawa's methods.

In order to be able to compare how suitable the application of the individual radiant determination methods is, it is necessary to determine the accuracy with which they approximate real meteor orbits. To verify the accuracy with which the orbit of a meteoroid with at least one node at 1 AU fits the original orbit of the parent body, we applied the Southworth-Hawkins D-criterion (Southworth and Hawkins, 1963). $D \leq 0.1$ indicates a very good fit of orbits, $0.1 < D \leq 0.2$ is considered to be a good fit of orbits, and $D > 0.2$ the fit is rather poor and the change of orbit unrealistic.

The optimal methods with the smallest values of D for given types of orbits are shown in two series of six plots. The new method of rotation around the line of apsides we propose is very appropriate in the region of small inclinations. There is no doubt that Hasegawa's ω-adjustment method (1990) has the widest application.

A comparison of the theoretical radiants with the observed radiants of seven known meteor showers is also presented.

Key words: comets – meteor streams

1. Introduction

Calculations of radiants of meteor showers are the first step in the search for a generic relationship between particular showers and their potential parent bodies. The methods now most frequently used to determine meteor radiants from

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orbital elements are modifications of the method presented by Porter (1952). Porter's method, which assumes the meteoroid velocity to be parallel with the parent body's velocity, is used almost universally, though the method itself has some limitations and is more reliable for low inclinations. Various authors, realizing these limitations, have derived other methods that are used only sporadically by themselves, as e.g. Steel and Baggaley (1985) in searching for radiants by rotating the line of apsides of the parent body's orbit, and most recently Hasegawa (1990) who constructs adjusted orbit with the help of an orthogonal projection of the cometary orbit onto the ecliptic. Hasegawa's approach has shown to be very advantageous in the case of the Orionid meteor shower radiant which, contrary to Porter's approach, gives the radiant in agreement with observations.

In this paper some additional methods resulting from mathematical modelling are presented and discussed together with the above methods. The reliability of the individual methods is illustrated in several plots, and the most suitable method for the given type of orbit is recommended.

2. Radiant determination methods

This paper deals only with the determining the radiants of meteors whose parent body is a comet. This also determines the examined range of individual orbital elements corresponding to the real distribution in the cometary population. However, the analysis also covers highly eccentric asteroidal orbits.

The individual orbital elements were examined within the following intervals:
- perihelion distance q from 0.1 to 1.2 AU,
- eccentricity e from 0.5 to 1.0,
- inclination i from 0° to 180°,
- argument of perihelion ω from 0° to 360°.

Change in the ascending node Ω was not taken into account, because the result is independent of Ω if the eccentricity of the Earth's orbit is neglected.

The common task of all the radiant determination methods is to modify the parent body's orbit to fit the meteoroid orbit, whose one node at least is at 1 AU. From this point of view these changes can be divided into two groups:

(A) Changes in size, form and orientation in the orbital plane of the parent body:

1. Adjustment of the orbit by variation of the perihelion distance - q-adjustment (Hasegawa, 1990).
2. Adjustment of the orbit by variation of the eccentricity.
3. Adjustment of the orbit by variation of both the perihelion distance and eccentricity considering a minimal change.
4. Adjustment of the orbit by variation of the argument of perihelion - rotation of the line of apsides (Steel-Baggaley, 1985).

(B) Changes concerning also the orientation of the orbit in space:

5. Adjustment of the orbit by rotation around the line of apsides.
6. Adjustment of the orbit by variation of the argument of perihelion and inclination - ω-adjustment (Hasegawa, 1990).
7. Parallel shift of the velocity vector (Porter, 1952).

3. Modelling

To be able to determine which method of determining the radiant is most suitable, it is necessary to determine the accuracy with which they approximate the real meteor orbit. As we did not want to prefer any of the above methods, the results were compared with the radiants obtained from observations. However, only a few radiants of meteor showers, whose parent comets are also known, are available, and, moreover, the spread of known cometary orbits from the point of view of the studied intervals of orbital elements is insufficient. The Southworth-Hawkins D-criterion (Southworth and Hawkins, 1963) was applied to test the fit of the meteoroid orbit with one node at 1 AU to the original orbit of the parent body.

For the individual methods listed above the respective element (or elements) was varied until one of the nodes reached $r = 1\ AU$. The D-value was calculated for the original and final orbit. Taking into account various analyses of the distribution of the D discriminant in meteor streams with different degree of dispersion of individual members, and considering standard meteor streams only, i.e. not excessively dispersed as e.g. the Taurids, $D \leq 0.1$ indicates a very good fit of orbits, $0.1 < D \leq 0.2$ is considered to indicate a good fit of the orbits, and $D > 0.2$ indicates the fit is rather poor, and the change of orbit unrealistic.

The model calculations were based on discrete values of q (0.1-1.2 AU, step 0.1 AU), e (0.5, 0.75, 1.0), ω (0°-360°, step 15°), i (5°-80°and 100°-175°, step 15°) and the corresponding parameter was changed. As we have to take into account the closest approaches along both arcs of the parent bodies' orbits (2 different streams can be associated), the argument of perihelion was investigated in the interval from 0°to 360°. It can be shown that the resulting course is symmetrical around the value $\omega = 180°$ (i.e. the resulting values for ω and $360° - \omega$ are the same). This is valid for all the methods. Therefore, only the results for the interval of ω from 0°to 180°are shown for all the methods.

It turned out to be very advantageous to distinguish between the arcs of the original orbit on the basis of the position of the ascending node. In this paper *the arc of the ascending node* is a part of the original orbit between the perihelion and aphelion including the ascending node; *the arc of the descending node* is the second half of the orbit. The transformation from the arcs thus defined to the

	ascending arc												
	180° 180°	165° 195°	150° 210°	135° 225°	120° 240°	105° 255°	90° 270°	75° 285°	60° 300°	45° 315°	30° 330°	15° 345°	0° 360°
1.2	ooo	···	···	···	···	···	···	···	···	···	···	···	···
1.1	●●●	ooo	ooo	o··	···	···	···	···	···	···	···	···	···
1.0	×××	●●●	●●●	●oo	o··	···	···	···	···	···	···	···	×××
0.9	●●●	●●●	●●●	●●●	●oo	o··	···	···	···	···	···	···	···
0.8	ooo	ooo	ooo	o●●	●●●	●oo	o··	···	···	···	···	···	···
0.7	···	···	···	·oo	o●●	●●●	●oo	o··	o··	···	···	···	···
0.6	···	···	···	···	·oo	o●●	●●●	●o·	●··	o··	···	···	···
0.5	···	···	···	···	···	·oo	o●×	●●o	×o·	●··	o··	o··	o··
0.4	···	···	···	···	···	···	·o●	o●●	●●o	●o·	●o·	●··	●··
0.3	···	···	···	···	···	···	··o	·o●	o●●	o●o	●●·	●o·	●o·
0.2	···	···	···	···	···	···	···	··o	·o●	·●●	o●o	o●o	o●o
0.1	···	···	···	···	···	···	···	···	··o	·o●	·o●	·●●	·●●
	0° 360°	15° 345°	30° 330°	45° 315°	60° 300°	75° 285°	90° 270°	105° 255°	120° 240°	135° 225°	150° 210°	165° 195°	180° 180°
	descending arc												

Figure 1. $q = q(\omega)$, Q - q-adjustment method.

	ascending arc												
	180° 180°	165° 195°	150° 210°	135° 225°	120° 240°	105° 255°	90° 270°	75° 285°	60° 300°	45° 315°	30° 330°	15° 345°	0° 360°
1.2	···	···	···	···	···	···	···	···	···	···	···	···	···
1.1	···	···	···	···	···	···	···	···	···	···	···	···	···
1.0	×××	···	···	···	···	···	···	···	···	···	···	···	···
0.9	···	···	···	●··	···	···	···	···	···	···	···	···	×××
0.8	···	···	···	···	o●·	o··	···	···	···	···	···	···	···
0.7	···	···	···	···	···	o●·	●··	o··	···	···	···	···	···
0.6	···	···	···	···	···	··o	o●·	●··	o··	o··	···	···	···
0.5	···	···	···	···	···	···	··×	o●·	×··	●··	o··	o··	o··
0.4	···	···	···	···	···	···	···	·o●	o●·	●··	●··	●··	●··
0.3	···	···	···	···	···	···	···	···	·oo	o●·	oo·	●o·	●··
0.2	···	···	···	···	···	···	···	···	··o	·oo	·●·	o●·	o●·
0.1	···	···	···	···	···	···	···	···	···	··o	·o●	·●o	·●o
	0° 360°	15° 345°	30° 330°	45° 315°	60° 300°	75° 285°	90° 270°	105° 255°	120° 240°	135° 225°	150° 210°	165° 195°	180° 180°
	descending arc												

Figure 2. $q = q(\omega)$, E - method of varying the eccentricity.

ascending arc

	180° 180°	165° 195°	150° 210°	135° 225°	120° 240°	105° 255°	90° 270°	75° 285°	60° 300°	45° 315°	30° 330°	15° 345°	0° 360°
1.2	···	···	···	···	···	···	···	···	···	···	···	···	···
1.1	···	···	···	···	···	···	···	···	···	···	···	···	···
1.0	×××	○○·	···	···	···	···	···	···	···	···	···	○○·	×××
0.9	···	···	○○○	●●○	○··	···	···	···	···	···	···	···	···
0.8	···	···	···	○○○	●●○	●··	···	···	···	···	···	···	···
0.7	···	···	···	···	·○○	●●○	●··	○··	···	···	···	···	···
0.6	···	···	···	···	···	·○●	○●·	●··	○··	···	···	···	···
0.5	···	···	···	···	···	···	·○×	○●·	×··	○··	···	···	···
0.4	···	···	···	···	···	···	···	·○●	·●·	●··	●··	○··	···
0.3	···	···	···	···	···	···	···	··○	·○○	·●·	···	···	···
0.2	···	···	···	···	···	···	···	···	··○	·○○	·●·	·○·	···
0.1	···	···	···	···	···	···	···	···	···	··○	··○	···	···
	0° 360°	15° 345°	30° 330°	45° 315°	60° 300°	75° 285°	90° 270°	105° 255°	120° 240°	135° 225°	150° 210°	165° 195°	180° 180°

descending arc

Figure 3. $q = q(\omega)$, W - method of variation of the argument of perihelion.

ascending arc

	180° 180°	165° 195°	150° 210°	135° 225°	120° 240°	105° 255°	90° 270°	75° 285°	60° 300°	45° 315°	30° 330°	15° 345°	0° 360°
1.2	○○○	···	···	···	···	···	···	···	···	···	···	···	···
1.1	●●●	○○○	○○○	○··	···	···	···	···	···	···	···	···	···
1.0	×××	●●●	●●●	●○○	○··	···	···	···	···	···	···	···	×××
0.9	●●●	●●●	●●●	●●●	●○○	○··	···	···	···	···	···	···	···
0.8	○○○	○○○	○○○	○●●	●●●	●○○	○··	○··	···	···	···	···	···
0.7	···	···	···	·○○	○●●	●●●	●○○	○··	○··	···	···	···	···
0.6	···	···	···	···	·○○	○●●	●●●	●○·	●··	○··	○··	○··	○··
0.5	···	···	···	···	···	·○○	○●×	●●○	×○·	●○·	●··	○··	○··
0.4	···	···	···	···	···	···	·○●	○●●	●●○	●○·	●○·	●○·	●··
0.3	···	···	···	···	···	···	··○	·○●	○●●	○●○	●●·	●○·	●○·
0.2	···	···	···	···	···	···	···	··○	·○●	○●●	○●○	○●○	○●○
0.1	···	···	···	···	···	···	···	···	··○	·○●	·●●	○●●	○●●
	0° 360°	15° 345°	30° 330°	45° 315°	60° 300°	75° 285°	90° 270°	105° 255°	120° 240°	135° 225°	150° 210°	165° 195°	180° 180°

descending arc

Figure 4. $q = q(\omega)$, B - method of variation of both the perihelion distance and eccentricity.

	ascending arc												
	180°	165°	150°	135°	120°	105°	90°	75°	60°	45°	30°	15°	0°
	180°	195°	210°	225°	240°	255°	270°	285°	300°	315°	330°	345°	360°
1.2	···	···	···	···	···	···	···	···	···	···	···	···	···
1.1	···	···	···	···	···	···	···	···	···	···	···	···	···
1.0	×××	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●	×××
0.9	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●
0.8	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●
0.7	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●
0.6	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●
0.5	●●●	●●●	●●●	●●●	●●●	●●●	●●×	●●●	×●●	●●●	●●●	●●●	●●●
0.4	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●
0.3	·●●	·●●	·●●	·●●	·●●	·●●	·●●	·●●	·●●	·●●	·●●	·●●	·●●
0.2	·●●	·●●	·●●	·●●	·●●	·●●	·●●	·●●	·●●	·●●	·●●	·●●	·●●
0.1	··●	··●	··●	··●	··●	··●	··●	··●	··●	··●	··●	··●	··●
	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°
	360°	345°	330°	315°	300°	285°	270°	255°	240°	225°	210°	195°	180°
	descending arc												

Figure 5. $q = q(\omega)$, H - ω-adjustment method, inclination = 5°and 175°.

	ascending arc												
	180°	165°	150°	135°	120°	105°	90°	75°	60°	45°	30°	15°	0°
	180°	195°	210°	225°	240°	255°	270°	285°	300°	315°	330°	345°	360°
1.2	···	···	···	···	···	···	···	···	···	···	···	···	···
1.1	···	···	···	···	···	···	···	···	···	···	···	···	···
1.0	×××	···	···	···	···	···	···	···	···	···	···	···	×××
0.9	●●●	●●●	●●●	●●●	●●●	●●●	●○○	○○○	○○○	○○○	○○○	○○○	●●●
0.8	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●○	●○○	●○○	●●○	●●●
0.7	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●
0.6	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●
0.5	●●●	●●●	○●●	●●●	●●●	●●●	●●×	●●●	×●●	●●●	●●●	●●●	●●●
0.4	●●●	○●●	○●●	○●●	○●●	○●●	○●●	●●●	●●●	●●●	●●●	●●●	●●●
0.3	·●●	·○●	·○●	·○●	·○●	·●●	·●●	·●●	·●●	·●●	·●●	·●●	·●●
0.2	·●●	·○○	·○○	·○○	·○○	·○●	·○●	·○●	·●●	·●●	·●●	·●●	·●●
0.1	··●	··○	··○	··○	··○	··○	··○	··●	··●	··●	··●	··●	··●
	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°
	360°	345°	330°	315°	300°	285°	270°	255°	240°	225°	210°	195°	180°
	descending arc												

Figure 6. $q = q(\omega)$, A - method of rotation around the line of apsides, inclination = 5°and 175°.

preperihelion or postperihelion arcs depends on the position of the perihelion with regard to the ecliptic plane (see Table 8).

The results of the methods of adjusting the orbits by changes in the orbital planes of the parent bodies are shown in Fig. 1-4. The circles indicate the degree of fit the method provides. Black circles indicate the method provides a very good fit ($D \leq 0.1$), the white circles indicate that the method is acceptable ($0.1 < D \leq 0.2$), points indicate that the method is unacceptable, and "×" indicates that parent body's orbit crosses the Earth's orbit.

The results for the two methods of adjusting the orbit by changes in space are shown as examples in Fig. 5-6. The plots covering all three methods and all chosen inclinations cannot be shown here because of the lack of place. But, the next section presents the best methods for all the chosen types of orbits.

4. Methods recommended for various types of orbits

The optimal methods with the smallest values of D for given types of orbits are shown in two series of six plots (Figs. 7-18). Maximum D-values in the first and second series are 0.1 and 0.2, respectively. All these plots are symmetrical with respect to $i = 90°$. That is why we have common plots for $i = 5°$ and $i = 175°$, and so on.

In the $q = q(\omega)$ plots, each point is marked with three letters which, from left to right, represent the eccentricity of 0.5, 0.75, 1.0.

The individual letters represent:

Q - q-adjustment method,
B - method of variation of both the perihelion distance and eccentricity,
W - method of variation of the argument of perihelion,
H - ω-adjustment method,
A - method of rotation around the line of apsides,
P - Porter's method of the velocity vector shift.

The method of varying the eccentricity (E) also fulfils the chosen values of D (0.1 and 0.2) for some types of orbits. But, this method is not included in the plots (7-18), because, of all the analysed methods this does not yield the smallest values of D in any combination of orbital elements.

There are few combinations of elements in which the values of D are the same or very similar to each other for two or more of the methods studied. The values of D are considered to be the same if the difference between them is smaller than 0.001. Priority is then given to the less complicated method (in the order Q, E, W, H, A, B, P). This choice is justified by the less pretentious mathematical elaboration and simultaneously by the same quality of results.

ascending arc

	180° 180°	165° 195°	150° 210°	135° 225°	120° 240°	105° 255°	90° 270°	75° 285°	60° 300°	45° 315°	30° 330°	15° 345°	0° 360°
1.2	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇
1.1	QQQ	111	111	111	111	111	111	111	111	111	111	111	111
1.0	×××	QQQ	PPP	PPH	PHH	HHH	HHH	HHH	HHH	HHH	HHH	HHH	×××
0.9	HHH	PPP	PPP	APP	PPP	PPP	HHH	HHH	HHH	HHH	HHH	HHH	HHH
0.8	HHH	APP	APP	APP	AAP	APP	HHH	HHH	HHH	HHH	HHH	HHH	HHH
0.7	HHH	AAA	AAA	AAA	AAA	AAA	HHH	HHH	HHH	HHH	HHH	HHH	HHH
0.6	HHH	HAA	HAA	HAA	HAA	HAA	HHH	AHH	AHH	AHH	AHH	AHH	HHH
0.5	HHH	HHH	HHH	HHH	HHH	HHH	HH×	AAH	×AH	AAH	AAH	AAH	HHH
0.4	HHH	HHH	HHH	HHH	HHH	HHH	HHH	PAA	PAA	PAA	PAA	PAA	BHH
0.3	1HH	1HH	1HH	1HH	1HH	1HH	1HH	1AA	1AA	1AA	BAA	BAA	BHH
0.2	◇HH	◇HH	◇HH	◇HH	◇HH	◇HH	◇HH	◇HA	◇PA	◇PA	◇QA	◇PA	◇HH
0.1	◇◇H	◇◇H	◇◇H	◇◇H	◇◇H	◇◇H	◇◇H	◇◇H	◇◇A	◇◇A	◇BA	◇BA	◇BH
	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°
	360°	345°	330°	315°	300°	285°	270°	255°	240°	225°	210°	195°	180°

descending arc

Figure 7. $q = q(\omega), D \leq 0.1$, inclination = 5° and 175°.

ascending arc

	180° 180°	165° 195°	150° 210°	135° 225°	120° 240°	105° 255°	90° 270°	75° 285°	60° 300°	45° 315°	30° 330°	15° 345°	0° 360°
1.2	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇
1.1	QQQ	111	111	111	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	111	111	111
1.1	×××	QQQ	QQQ	Q11	111	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	111	HHH	×××
0.9	QQQ	QQQ	QQQ	QQQ	B11	111	1◇◇	◇◇◇	H◇◇	HHH	HHH	111	111
0.8	◇◇◇	111	111	1QQ	QQQ	B11	111	HH◇	HHH	◇HH	◇◇◇	◇◇◇	◇◇◇
0.7	◇◇◇	◇◇◇	◇11	111	1AA	BAA	B11	HHH	1HH	◇◇◇	◇◇◇	◇◇◇	◇◇◇
0.6	◇◇◇	◇◇◇	◇◇◇	◇◇1	111	HAA	BHH	BHH	B1◇	1◇◇	◇◇◇	◇◇◇	◇◇◇
0.5	◇◇◇	◇◇◇	◇◇◇	H◇◇	H11	HHH	1H×	BAH	×11	B1◇	B◇◇	◇◇◇	◇◇◇
0.4	1◇◇	◇◇◇	H◇◇	H◇◇	◇H◇	◇HH	◇1H	1BA	BB1	B11	B◇◇	B◇◇	B◇◇
0.3	1◇◇	1◇◇	1◇◇	◇H◇	◇HH	◇◇H	◇◇1	◇1A	◇BA	◇B1	BB◇	B1◇	B◇◇
0.2	◇1◇	◇H◇	◇H◇	◇HH	◇◇H	◇◇◇	◇◇◇	◇◇1	◇1B	◇BB	◇Q1	◇B◇	◇B◇
0.1	◇◇◇	◇◇◇	◇◇H	◇◇H	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇1	◇◇B	◇BB	◇BB	◇BB
	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°
	360°	345°	330°	315°	300°	285°	270°	255°	240°	225°	210°	195°	180°

descending arc

Figure 8. $q = q(\omega), D \leq 0.1$, inclination = 20° and 160°.

	ascending arc												
	180°	165°	150°	135°	120°	105°	90°	75°	60°	45°	30°	15°	0°
	180°	195°	210°	225°	240°	255°	270°	285°	300°	315°	330°	345°	360°
1.2	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄
1.1	QQQ	111	111	1⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	111	111
1.0	×××	QQQ	QQQ	Q11	1⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	111	×××
0.9	QQQ	QQQ	QQQ	QQQ	B11	1⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	HHH	⋄HH	111	111
0.8	⋄⋄⋄	111	111	1QQ	QQQ	B11	1⋄⋄	H⋄⋄	HHH	⋄⋄H	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄
0.7	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄11	1BQ	BQB	B11	HHH	⋄⋄H	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄
0.6	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄11	HBQ	BBB	B1H	B⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄
0.5	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	H⋄⋄	⋄H1	1B×	BB1	×1⋄	B⋄⋄	B⋄⋄	⋄⋄⋄	⋄⋄⋄
0.4	⋄⋄⋄	⋄⋄⋄	H⋄⋄	H⋄⋄	⋄H⋄	⋄HH	⋄1B	⋄BB	BB1	B1⋄	B⋄⋄	B⋄⋄	B⋄⋄
0.3	1⋄⋄	1⋄⋄	⋄⋄⋄	⋄H⋄	⋄HH	⋄⋄H	⋄⋄⋄	⋄⋄B	⋄BB	⋄B⋄	BB⋄	B⋄⋄	B⋄⋄
0.2	⋄⋄⋄	⋄⋄⋄	⋄H⋄	⋄⋄H	⋄⋄H	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄B	⋄BB	⋄Q⋄	⋄B⋄	⋄B⋄
0.1	⋄⋄⋄	⋄⋄⋄	⋄⋄H	⋄⋄H	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄B	⋄BB	⋄BB	⋄BB
	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°
	360°	345°	330°	315°	300°	285°	270°	255°	240°	225°	210°	195°	180°
	descending arc												

Figure 9. $q = q(\omega), D \leq 0.1$, inclination = 35° and 145°.

	ascending arc												
	180°	165°	150°	135°	120°	105°	90°	75°	60°	45°	30°	15°	0°
	180°	195°	210°	225°	240°	255°	270°	285°	300°	315°	330°	345°	360°
1.2	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄
1.1	QQQ	111	111	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	111
1.0	×××	QQQ	QQQ	Q11	1⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	111	×××
0.9	QQQ	QQQ	QQQ	QQQ	B11	1⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	HH⋄	⋄⋄H	111	111
0.8	⋄⋄⋄	111	111	1QQ	QQQ	B11	1⋄⋄	H⋄⋄	HHH	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄
0.7	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄11	1BQ	BQB	B1⋄	HH⋄	⋄⋄H	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄
0.6	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄1	HBQ	BBB	B1H	B⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄
0.5	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	H⋄⋄	⋄H1	⋄B×	BB1	×⋄⋄	B⋄⋄	B⋄⋄	⋄⋄⋄	⋄⋄⋄
0.4	⋄⋄⋄	⋄⋄⋄	H⋄⋄	H⋄⋄	⋄H⋄	⋄HH	⋄⋄B	⋄BB	BB⋄	B⋄⋄	B⋄⋄	B⋄⋄	B⋄⋄
0.3	1⋄⋄	1⋄⋄	⋄⋄⋄	⋄H⋄	⋄⋄H	⋄⋄⋄	⋄⋄⋄	⋄⋄B	⋄BB	⋄B⋄	BB⋄	B⋄⋄	B⋄⋄
0.2	⋄⋄⋄	⋄⋄⋄	⋄H⋄	⋄⋄⋄	⋄⋄H	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄B	⋄BB	⋄Q⋄	⋄B⋄	⋄B⋄
0.1	⋄⋄⋄	⋄⋄⋄	⋄⋄H	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄⋄	⋄⋄B	⋄BB	⋄BB	⋄BB
	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°
	360°	345°	330°	315°	300°	285°	270°	255°	240°	225°	210°	195°	180°
	descending arc												

Figure 10. $q = q(\omega), D \leq 0.1$, inclination = 50° and 130°.

ascending arc

	180°	165°	150°	135°	120°	105°	90°	75°	60°	45°	30°	15°	0°
	180°	195°	210°	225°	240°	255°	270°	285°	300°	315°	330°	345°	360°
1.2	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇
1.1	QQQ	111	111	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	111
1.0	×××	QQQ	QQQ	Q11	1◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	×××
0.9	QQQ	QQQ	QQQ	QQQ	B11	1◇◇	◇◇◇	◇◇◇	◇◇◇	HH◇	◇◇◇	◇◇◇	111
0.8	◇◇◇	111	111	1QQ	QQQ	B1◇	1◇◇	H◇◇	HH◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇
0.7	◇◇◇	◇◇◇	◇◇◇	◇11	1BQ	BQB	B1◇	HH◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇
0.6	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇1	HBQ	BBB	B◇H	B◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇
0.5	◇◇◇	◇◇◇	◇◇◇	◇◇◇	H◇◇	◇H1	◇B×	BB1	×◇◇	B◇◇	B◇◇	◇◇◇	◇◇◇
0.4	◇◇◇	◇◇◇	H◇◇	H◇◇	◇H◇	◇◇H	◇◇B	◇BB	BB◇	B◇◇	B◇◇	B◇◇	B◇◇
0.3	1◇◇	◇◇◇	◇◇◇	◇H◇	◇◇◇	◇◇◇	◇◇◇	◇◇B	◇BB	◇B◇	BB◇	B◇◇	B◇◇
0.2	◇◇◇	◇◇◇	◇H◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇B	◇BB	◇Q◇	◇B◇	◇B◇
0.1	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇B	◇BB	◇BB	◇BB
	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°
	360°	345°	330°	315°	300°	285°	270°	255°	240°	225°	210°	195°	180°

descending arc

Figure 11. $q = q(\omega), D \leq 0.1$, inclination = 65° and 115°.

ascending arc

	180°	165°	150°	135°	120°	105°	90°	75°	60°	45°	30°	15°	0°
	180°	195°	210°	225°	240°	255°	270°	285°	300°	315°	330°	345°	360°
1.2	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇
1.1	QQQ	111	111	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	111
1.0	×××	QQQ	QQQ	Q11	1◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	×××
0.9	QQQ	QQQ	QQQ	QQQ	B11	1◇◇	◇◇◇	◇◇◇	◇◇◇	HH◇	◇◇◇	◇◇◇	111
0.8	◇◇◇	111	111	1QQ	QQQ	B1◇	1◇◇	H◇◇	HH◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇
0.7	◇◇◇	◇◇◇	◇◇◇	◇11	1BQ	BQB	B1◇	HH◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇
0.6	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇1	HBQ	BBB	B◇H	B◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇
0.5	◇◇◇	◇◇◇	◇◇◇	◇◇◇	H◇◇	◇H1	◇B×	BB◇	×◇◇	B◇◇	B◇◇	◇◇◇	◇◇◇
0.4	◇◇◇	◇◇◇	H◇◇	H◇◇	◇H◇	◇◇H	◇◇B	◇BB	BB◇	B◇◇	B◇◇	B◇◇	B◇◇
0.3	1◇◇	◇◇◇	◇◇◇	◇H◇	◇◇◇	◇◇◇	◇◇◇	◇◇B	◇BB	◇B◇	BB◇	B◇◇	B◇◇
0.2	◇◇◇	◇◇◇	◇H◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇B	◇BB	◇Q◇	◇B◇	◇B◇
0.1	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇◇	◇◇B	◇BB	◇BB	◇BB
	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°
	360°	345°	330°	315°	300°	285°	270°	255°	240°	225°	210°	195°	180°

descending arc

Figure 12. $q = q(\omega), D \leq 0.1$, inclination = 80° and 100°.

	ascending arc												
	180°	165°	150°	135°	120°	105°	90°	75°	60°	45°	30°	15°	0°
	180°	195°	210°	225°	240°	255°	270°	285°	300°	315°	330°	345°	360°
1.2	QQQ	***	***	***	***	***	***	***	***	***	***	***	***
1.1	QQQ	QQQ	QQQ	PP2	P22	P22	P22	222	222	222	222	222	222
1.0	×××	QQQ	PPP	PPH	PHH	HHH	HHH	HHH	HHH	HHH	HHH	HHH	×××
0.9	HHH	PPP	PPP	APP	PPP	PPP	HHH	HHH	HHH	HHH	HHH	HHH	HHH
0.8	HHH	APP	APP	APP	AAP	APP	HHH	HHH	HHH	HHH	HHH	HHH	HHH
0.7	HHH	AAA	AAA	AAA	AAA	AAA	HHH	HHH	HHH	HHH	HHH	HHH	HHH
0.6	HHH	HAA	HAA	HAA	HAA	HAA	HHH	AHH	AHH	AHH	AHH	AHH	HHH
0.5	HHH	HHH	HHH	HHH	HHH	HHH	HH×	AAH	×AH	AAH	AAH	AAH	HHH
0.4	HHH	HHH	HHH	HHH	HHH	HHH	HHH	PAA	PAA	PAA	PAA	PAA	BHH
0.3	2HH	2HH	2HH	2HH	2HH	2HH	PHH	PAA	PAA	BAA	BAA	BAA	BHH
0.2	*HH	*HH	*HH	*HH	*HH	*HH	*HH	*HA	*PA	BPA	BQA	BPA	BHH
0.1	**H	**H	**H	**H	**H	**H	*PH	*PH	*PA	*BA	*BA	BBA	BBH
	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°
	360°	345°	330°	315°	300°	285°	270°	255°	240°	225°	210°	195°	180°
	descending arc												

Figure 13. $q = q(\omega), D \leq 0.2$, inclination $= 5°$and $175°$.

	ascending arc												
	180°	165°	150°	135°	120°	105°	90°	75°	60°	45°	30°	15°	0°
	180°	195°	210°	225°	240°	255°	270°	285°	300°	315°	330°	345°	360°
1.2	QQQ	***	***	***	***	***	***	***	***	***	***	***	***
1.1	QQQ	QQQ	QQQ	B22	***	***	***	***	***	***	222	222	222
1.0	×××	QQQ	QQQ	QQQ	B22	***	***	***	***	***	HHH	HHH	×××
0.9	QQQ	QQQ	QQQ	QQQ	BBB	B22	2**	HH*	HHH	HHH	HHH	HHH	222
0.8	QQQ	QQQ	QQQ	QQQ	QQQ	BBP	BH2	HHH	HHH	HHH	HHH	***	***
0.7	***	***	*22	2AA	BAA	BAA	BHH	HHH	HHH	*HH	***	***	***
0.6	***	***	***	H*2	HAA	HAA	BHH	BHH	BHH	B*H	B**	B**	B**
0.5	***	***	H**	HH*	HHH	HHH	BH×	BAH	×AH	BB*	B**	B**	B**
0.4	2**	H**	H**	HHH	HHH	*HH	*HH	BBA	BBA	BB2	BB*	BB*	B**
0.3	2**	2H*	2H*	*HH	*HH	*HH	**H	*BA	BBA	BBB	BB*	BB*	BB*
0.2	*H*	*H*	*HH	*HH	*HH	**H	***	**B	*BB	BBB	BQB	BBB	BBB
0.1	***	**H	**H	**H	**H	***	***	***	**B	*BB	*BB	BBB	BBB
	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°
	360°	345°	330°	315°	300°	285°	270°	255°	240°	225°	210°	195°	180°
	descending arc												

Figure 14. $q = q(\omega), D \leq 0.2$, inclination $= 20°$and $160°$.

ascending arc

	180°	165°	150°	135°	120°	105°	90°	75°	60°	45°	30°	15°	0°
	180°	195°	210°	225°	240°	255°	270°	285°	300°	315°	330°	345°	360°
1.2	QQQ	***	***	***	***	***	***	***	***	***	***	***	***
1.1	QQQ	QQQ	QQQ	B**	***	***	***	***	***	***	***	222	222
1.0	×××	QQQ	QQQ	QQQ	B**	***	***	***	***	***	***	WHH	×××
0.9	QQQ	QQQ	QQQ	QQQ	BBB	B**	***	***	HH*	HHH	HHH	222	222
0.8	QQQ	QQQ	QQQ	QQQ	QQQ	BBB	B**	HH*	HHH	HHH	***	***	***
0.7	***	***	***	*QQ	BBQ	BQB	BBB	HHH	BHH	***	***	***	***
0.6	***	***	***	***	HBB	HBQ	BBB	BHH	B*H	B**	B**	B**	B**
0.5	***	***	***	H**	HH*	HHB	BB×	BBB	×B*	BB*	B**	B**	B**
0.4	***	H**	H**	HH*	*HH	*HH	*BB	BBB	BBB	BB*	BB*	BB*	B**
0.3	2**	2**	*H*	*H*	*HH	**H	**B	*BB	BBB	BBB	BB*	BB*	BB*
0.2	***	*H*	*H*	*HH	**H	***	***	**B	*BB	BBB	BQB	BBB	BBB
0.1	***	***	**H	**H	***	***	***	***	**B	*BB	*BB	BBB	BBB
	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°
	360°	345°	330°	315°	300°	285°	270°	255°	240°	225°	210°	195°	180°

descending arc

Figure 15. $q = q(\omega), D \leq 0.2$, inclination = 35°and 145°.

ascending arc

	180°	165°	150°	135°	120°	105°	90°	75°	60°	45°	30°	15°	0°
	180°	195°	210°	225°	240°	255°	270°	285°	300°	315°	330°	345°	360°
1.2	QQQ	***	***	***	***	***	***	***	***	***	***	***	***
1.1	QQQ	QQQ	QQQ	B**	***	***	***	***	***	***	***	***	222
1.0	×××	QQQ	QQQ	QQQ	B**	***	***	***	***	***	***	WHH	×××
0.9	QQQ	QQQ	QQQ	QQQ	BBB	B**	***	***	H**	HHH	HHH	222	222
0.8	QQQ	QQQ	QQQ	QQQ	QQQ	BBB	B**	HH*	HHH	*HH	***	***	***
0.7	***	***	***	*QQ	BBQ	BQB	BBB	HHH	BHH	***	***	***	***
0.6	***	***	***	***	HBB	HBQ	BBB	BHH	B**	B**	B**	B**	B**
0.5	***	***	***	H**	H**	HHB	BB×	BBB	×B*	BB*	B**	B**	B**
0.4	***	***	H**	H**	*H*	*HH	*BB	BBB	BBB	BB*	BB*	BB*	B**
0.3	2**	2**	***	*H*	*HH	**H	**B	*BB	BBB	BBB	BB*	BB*	BB*
0.2	***	*H*	*H*	*HH	**H	***	***	**B	*BB	BBB	BQB	BBB	BBB
0.1	***	***	**H	**H	***	***	***	***	**B	*BB	*BB	BBB	BBB
	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°
	360°	345°	330°	315°	300°	285°	270°	255°	240°	225°	210°	195°	180°

descending arc

Figure 16. $q = q(\omega), D \leq 0.2$, inclination = 50°and 130°.

ascending arc

	180°	165°	150°	135°	120°	105°	90°	75°	60°	45°	30°	15°	0°
	180°	195°	210°	225°	240°	255°	270°	285°	300°	315°	330°	345°	360°
1.2	QQQ	***	***	***	***	***	***	***	***	***	***	***	***
1.1	QQQ	QQQ	QQQ	B**	***	***	***	***	***	***	***	***	222
1.0	×××	QQQ	QQQ	QQQ	B**	***	***	***	***	***	***	WH*	×××
0.9	QQQ	QQQ	QQQ	QQQ	BBB	B**	***	***	H**	HHH	HHH	***	222
0.8	QQQ	QQQ	QQQ	QQQ	QQQ	BBB	B**	H**	HHH	*HH	***	***	***
0.7	***	***	***	*QQ	BBQ	BQB	BBB	HHH	BHH	***	***	***	***
0.6	***	***	***	***	HBB	HBQ	BBB	BBH	B**	B**	B**	B**	B**
0.5	***	***	***	H**	H**	HHB	BB×	BBB	×B*	BB*	B**	B**	B**
0.4	***	***	H**	H**	*H*	*HH	*BB	BBB	BBB	BB*	BB*	BB*	B**
0.3	2**	***	***	*H*	*HH	**H	**B	*BB	BBB	BBB	BB*	BB*	BB*
0.2	***	*H*	*H*	*HH	**H	***	***	**B	*BB	BBB	BQB	BBB	BBB
0.1	***	***	**H	**H	***	***	***	***	**B	*BB	*BB	BBB	BBB
	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°
	360°	345°	330°	315°	300°	285°	270°	255°	240°	225°	210°	195°	180°

descending arc

Figure 17. $q = q(\omega), D \leq 0.2$, inclination = 65° and 115°.

ascending arc

	180°	165°	150°	135°	120°	105°	90°	75°	60°	45°	30°	15°	0°
	180°	195°	210°	225°	240°	255°	270°	285°	300°	315°	330°	345°	360°
1.2	QQQ	***	***	***	***	***	***	***	***	***	***	***	***
1.1	QQQ	QQQ	QQQ	B**	***	***	***	***	***	***	***	***	222
1.0	×××	QQQ	QQQ	QQQ	B**	***	***	***	***	***	***	WW*	×××
0.9	QQQ	QQQ	QQQ	QQQ	BBB	B**	***	***	H**	HHH	HHH	***	222
0.8	QQQ	QQQ	QQQ	QQQ	QQQ	BBB	B**	H**	HHH	HHH	***	***	***
0.7	***	***	***	*QQ	BBQ	BQB	BBB	HHH	BHH	***	***	***	***
0.6	***	***	***	***	HBB	HBQ	BBB	BBH	B**	B**	B**	B**	B**
0.5	***	***	***	H**	H**	HHB	BB×	BBB	×B*	BB*	B**	B**	B**
0.4	***	H**	H**	H**	*H*	*HH	*BB	BBB	BBB	BB*	BB*	BB*	B**
0.3	2**	***	***	*H*	*HH	**H	**B	*BB	BBB	BBB	BB*	BB*	BB*
0.2	***	*H*	*H*	*HH	**H	***	***	**B	*BB	BBB	BQB	BBB	BBB
0.1	***	***	**H	**H	***	***	***	***	**B	*BB	*BB	BBB	BBB
	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°
	360°	345°	330°	315°	300°	285°	270°	255°	240°	225°	210°	195°	180°

descending arc

Figure 18. $q = q(\omega), D \leq 0.2$, inclination = 80° and 100°.

Besides the letters in the plots representing particular methods, the following additional symbols are used:

1 - no method yields $D \leq 0.1$, and a part of the orbit is closer to the Earth's orbit than 0.2 AU,

2 - no method yields $D \leq 0.2$, and a part of the orbit is closer to the Earth's orbit than 0.2 AU,

$\diamond$ - no method yields $D \leq 0.1$, and the whole orbit is far from the Earth's orbit (more than 0.2 AU),

$*$ - no method yields $D \leq 0.2$, and the whole orbit is far from the Earth's orbit (more than 0.2 AU),

$\times$ - the cometary orbit crosses the Earth's orbit.

It was necessary to introduce the distance condition of the whole orbit from the Earth's orbit (0.2 AU) to distinquish whether the orbit geometry or the absence of suitable method was the cause of the D-criterion condition not being fulfilled. By comparing figures 7-18 one can see that the new method of rotation around the line of apsides we have proposed is very appropriate in the region of small inclinations. Indubitably, Hasegawa's ω-adjustment method has the widest application.

The user of Porter's method has to take into account that there are a few special orbits (if the function of the distance between the Earth's and parent body's orbits has only one local minimum) for which a solution only on one of the arcs is real. For such orbits the solution on the second arc is artificial, and connected with the apparent motion of the radiant during the year. Moreover, there are cases (e.g. the stream of α-Capricornids - see Table 2) in which this method does not yield usable results, in spite of the low inclined orbit. By comparing Tables 1-7 one can also see that there is no correlation between the D-discriminant and the agreement of the theoretical radiant with the real radiant if Porter's method is applied.

5. Comparison of the theoretical with the observed radiants of known meteor showers

A comparison of the theoretical with the observed radiants was made for the Lyrids, α-Capricornids, Perseids (both for the elements of Comet 1862 III and Comet 1992 t), Draconids, Orionids, Leonids and Geminids. The list of the observed radiants summarized by Cook (1973) was the main source. Some of the radiants were used in a more precise form published later: the Lyrids (Lindblad and Porubčan, 1991) and α-Capricornids (Neslušan, Porubčan and Svoreň, 1992). The orbital elements of the parent comets were taken from the Catalogue of Cometary Orbits (Marsden, 1989) and those of the periodic comet Swift-Tuttle (1992 t) from the Minor Planet Circular (Marsden, 1992).

In modelling the eccentricity of the Earth orbit was neglected, and this simplification may slightly affect the relations between the individual methods. The error incurred thereby does not exceed ± 2 %.

The comparisons of the viability of the individual methods are shown in Tables 1-7. The same abbreviations for the methods are used as in Section 4,

Table 1. LYRIDS - applicability of individual methods

method	D	γ	Δl	Δv
Q	0.003	0.4	0.7	0.5
E	0.064	0.9	0.7	1.1
W	0.010	0.6	0.7	0.5
B	0.003	0.4	0.7	0.5
A	0.190	8.4	5.3	3.4
H	0.010	0.6	0.5	0.5
P	0.008	0.5	0.7	0.5

Table 2. α-CAPRICORNIDS - applicability of individual methods

method	D	γ	Δl	Δv
Q	0.366	20.9	45.3	4.8
E	0.487	14.6	45.3	14.6
W	0.756	47.4	45.3	1.9
B	0.313	17.7	45.3	4.2
A	0.060	8.8	9.6	1.9
H	0.060	8.6	9.5	1.9
P	1.638	118.6	8.5	40.7

Table 3. PERSEIDS - applicability of individual methods

	Elements 1862 III				Elements 1992 t			
method	D	γ	Δl	Δv	D	γ	Δl	Δv
Q	0.017	1.0	0.1	0.5	0.012	0.5	0.6	0.5
E	0.452	4.7	0.1	4.1	0.356	3.2	0.6	3.1
W	0.079	2.6	0.1	0.6	0.057	1.6	0.6	0.6
B	0.017	1.0	0.1	0.5	0.012	0.5	0.6	0.5
A	-	-	-	-	-	-	-	-
H	0.073	1.1	1.8	0.6	0.049	4.5	3.2	0.6
P	0.040	0.5	0.1	0.1	0.029	4.0	4.5	0.0

γ is the angle between the directions to the theoretical and observed radiants in degrees, Δl is the absolute value of the difference between the theoretical and observed solar longitudes in degrees, and Δv is the absolute value of the difference between the theoretical and observed geocentric velocities in $km\ s^{-1}$.

Table 4. DRACONIDS - applicability of individual methods

method	D	γ	Δl	Δv
Q	0.032	2.5	1.6	0.4
E	-	-	-	-
W	-	-	-	-
B	0.032	2.5	1.6	0.4
A	-	-	-	-
H	-	-	-	-
P	0.061	3.1	2.0	0.1

Table 5. ORIONIDS - applicability of individual methods

method	D	γ	Δl	Δv
Q	0.262	21.1	30.1	3.8
E	0.537	40.3	30.1	8.5
W	0.518	29.8	30.1	0.1
B	0.245	24.4	30.1	5.2
A	0.166	2.3	2.3	0.2
H	0.158	0.5	0.3	0.3
P	0.208	4.1	2.2	0.5

Table 6. LEONIDS - applicability of individual methods

method	D	γ	Δl	Δv
Q	0.014	0.2	0.1	0.6
E	-	-	-	-
W	0.135	2.5	0.1	0.8
B	0.014	0.2	0.1	0.6
A	0.165	8.3	8.8	0.1
H	0.044	5.5	8.2	0.7
P	0.031	0.4	0.2	0.0

Table 7. GEMINIDS - applicability of individual methods

method	D	γ	Δl	Δv
Q	0.019	2.0	3.9	0.5
E	0.039	1.8	3.9	2.5
W	0.062	4.2	3.9	1.0
B	0.017	1.8	3.9	0.0
A	0.050	0.4	0.4	0.7
H	0.026	0.8	0.2	1.0
P	0.071	1.5	0.8	1.3

6. Mathematical appendix

In all the above methods, the Earth's orbit is considered to be circular. The orbit of the parent body - the original orbit - is described by the elements: q_a - perihelion distance, e_a - eccentricity, ω_a - argument of perihelion, Ω_a -longitude of the ascending node, i_a - inclination. The final adjusted orbit intersecting the Earth's orbit is analogously described by the elements: $q_b, e_b, \omega_b, \Omega_b, i_b$.

Both the arc of the ascending node and of the arc of the descending node have been investigated separately. Table 8 enables transformation from the ascending/descending arc to the preperihelion/postperihelion arc.

Table 8. Transformations among the arcs

	$0° \leq \omega_a < 180°$	$180° \leq \omega_a < 360°$
ascending arc	preperihelion arc	postperihelion arc
descending arc	postperihelion arc	preperihelion arc

6.1. Methods of variation of the orbit in the orbital plane

The methods of this group make use of the fact that one node of the adjusted orbit is at distance $r_1 = 1AU$ from the Sun, justifying the equation

$$r_1 = \frac{q_b(1+e_b)}{1+e_b \cos f_x} \qquad (1)$$

where f_x is the true anomaly of the point of intersection of the adjusted orbit with the Earth's orbit.

If the perihelion distance or/and eccentricity are varied, $f_x + \omega_a = 360°$ holds for the above mentioned point of intersection on the ascending arc, and $f_x + \omega_a = 180°$ for the intersecting point on the descending arc.

If the argument of perihelion is varied, the chosen arc (ascending or descending) either does not intersect the Earth's orbit (in equation (1) we shall obtain $|\cos f_x| > 1$) or it intersects the Earth's orbit twice. Equation (1) yields angle f_x within the interval 0°-180°, designated as f_o. On the ascending arc then real $f_x = 360° - f_o$ if $0 \leq \omega_a < 180°$ and $f_x = f_o$ if $180° \leq \omega_a < 360°$. The first solution is an orbit with $\omega_b = 360° - f_x$ and the second solution an orbit with $\omega_b = 180° - f_x$. Similary, on the descending arc $f_x = f_o$ if $0° \leq \omega_a < 180°$ and $f_x = 360° - f_o$ if $180° \leq \omega_a < 360°$. The first solution is $\omega_b = 360° - f_x$ and the second solution $\omega_b = 180° - f_x$.

If two elements - q and e - were varied, the perihelion distance referring to the individual discrete values of the eccentricity within the interval from $e - \Delta e$ to $e + \Delta e$ was found step by step from equation (1) (again $f_x + \omega_a = 360°$ on the ascending arc and $f_x + \omega_a = 180°$ on the descending arc) and, the corresponding value of the Southworth-Hawkins D-criterion was then calculated. The range of the eccentricity variation was defined by $\Delta e = D_{max}$, where D_{max} is the maximum value of D that can still be accepted as suitable for the adjusted orbit. The values at which D was minimum were taken for q_b and e_b.

6.2. Methods of variation and rotation of the orbit in space

6.2.1. Adjustement of the orbit by rotation around the line of apsides

The perihelion distance and the eccentricity of the orbit do not change, i.e. $q_b = q_a$, $e_b = e_a$. The argument of perihelion is expressed in a similar way as in Section 6.1

$$\cos f_o = \frac{q_b(1 + e_b)}{e_b \; r_1} - \frac{1}{e_b} \tag{2}$$

If $|\cos f_o| > 1$, no point of intersection with the Earth's orbit will be obtained by rotating the original orbit around the line of apsides because the orbit has no point at distance 1 AU from the Sun.

Angle f_o is within the interval 0°-180°. In searching for a point of intersection on the ascending arc $f_x = 360° - f_o$ if $0° \leq \omega_a < 180°$, $f_x = f_o$ if $180° \leq \omega_a < 360°$ and, consequently, $\omega_b = 360° - f_x$. In searching for a point of intersection on the descending arc $f_x = f_o$ for $0° \leq \omega_a < 180°$, $f_x = 360° - f_o$ for $180° \leq \omega_a < 360°$ and then $\omega_b = 180° - f_x$.

The rectangular ecliptical coordinates of the perihelion x_q, y_q, z_q of the original orbit satisfy the relations

$$\left.\begin{aligned} x' &= \cos\omega_a \cos\Omega_a - \sin\omega_a \cos i_a \sin\Omega_a \\ y' &= \cos\omega_a \sin\Omega_a + \sin\omega_a \cos i_a \cos\Omega_a \\ z' &= \sin\omega_a \sin i_a \end{aligned}\right\} \tag{3}$$

where $x = x_q/q_a, y' = y_q/q_a, z' = z_q/q_a$.

As the perihelion of the adjusted orbit is identical with the perihelion of the original orbit

$$\left.\begin{aligned} \cos\omega_b \cos\Omega_b - \sin\omega_b \cos i_b \sin\Omega_b &= x' \\ \cos\omega_b \sin\Omega_b + \sin\omega_b \cos i_b \cos\Omega_b &= y' \\ \sin\omega_b \sin i_b &= z' \end{aligned}\right\} \tag{4}$$

Inclination i_b can be obtained from the last equation in (4) as

$$\sin i_b = \frac{\sin\omega_a \sin i_a}{\sin\omega_b} \tag{5}$$

The final adjusted orbit with its node at $r_1 = 1\,AU$ from the Sun, also has to satisfy the condition: $\sin^2\omega_b \geq \sin^2\omega_a \sin^2 i_a$. Two real-valued solutions on both the arcs for i_b can then be found as

$$\cos i_b = \pm\sqrt{1 - \sin^2 i_b} \tag{6}$$

and, consequently, this results in two corresponding values for the longitude of the ascending node Ω_b. The values are obtained from (4) explicitly

$$\sin\Omega_b = \frac{y' \cos\omega_b - x' \sin\omega_b \cos i_b}{\cos^2\omega_b + \sin^2\omega_b \cos^2 i_b} \tag{7}$$

$$\cos\Omega_b = \frac{x' \cos\omega_b + y' \sin\omega_b \cos i_b}{\cos^2\omega_b + \sin^2\omega_b \cos^2 i_b} \tag{8}$$

Discriminant D is computed for both pairs of i_b, Ω_b and the resulting elements are those corresponding to the smaller value of D.

6.2.2. The argument of perihelion adjustment

The true anomaly of the point of intersection of the adjusted orbit with the Earth's orbit, f_b, for $q_b = q_a$ and $e_b = e_a$ may be determined from the equation

$$\cos f_b = \frac{q_b(1 + e_b)}{e_b\, r_1} - \frac{1}{e_b} \tag{9}$$

Again, if $|\cos f_b| > 1$, this method is not applicable.

Since the method involves the projection of the original orbit, the ascending arc of the original orbit corresponds to the ascending arc of the adjusted orbit, and *vice versa*. If the point of intersection lies on the ascending arc $f_b \epsilon \langle 180°, 360°)$, and if it lies on the descending arc $f_b \epsilon \langle 0°, 180°)$.

Substituting $u = \omega_a + f_b$, the longitude of the Earth, $L_\otimes$, may be determined from the equations

$$\left.\begin{aligned} \cos\beta \cos(L_\otimes - \Omega_a) &= \cos u \\ \cos\beta \sin(L_\otimes - \Omega_a) &= \sin u \cos i_a \\ \sin\beta &= \sin u \sin i_a \end{aligned}\right\} \tag{10}$$

If $-90° < u < 90°$, then

$$\left.\begin{aligned} \omega_b &= -f_b \\ \Omega_b &= L_\otimes \end{aligned}\right\} \tag{11}$$

but if $90° < u < 270°$, then

$$\left.\begin{aligned} \omega_b &= 180° - f_b \\ \Omega_b &= L_\otimes + 180° \end{aligned}\right\} \tag{12}$$

In the special cases of $u = 90°$ or $u = 270°$ we have to compute the pair ω_b, Ω_b both from equations (11) and from equations (12), and then choose the solution with the smaller value of the D-discriminant.

Inclination i_b may be computed from

$$\sin i_b = \pm \sin i_a \cos(L_\otimes - \Omega_a) \tag{13}$$

The sign to choose is that at which both inclinations i_a, i_b are in the same quadrant.

6.2.3. Porter's method of the velocity vector shift

The places of the closest approaches of the parent body's orbit to the Earth's orbit for the ascending and descending arcs are computed numerically. Each of these positions is described by rectangular ecliptical coordinates x, y, z, radius-vector r and true anomaly f. Similarly, the position of the Earth at the moment of its closest approach to the parent body's orbit is represented by coordinates $x_z, y_z (z_z = 0, r_1 = \sqrt{x_z^2 + y_z^2} = 1\ AU)$. To obtain the radiant, we have to shift the velocity vector of the parent body to the position of the Earth.

The velocity vector may be determined as follows. Let ϕ_a be the angle between the velocity vector and the positional vector of the parent body $\boldsymbol{r} = (x, y, z)$, then

$$\sin \phi_a = \frac{1 + e_a \cos f}{\sqrt{1 + e_a^2 + 2e_a \cos f}} \tag{14}$$

$$\cos \phi_a = -\frac{e_a \sin f}{\sqrt{1 + e_a^2 + 2e_a \cos f}} \tag{15}$$

The absolute value of the velocity vector v may be found from

$$\frac{v}{k} = \sqrt{\frac{2}{r} - \frac{1 - e_a}{q_a}} \tag{16}$$

where k is Gauss constant.

The velocity vector components in the ecliptic coordinate system are given by

$$\left.\begin{aligned} v_x &= -v[\cos(f+\omega_a-\phi_a)\cos\Omega_a - \sin(f+\omega_a-\phi_a)\cos i_a \sin\Omega_a] \\ v_y &= -v[\cos(f+\omega_a-\phi_a)\sin\Omega_a + \sin(f+\omega_a-\phi_a)\cos i_a \cos\Omega_a] \\ v_z &= -v\sin(f+\omega_a-\phi_a)\sin i_a \end{aligned}\right\} \quad (17)$$

The ecliptical longitude of the Earth, λ_z, at the time of the closest approach can be obtained from

$$\sin\lambda_z = \frac{y_z}{r_1} \quad (18)$$

$$\cos\lambda_z = \frac{x_z}{r_1} \quad (19)$$

If $v_z \geq 0$, then $\Omega_b = \lambda_z$, otherwise $\Omega_b = \lambda_z + 180°$.

Inclination i_b may be calculated from

$$\tan i_b = \frac{v_z}{-v_x \sin\Omega_b + v_y \cos\Omega_b} \quad (20)$$

If $i_b = 0°$, we have to correct the value of Ω_b to obtain the correct value of the D-discriminant. The correction is necessary due to the method of calculating D. For the ascending arc $\Omega_b = \omega_a$ and for the descending arc $\Omega_b = \omega_a + 180°$.

Let ϕ_b be the angle between the shifted velocity vector and the positional vector of the Earth $\boldsymbol{r}_z = (x_z, y_z, 0)$, then $\phi_b = 180° - \gamma_b$, where angle $\gamma_b \epsilon \langle 0°, 180° \rangle$ may be derived from

$$\cos\gamma_b = \frac{x_z\ v_x + y_z\ v_y}{r_1\ v} \quad (21)$$

Consequently, eccentricity e_b is given by

$$e_b = \sqrt{1 - \frac{v^2}{k^2}\left(\frac{2}{r_1} - \frac{v^2}{k^2}\right)\sin^2\phi_b}, \quad (22)$$

and the perihelion distance of the adjusted orbit

$$q_b = \frac{1-e_b}{\frac{2}{r_1} - \frac{v^2}{k^2}} \quad (23)$$

The argument of perihelion of the adjusted orbit can be obtained from equation (2) where $0° \leq f_o < 180°$. For the ascending arc $f_x = 360° - f_o$ if $0° \leq \omega_a < 180°$ and $f_x = f_o$ if $180° \leq \omega_a < 360°$ and, consequently, $\omega_b = 360° - f_x$. For the descending arc $f_x = f_o$ if $0° \leq \omega_a < 180°$ and $f_x = 360° - f_o$ if $180° \leq \omega_a < 360°$, consequently, $\omega_b = 180° - f_x$.

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