ON THE MOMENTS OF INERTIA AND RADII OF THE WHITE DWARFS AND POLYTHROPIC STARS

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The rotational evolution of the accreting component in close binary system (CBS) is one of the most exciting problems in astrophysics. Different physical forces acting onto the neutron stars were reviewed by Lipunov (1987). Similar problems one may find in the cataclysmic variables (*CV*) with a degenerate white dwarf (*WD*) as a primary star (see. eg. the reviews of Lamb (1985) and Andronov (1987a) on magnetic *CV*'s). However, despite many works were devoted to the 'mass-radius' (*MR*) relation for *WD*'s (eg. Brancewich, 1970; Nauenberg, 1972 and refs. therein), the precise computational results for the moments of inertia I are unfortunately absent, only the characteristic value of $10^{50}\mathrm{g\cdot cm^2}$ is usually assumed.

Here we present the table of the *WD*' parameters including the moments of inertia I. To compute the models, we follow Brancewich (1970), who used the equation of state derived by Salpeter (1961). Thus the equations of the internal structure may be written as follows:

$$dx/dr = - G M_r r^{-2} \rho / (dP/dx),$$

$$dM_r/dr = 4 \pi r^2 \rho,$$

$$dI_r/dr = 8 \pi r^4 \rho / 3,$$

where G is the gravitational constant; M_r and I_r - the mass and the moment of inertia of the part of the star inside a sphere of radius r, respectively; $\rho=Bx^3$ is the local density; B is the constant; $P=P(x)$ - is the pressure being dependent on the dimensionless parameter x. The Runge-Kutta method was used to solve this system of differential equations for the parameter μ_e=2.002 (helium-rich *WD*).

In Table 1, one may find the dependence of the parameters of the *WD* on its mass, including the parameter of the internal structure $\alpha=I/MR^2$. R_{100} is the *WD*'s radius in units of 0.01 $R_\odot$ (=$6.96\cdot10^8$ cm). The value of

α monotonically decreases with an increasing mass, changing from 0.21 to 0.12, while M is in reasonable interval from 0.10 to 1.38 $M_\odot$. The moment of inertia in this sub-interval changes at a factor of 13, being largest at $M \simeq 0.4M_\odot$. It may be approximated by a following best fit expression for $I_{50} = I/10^{50}$ g·cm^2 :

$$I_{50} = 4.095 - 2.795\,(M/M_\odot) - 3.207\cdot\exp(-2.455\cdot M/M_\odot),$$

which is correct within 1 per cent in a wide range from 0.39 to 1.38$M_\odot$. However, the extrapolation till 0.15 $M_\odot$ has the same accuracy, and we may conclude that the written above expression has large systematic errors only at very small masses ($M < 0.1M_\odot$) and near the Chandrasekhar limit. For the mass interval from 0.6 to 1 $M_\odot$, where the largest part of the WD's is being observed, one may use the value $I \simeq 1.7\cdot10^{50}$ g·cm^2 (cf. Andronov, 1987b) with an arbitrary accuracy of 5 per cent.

For the 'M-R' relation we used the form of Nauenberg (1972) and derived the best fit coefficients for the expression

$$R = R_*\cdot(\,(M/M_*)^{-p} - (M/M_*)^{p}\,)^{-q}.$$

Our results for the range $0.1M_\odot \leqslant M \leqslant 1.38M_\odot$ are the following: $M_*=1.44M_\odot$, p=2/3, q=0.465 and $R_*= 0.01153\ R_\odot$ (mean-squared deviation is $7\cdot10^{-6}R_\odot$). Here we changed the value of q and, consequently, R_*. For p= 2/3 and q=1/2, the best fit values of the other coefficients are following : $R_*=0.010616\ R_\odot$ and $M_*= 1.483\ M_\odot$ (mean-squared deviation is $1.2\cdot10^{-4}R_\odot$).

Table 1

Parameters of White Dwarfs as functions of the central value of x

x_c	R_{100}	$M/M_\odot$	I_{50}	α	x_c	R_{100}	$M/M_\odot$	I_{50}	α
0.04	4.789	0.0028	0.1704	0.2751	1.0	1.369	0.4979	1.7522	0.1947
0.06	4.504	0.0067	0.3270	0.2495	1.5	1.091	0.7225	1.5286	0.1843
0.08	4.175	0.0116	0.4625	0.2372	2.0	0.920	0.8833	1.2601	0.1748
0.1	3.888	0.0172	0.5799	0.2313	2.5	0.800	0.9981	1.0283	0.1669
0.2	2.972	0.0544	1.0057	0.2170	3.	0.712	1.0816	0.8436	0.1595
0.3	2.486	0.1018	1.2855	0.2119	4.	0.586	1.1914	0.5860	0.1485
0.4	2.175	0.1556	1.4806	0.2086	5.	0.501	1.2579	0.4254	0.1397
0.5	1.953	0.2131	1.6148	0.2060	6.	0.439	1.3011	0.3207	0.1326
0.65	1.714	0.3016	1.7311	0.2026	8.	0.353	1.3517	0.1990	0.1225
0.8	1.541	0.3887	1.7728	0.1991	10.	0.296	1.3791	0.1345	0.1154
					50.	0.072	1.4370	0.0061	0.0849

For comparison, we computed also the parameters of the polythropic stars, which may be described by the second order Lane-Emden equation :

$$d^2\theta/dr^2 + 2\, r^{-1}\, d\theta/dr = -\,\theta^n$$

(eg. Tassoul, 1978). The values of central pressure P_c, density ρ_c and temperature T_c of the polythropic star may be calculated by using the values of its mass and radius as well as the value of r_o being the root of the expression $\theta(r_o)=0$, and on the derivative $\theta'(r_o)$ at this point. Their values for the polythropic star with the mass and radius equal to that of the Sun are listed in Table 2. For other stars, $P_c \sim \mathcal{M}^2/R^4$, $\rho_c \sim \mathcal{M}/R^3$, and $T_c/\mu \sim \mathcal{M}/R$. Despite the values n > 3 are not physically real (because the total energy is positive, so the configuration is highly unstable), we also included these values for comparison.

Another important parameter of the internal structure, which is proportional to the mean viscosity (Eq.(6) of Press et al.(1975)), is

$$\beta = 14\cdot\mathcal{M}^{-1}R^{-11} \int_0^R \rho(r)\, r^{13}\, dr,$$

where $\rho(r)$ is the local density at the distance r from the stellar centrum. For polythropic star, it may be written as $\beta(n)=7F(n)/2\pi r_o{}^{11}F_{13}(n)$, where

$$F_k(n) = \int_0^{r_o} \theta^n(r)\, r^k\, dr.$$

Table 2

Central pressure, density, temperature and the parameter β for polythropic stars (E k denotes 10^k)

n	P_c	ρ_c	T_c/μ	β
0.0	.134362 E 16	1.40908	.577652 E 7	.238732
0.5	.238869 E 16	2.58587	.559602 E 7	.121159
1.0	.442033 E 16	4.63570	.577657 E 7	.580094 E-1
1.5	.866892 E 16	8.44140	.622125 E 7	.257435 E-1
2.0	.184399 E 17	16.0671	.695258 E 7	.103794 E-1
2.5	.440015 E 17	32.9817	.808206 E 7	.369949 E-2
3.0	.124389 E 18	76.3476	.986997 E 7	.111767 E-2
3.5	.460493 E 18	215.425	.129495 E 8	.266188 E-3
4.0	.278660 E 19	877.025	.192482 E 8	.430423 E-4
4.5	.554017 E 20	8721.48	.384822 E 8	.307183 E-5

Table 3

Parameters of polythropes with different index n

n	r_o	$-\theta'(r_o)$	α	n	r_o	$-\theta'(r_o)$	α
0.0	2.449490	0.816497	0.400000	2.6	5.609382	0.068316	0.103988
0.1	2.504545	.735868	.384264	2.8	6.191322	.054269	.089145
0.3	2.622679	.603761	.354252	2.9	6.526374	.048092	.082119
0.5	2.752698	.499997	.325931	3.0	6.896849	.042430	.075358
0.8	2.973764	.380598	.286206	3.1	7.308484	.037251	.068861
1.0	3.141593	.318310	.261382	3.2	7.768310	.032524	.062630
1.2	3.328870	.266325	.237791	3.4	8.869580	.024320	.050972
1.4	3.538927	.222578	.215378	3.6	10.30159	.017614	.040397
1.5	3.653754	.203301	.204600	3.8	12.23351	.012232	.030922
1.6	3.775905	.185544	.194102	4.0	14.97155	.008018	.022574
1.8	4.045010	.154067	.173933	4.2	19.13161	.004833	.015386
2.0	4.352875	.127249	.154849	4.4	26.15892	.002553	.009408
2.2	4.708071	.104380	.136837	4.6	40.41324	.001060	.004718
2.4	5.121870	.084891	.119885	4.8	83.81284	.000245	.001454

REFERENCES

Andronov I.L., 1987a, Astrophys. Space Sci., 131, 557.

Andronov I.L., 1987b, Astron. Zhurn., 64, 97 (Transl.:Soviet Astronomy)

Brancewicz H., 1970, Acta Astron., 20, No. 4, 383.

Chandrasekhar S., 1939, An Introduction to the Study of Stellar Structure

Lamb D.Q., 1985, in:'Catacl.Variables and Low-Mass X-ray Binaries', ed. D.Q.Lamb and J.Patterson, D.Reidel, p. 179.

Lipunov V.M., 1987, Astrophysics of Neutron Stars (in Russian), Moscow, 'Nauka', 296 pp.

Nauenberg M., 1972, Astrophys. J., 175, 417.

Press W.H., Wiita P.J., Smarr L.L., 1975, Astrophys.J., 202, L 135.

Salpeter E.E., 1961, Astrophys.J., 134, 669.

Tassoul J.L., 1978, Theory of Rotating Stars, Princeton, NJ, 472 pp.