A note to comet ejection process from the Uranus-Neptune region

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Abstract. A simple energy balance of comet ejection from the Uranus-Neptune region is discussed. If we assume that the orbits of Uranus and Netune remain stable during the ejection process, the necessary energy loss of perturbing planets needs a compensation. Planetary perturbation leading to short-period orbits of a part of comets are suggested as an explanation. Considerations about the total energy conservation become more important if a high mass of Oort cloud is supposed.

Key words: comets - cosmogony - celestial mechanics

1. Introduction

Further we shall use the term "comet" instead of "cometary nucleus" for sake of simplicity.

The theory of primordial comet origin suggests that comets are a by-product of planet formation and that they were created mainly in the Uranus-Neptune region. Neptune (and Uranus to a lesser degree) ejected the comets either out from the Solar System, or into the Oort cloud region. (e.g. Fernandèz, 1985; Weissman, 1990)

The orbits of a part of the comets were probably changed into short-period orbits with semi-major axes smaller than the original ones. This group of comets disappeared early after this process and were usually not included in dynamic studies. However, this group is important from the point of view of energy conservation, because these comets compensated the orbital energy loss of the planet considered in the ejection process ("perturbing planet").

In this paper we analyse the energy transfer between the perturbing planet and the comets.

The total mass of the comet cloud is estimated from $\sim$14 to $\sim$1000 $M_{\oplus}$ (Earth masses), the best estimates being between 45 and 50 $M_{\oplus}$. During the

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history of the Solar System, the Oort cloud has lost between 40% and 80% of its population, so that the original mass must have been larger by about a factor of two to five. (Weissman, 1990)

2. Energy balance of the comet ejection process

We shall assume a comet whose orbit has semi-major axis a_o and whose total orbital energy in the solar gravitational field is expressed by equation

$$E_1 = -\frac{GmM_\odot}{2a_o} \tag{1}$$

where G is the gravitational constant, m the mass of the comet, $M_\odot$ the mass of the Sun. Assume the comet to be moved from the Uranus-Neptune region to the Oort cloud region after encounter with one of the outer planets. The orbital energy of the comet will increase by energy increment ΔE, and the planet will lose ΔE of its orbital energy in the solar gravitational field due to energy conservation. Consequently, the semi-major axis of the planet's orbit will decrease from $a_{\mathrm{P}o}$ to a_{P}. We can describe the diminishing of the semi-major axis as a function of energy loss in the solar gravitational field by differential equation

$$\frac{da_{\mathrm{P}}}{a_{\mathrm{P}}{}^2} = \frac{2}{GM_{\mathrm{P}}M_\odot}dE \tag{2}$$

where M_{P} is the mass of the planet.

If the total mass of the original comet population was $\Sigma m_i = M_{\mathrm{K}}$, the portion of the mass ejected by the planet p_e, the initial mean reciprocal value of the semi-major axis of the orbits of the ejected comets $< 1/a >_o$ and the final value $< 1/a >_e$, then the total energy imparted by the planet is

$$E_e = -\frac{1}{2}Gp_eM_{\mathrm{K}}M_\odot(< 1/a >_o - < 1/a >_e) \tag{3}$$

(In differentiating this equation, we can express the difference of energy ΔE in equation (2) in terms of the difference of mass $d(p_eM_{\mathrm{K}})$ ejected by the planet, as its semi-major axis decreases by difference da_{P}.) Consequently, we can determine the change of the planet's semi-major axis due to ejections of the comets by integrating equation (2) from $a_{\mathrm{P}o}$ to a_{P} and from 0 to E_e. Then

$$a_{\mathrm{P}} = \frac{a_{\mathrm{P}o}}{1-(2a_{\mathrm{P}o}E_e)/(GM_{\mathrm{P}}M_\odot)} \tag{4}$$

and the relative decrease in the size of the semi-major axis is

$$\frac{a_{\mathrm{P}} - a_{\mathrm{P}o}}{a_{\mathrm{P}o}} = \frac{1}{(GM_{\mathrm{P}}M_\odot)/(2a_{\mathrm{P}o}E_e) - 1} \tag{5}$$

We have neglected the value $< 1/a >_e$ in equation (5), because the semi-major axes of the Oort cloud comet orbits are much larger then those of the original comet population orbits.

Let k be the ratio of the reciprocal semi-major axis of Neptune $1/a_{N_o}$ to $< 1/a >_o$. Value $< 1/a >_o$ was either roughly equal to value $1/a_{N_o}$ for the comets in initially circular orbits, or it could have been lower or higher, if the original orbits were more eccentric. However value $< 1/a >_o$ could not have been lower by more than a factor of roughly five ($k = 5$) because major parts of the orbits would have then been located outside the Uranus-Neptune region. This would contradict the primordial theory of comet origin. And it could not be higher by more than a factor of roughly two ($k = 0.5$) because the comets were created in a cold environment far away from the Sun.

There is an implicit assumption that the distribution of comet orbits is always characterized by value $< 1/a >_o$, e.g. this assumption is valid for any value of the planet's semi-major axis between a_{P_o} and a_P. (On the contrary, if we assume that value $< 1/a >_o$ was proportional to the planet's semi-major axis, we would express that the relative decrease in the size of this semi-major axis would be larger than that given by relation (5).)

In the process of computation we need to know mass $p_e M_K$ of the comets, ejected by the perturbing planet. Let the portion of the comets in the Oort cloud ejected by this planet be equal u, and let the rest of the comets $1 - u$ be ejected from the Uranus-Neptune region by other planets. We shall denote h the ratio of the mass of the comets ejected into the Oort cloud region to the mass of the comets ejected away from the Solar System by the perturbing planet. If the present mass of the comets equals M_{KP} in the Oort cloud, and if it originally was n time larger (n is between 2 and 5 - see Introduction), then

$$p_e M_K = u n M_{KP}(1 + \frac{1}{h}) \quad , \tag{6}$$

The values u, h, $p_e M_K$ are given in Table 1. for all four Jovian planets.

We shall now illustrate the inevitability of existence of the short-period comet group on the case of the ejection by Neptune. If Neptune had only elongated the comet orbits, it would have lost orbital energy and its semi-major axis would have decreased. We can estimate the minimum, mean, and maximum energy losses by using the marginal or mean values of parameters n, k, and of mass M_{KP}. The relative decrease of the semi-major axis for these estimates is given in Table 2.

Considering the minimum estimate alone, Neptune's semi-major axis would have decreased by about 20%.

In the case of the maximum estimate it would be necessary to verify the validity of either the upper Oort cloud mass estimate, or the primordial theory of comet origin itself.

Should the energy loss approach the mean estimate, a comparatively large portion of the original comet orbits would have had to be changed to shorter-

planet	u	h	$p_e M_K$
Jupiter	0.08	0.03 ± 0.01	$2.75nM_{KP}$
Saturn	0.16	0.16 ± 0.04	$1.16nM_{KP}$
Uranus	0.24	1.3 ± 0.5	$0.43nM_{KP}$
Neptune	0.52	2.6 ± 0.7	$0.72nM_{KP}$

Table 1. Part $p_e M_K$ of the mass of the comets of the original comet population ejected by the perturbing planet (equation (6)). Value u is the ratio of the mass of the Oort cloud comets, which were ejected into this region by the perturbing planet to the total mass of the original comet population (Safronov, 1972). Value h is the ratio of the mass of the comets ejected into the Oort cloud region to the mass of the comets ejected by the planet away from the Solar System along hyperbolic orbits (Fernandèz, 1985). The original Oort cloud comet mass was n times larger than the present mass M_{KP} (see Introduction).

period orbits by Neptune (and also by the other planets). We are still speaking of a definitive change of the orbit. Consequently, this comet group became extinct due to physical disintegration, by passing several thousand times near the Sun.

Considering the deceleration of the comets by the perturbing planet, one comet loses energy

$$E_{1s} \leq \frac{GmM_\odot}{2a_{P_o}} \tag{7}$$

and all the decelerated comets add energy

$$E_s = \frac{1}{2} G p_s M_K M_\odot (< 1/a >_o - < 1/a >_s) \tag{8}$$

to the planet, p_s being a part of mass of the original comet population decelerated by the perturbing planet and $< 1/a >_s$ being the mean reciprocal semi-major axis of the final orbits of the decelerated comets.

In considering both, the acceleration and deceleration processes, energy E_e has to be replaced by the sum $E_e + E_s$ in relation (4), and we obtain the relation for the relative change of the semi-major axis of the planet's orbit:

$$\frac{a_P - a_{P_o}}{a_{P_o}} = \frac{1}{[GM_P M_\odot]/[2a_{P_o}(E_e + E_s)] - 1} \tag{9}$$

This relation gives the constraints imposed on the distribution of the original comet orbits in the Uranus-Neptune region. In modelling the ejection process, we assume a particular semi-major axes distribution of comet orbits, whose mean reciprocal semi-major axis is $< 1/a >_o$. The result of the modelling are elongated orbits with mean reciprocal semi-major axis $< 1/a >_e$ and shortened orbits with $< 1/a >_s$. We can calculate the energies E_e, E_s from relations (3), (8), and the

energy loss estimate	n [1]	$M_{\rm KP}$ [$M_\oplus$]	k [1]	$(a_{\rm N} - a_{\rm No})/a_{\rm No}$ [%]
minimum	2	14	5	-19.0
mean	3	50	2	-75.8
maximum	5	1000	0.5	-99.8

Table 2. Relative decrease $(a_{\rm N} - a_{\rm No})/a_{\rm No}$ ($a_{\rm No}$ is the original and $a_{\rm N}$ the resulting semi-major axis of the orbit) of the semi-major axis of Neptune's orbit due to the comets of the original comet population being ejected by this planet. The decrease is calculated from equation (5) for the minimum, mean, and maximum estimates of possible orbital energy loss. The present Oort cloud comet mass $M_{\rm KP}$ is given in units of the Earth's mass ($M_\oplus$). The original mass of the Oort cloud comets was larger by factor n. Value k is the ratio of the reciprocal semi-major axis of Neptune, $1/a_{\rm No}$, and of the mean reciprocal semi-major axis, $< 1/a >_{\rm o}$, of the original orbits of the comets in the Uranus-Neptune region. The decrease in the semi-major axis of Neptune would have occured had Neptune only elongated the comets' orbits.

relative change of the semi-major axis of the planet's orbit from relation (9). The values $< 1/a >_{\rm o}$, $< 1/a >_{\rm e}$, $< 1/a >_{\rm s}$ must form a configuration which would render the relative change small. (The portions of the accelerated and decelerated masses, $p_{\rm e}$ and $p_{\rm s}$, are the consequence of the original distribution of the semi-major axes of orbits characterized by the value $< 1/a >_{\rm o}$.)

This conclusion need not be true, if a mutual resonance mechanism were to apply between the planets (Patterson, 1987). The process of comet ejection lasted 1.3 billions years in the case of Neptune (Fernandèz, 1985). As a consequence Neptune was losing its energy slowly, and this loss could have been compensated by the resonance action of the other planets, whereby the resonance would be conserved. The energy loss could have been distributed proportionally to all planets and, therefore, need not have been large. However, this idea is just a speculation. The question of the efficiency of this mechanism remains opened.

We should like to add, that the constraints imposed on the original distribution of orbits would have been negligible, had the total mass of the original comet population been lower than the mass of the planet, which changed the comet orbits. For example, if we assume the Oort cloud mass to be between $\sim$0.1 and $\sim$1 $M_\oplus$ (which is what was assumed since Oort's discovery of the comet cloud until the end of the seventies) and the total mass of the original comet population to be about one order higher, then the absolute values of the energies $|E_{\rm e}|$, $|E_{\rm s}|$ in relation (9) would be low enough to make the change in the relative size of the semi-major axis of Neptune insignificant for any configuration of $< 1/a >_{\rm o}$, $< 1/a >_{\rm e}$, $< 1/a >_{\rm s}$.

3. Conclusion

We have assumed that the sum of the orbital energies of the comets ejected by the perturbing planet, and of the orbital energy of this planet is the same before as well as after the process of ejection.

As a consequence of the energy conservation law, the semi-major axis of the perturbing planet is sensitive to the total mass of all ejected comets. For example, if this mass was equal to 28 Earth's masses (minimum estimate of this mass) and the planets had only elongated the comet orbits, Neptune's semi-major axis would have decreased by about 20%.

The ejection process lasted 1.3 billion years in case of Neptune ejection. Supposing the Neptune's semi-major axis remains unchanged during this period, as it is commonly accepted in most scenarios of the Solar System evolution, some amount of comets directed to shorter-period orbits was needed to balance the influence of energy loss due to ejection of comets to very long-period orbits.

The energy transfer would have been insignificant if the total mass of the original comet population had been much lower than the mass of the perturbing planet. However, this mass was probably higher according to recent estimates of the Oort cloud mass.

Our conclusion could be invalid if some one were to prove the existence of the mutual resonance mechanism between the planets.

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