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Abstract

A simple program for simultaneous solution of light- and RV- curves in binary or triple stellar systems and the underlying assumptions are described.

1 Introduction

The synthesis of light curves and of the spectral changes of eclipsing binaries includes many physically interesting problems, the solution of which requires much larger effort and more computations than simple plane-parallel atmospheres. However, the comparison of such theories with observations needs to find approximate parameters of the observed binary at first, what can be done by an analytic program based on quite simple model. This purpose, as well as practical needs of interpretations of photometric observations led to development of the program FOTEL for solution of photometric elements, which is a supplement to the set of programs (mainly the SPEL for the solution of the spectroscopic elements written by J. Horn, but also the programs for searching for period etc.) used at the Ondřejov observatory for data processing. The PC-version of FOTEL and a brief manual to it are presented in this paper.

2 Physical model

FOTEL is designed to look for eccentric orbits of binary stars with a possible third component. Some orbital elements are well defined by light curves (e.g. inclination i), while others can be determined also (or even much better – e.g. eccentricity e or mass ratio q) from radial velocities. It is thus favourable to solve simultaneously photometric and spectroscopic elements (Wilson 1979). Consequently, the *Type of variable* must be given for each point of the data to be fitted, which determines whether the observed *Value* is RV of primary or secondary component (*Type* $= 1, 2$ – in agreement with SPEL) and it is fitted by

$$RV_{1,2} = K_{1,2}[\cos(\omega + v) + e\ \cos(\omega)] + K_3[\cos(\omega' + v') + e' \cos(\omega')] + \gamma\ , \tag{1}$$

where $K_2 = -K_1/q$, or it is magnitude in a particular filter (*Type* $= 3, ..., 12$). Light-time effect due to motion around the third component is taken into account for both photometric values and RV's. FOTEL is thus generalization of SPEL, however, SPEL is faster for the case of a simple binary orbit, and it is recommended as a zero order approximation even in the cases of some secular changes.

The shapes of both stars are changing during the orbital period due to varying tidal force in the eccentric orbit. In FOTEL it is approximated by the three-axial ellipsoid with the semiaxes

$$A = R\left[1 - \frac{2}{3}\frac{1+q}{q}P^2R^3\right]^{1/2}\left[1 + \frac{R^3}{q}\left(-\frac{2}{r^3} - (1+q)P^2\right)\right]^{-1/2} \tag{2}$$

$$B = R\left[1 - \frac{2}{3}\frac{1+q}{q}P^2R^3\right]^{1/2}\left[1 + \frac{R^3}{q}\left(\frac{1}{r^3} - (1+q)P^2\right)\right]^{-1/2} \tag{3}$$

$$C = R\left[1 - \frac{2}{3}\frac{1+q}{q}P^2R^3\right]^{1/2}\left[1 + \frac{R^3}{qr^3}\right]^{-1/2} \tag{4}$$

in directions toward the companion, perpendicular to it in the orbital plane and perpendicular to the orbital plane, respectively. Here r is the instantaneous distance between centers of the stars, P is the synchronization index, i.e. the ratio of rotational and orbital frequency, and R is effective radius $\left(\simeq (ABC)^{1/3}\right)$. R changes according to

$$\frac{q}{2R^2} + \frac{2}{3}(1+q)P^2R + \frac{4\kappa - 5}{\kappa - 1}\,\frac{1}{2qr^2} = const., \tag{5}$$

where κ is the polytropic index of the star ($p \sim \rho^\kappa$; note, that R is constant for κ=1.25) – cf. Hadrava (1987).

The instantaneous intensity of the star is taken proportional to the area of its apparent disc, i.e. to

$$R_a^2 = [A^2B^2\cos^2 i + (B^2 + (A^2 - B^2)\cos^2(\omega + v))C^2\sin^2 i]^{1/2}\,. \tag{6}$$

The geometry of eclipses is simplified to occultation of two circular discs with apparent radii R_a. A linear limb darkening u is taken into account.

The epoch of periastron is taken in FOTEL as the beginning of phase calculation, however, phases and epochs of RV_{max}, RV_{min}, conjunctions in the direction of the node line and photometric minima are also given in the output. The minima differ from conjunctions due to the relative motion of stars perpendicularly to the node line. Consequently, the closest approach of centers of visible discs occurs if

$$(\omega + v) = e\,\cos\,\omega\,\cot^2 i \mp e^2\,\cos\,\omega\,\sin\,\omega\,\frac{\cos^2 i}{\sin^4 i} + o(e^3)\,, \tag{7}$$

where the signum is given by $\mathrm{sign}(\sin(\omega + v)) = \pm 1$. The effective radii of components in periastron are assumed to be given in the input. The zero-points of light-curves are adjusted to intensities of uneclipsed discs with these radii.

The reflection is included according to the formula

$$L_{ref} = K\frac{BC}{r^2}[\sin\alpha + (\pi - \alpha)\cos\alpha]\,, \tag{8}$$

where

$$\alpha = \arccos[\sin i\sin(\omega + v)] \tag{9}$$

is the angle between the line of sight and the direction toward the companion. The effective monochromatic reflection coefficients K (given in FOTEL in units of apparent luminosity of the binary in the zero-point of the light-curve) are defined by this relation, which can be obtained by integration

$$L_{ref} = \int \Delta I\cos\psi\, d^2\sigma \tag{10}$$

of the increase

$$\Delta I = 2kI_0\cos\theta \tag{11}$$

of intensity (k is the effective albedo, I_0 is the incident intensity, θ is the zenith distance of the companion at the given point of the stellar disc) over the visible ($\cos\psi \geq 0$) part of the illuminated ($\cos\theta \geq 0$) stellar surface

$$d^2\sigma = R^2 d\phi\, d\cos\theta\,. \tag{12}$$

Note, that the term

$$\cos\psi = \cos\alpha\cos\theta + \sin\alpha\sin\theta\cos\phi \tag{13}$$

giving the projection of $d^2\sigma$ to the line of sight has been omitted by Formiggini and Leibowitz (1990; $\cos\gamma$ in their notation), hence their reflection with coefficient $\mu = 1$ actually corresponds to the zero limb darkening. In general, the reflection reduces the limb darkening, what could be taken into account according to Vaz and Nordlund (1985). However, such treatment requires to specify the spectral band of each photometric *Type of variable*, hence the limb darkening $\Delta I = \Delta I(\cos\psi)$ of the reflected light is neglected at all in the present version of FOTEL.

3 Numerical solution

The solution is based on minimization of the sum $\Sigma(O-C)^2$ as a function of the elements (Σ means summation over all points in the *Input file* with their individual weights). The photometry must be given in magnitudes and RV's, which can be given in an arbitrary unit, are divided by the initial value of K_1 at the beginning of each step to be commensurable with photometry for the simultaneous solution. The data can consist of several *Data sets*, for which individual zero-points are calculated (e.g. γ-velocities for different spectrographs like in SPEL, or systematic shifts for differential photometry with respect to different comparison stars). The contributions of separate *Data sets* to the total sum are given at the beginning and the end of each step and each of them can be multiplied by an arbitrarily chosen common weight of the *Data set* to diminish or to strengthen its influence on the solution. Any *Data sets* distinguished in the *Input file* can be merged during the calculation if their common zero-point is required.

The minimization is carried out using the simplex method as it has been described by Kallrath and Linnel (1987). The number of transformations of the simplex in each step is 10-times the number (which is 10 at maximum) of elements being converged in the step. Zero-points of individual *Data sets* and K_1-velocity are calculated directly by least square method and hence they are not included in this number.

Following SPEL, the values E of elements are transformed for the optimization by logarithmic transformation

$$\begin{array}{rcll} P & = & \ln\left(\frac{E-E_{min}}{E_{max}-E}\right) & \text{for} \quad E \in (E_{min}, E_{max}) \\ & = & \ln(E-E_{min}) & \text{for} \quad E_{max} = +\infty \\ & = & E & \text{for} \quad E_{max} = +\infty,\ E_{min} = -\infty \end{array} , \qquad (14)$$

which depicts their physically meaningful region to the whole interval $(-\infty, +\infty)$. This enables to avoid meaningless solutions like those with negative luminosities or with a sum of radii larger than the distance of components etc. Actually just these transformed parameters P are the independent variables in which $\Sigma(O-C)^2$ is minimized. The initial steps of elements are multiplied by $\frac{dP}{dE}$ to give steps of parameters. The values of P's are also displayed in the protocol on the convergence, where the sequential number of simplex operation is followed by its code (A means reflection, B expansion, C contraction and D shrinkage), then by the number of the worst point of the simplex, its value of the sum and transformed values of its coordinates (in the order in which they were introduced in *element-parameters*). Last two rows of the protocol without the code of operation give the number and value of the worst point followed by the final steps of parameters (which are used as initial in the following step) and number, value and parameters of the best point. Values of steps and elements are then transformed back to their physically meaningful form and rewritten.

The built-in *Help* contains further explanations on the use of the program.

References

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