THE STRAIGHTFORWARD SOLUTION OF LEVEL POPULATIONS IN PURE HYDROGEN PLASMA

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Abstract

The populations of $n-$states in pure hydrogen plasma are computed by explicit use of all formulae for radiative bound-bound and bound-free transitions in hydrogen atom surrounded by a Planckian radiation field of given temperature. Introduction of a new method called level grouping makes it possible to overcome large demands on computing time and memory capacity due to inclusion of very high (up to 500) levels.

The algorithm was tested for the electron temperature $T_e = 10^4$ K, electron density $N_e = 10^4$ cm^{-3} and the Planckian radiation temperatures $T = 2 \cdot 10^4$, $4 \cdot 10^4$ and $8 \cdot 10^4$ K corresponding to conditions in H_{II} regions. Comparison of our results with Seaton's cascade-matrix solution (independent of the incident radiation) of an optically thin nebula shows that the new method gives significantly higher departure coefficients for all temperatures of radiation. The importance of considering also the upward radiative transitions is thus stressed. That suggests that the Seaton's cascade-matrix method can be understood as an approximation corresponding to the radiation with infinite temperature.

1 Introduction

Observations of radio frequency recombination lines emitted in H_{II} regions lead to the conclusion that these high-level transition lines are formed under conditions considerably different from LTE. Hoang Binh (1985) and Martín Pintado et al. (1989) suppose that observed differences could be explained by a line amplification resulting from a maser emission in diluted hydrogen plasma. To verify such a explanation it is necessary to determine populations of a large number of quantum levels. For simplicity we confine ourselves to the pure hydrogen plasma.

The problem of the calculation of the statistical equilibrium produced in an assembly of hydrogen atoms in gas nebulae is very complex and has been solved by various approximations since the beginning of this century. In a number of papers (e.g. Baker and Menzel 1938, Seaton 1959, Burgess 1958) a simplified model of the hydrogen atom is described, in which the influence of collisional transitions on the population of discrete levels is neglected. This case has been solved in two approximations introduced by Baker and Menzel (1938).

In their case A, which is suitable for nebulae optically thin in Lyman lines, an hydrogen atom, whose n-th level is populated by recombination of free electrons and cascade from higher levels and depopulated by radiative transitions to lower levels is treated. One neglects the possibility of excited states being populated by absorption of radiation produced in the star and nebula itself.

Case B is better suited for nebulae, which are very opaque to Lyman line radiation. One assumes that the absorption of a Lyman quantum originated in the nebula is counterbalanced by its spontaneous emission. The flux of stellar Lyman line radiation is neglected. In both cases the only source of free electrons is photoionization from the first level.

Later papers (Pengelley 1964, Seaton 1964, Brocklehurst 1970) take into account also the excitation of discrete levels caused by collisions, but again only in A and B approximations.

All solutions are based on the idea of the cascade matrix introduced by Seaton (1959), which, however, does not involve the influence of incident stellar radiation and neglects all excitations and ionizations in atom. Therefore a simple model of the hydrogen plasma with the given electron density and temperature surrounded by the diluted Planckian radiation field has been devised. The model includes all radiative

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bound-bound and bound-free transitions in hydrogen atom. The collisional transitions are neglected. The details of numerical solution will be published elsewhere, this paper should only draw attention to problems connected with the cascade matrix approximation.

2 Basic Steps of the Solution

The solution of the model consists of following computational steps:

- Computation of rates for bound—bound transitions (including photoexcitations) between finite number of bound levels. The explicit expression for Gaunt factors according Menzel and Pekeris (1935) are used.
- Computation of rates for bound—free transitions between bound levels and the continuum. Photoionization rates are obtained by the direct numerical integration of the absorption cross-section over an infinite range of electron energies, recombination rates follows from a similar integration of recombination coefficient.
- Solution of equations of statistical equilibrium. The system of equations of detailed balance for each level is written in matrix form and then solved by Gauss elimination method. Thus the level populations, corresponding to the normalization condition defining the sum of all level and continuum populations equal to unity, are obtained.
- Computation of b-factors, showing the departure from LTE population of given level.

3 The Method of Level Grouping

The solution of equations of statistical equilibrium (ESE) for a large number of levels is limited by the available computer memory and execution speed. The solution of ESE for n levels requires $\approx n^3$ floating point operations and the storage of such a system occupies an array of $n \times n$ real numbers. Therefore the explicit solution for several hundred levels, even on large supercomputers, is practically impossible. This is why the method of level grouping has been devised.

The main idea of the method lies in coupling of several neighbouring levels into one single group, for which its population is being solved. For each pair of groups the rates of transitions between groups are defined analogically to those between levels. Thus it suffices to solve the ESE only for much smaller number of groups.

Inside each group the distribution of populations has been assumed to be the same as under LTE with some temperature called a group temperature. Having solved the ESE system for groups, we calculate new temperatures from their occupation numbers and this procedure is repeated. Having reached the required precision the populations of levels are finally determined supposing that the Boltzmann distribution holds inside the group.

4 Results of Calculations

4.1 The LTE Test of Accuracy

The accuracy of the method described above was examined by means of the LTE model, characterized by dilution factor equal to unity and uniform electron and radiation temperature $T = T_e$, which should give all departure coefficients equal to unity. The calculations were accomplished with 500 atom levels distributed in various manners into groups. The number of groups was restricted to 200 due to the available memory storage. Therefore the first test was carried out with 200 separate levels and one group containing levels 201–500. In the next test the bound levels were divided into 64 groups such that the difference of excitation energy between the first and the last level in the group was the same. The last group has always represented the continuum. The physical parameters of the models were chosen similar to those in solar photosphere: $N_e = 10^{12}$ cm^{-3}, $T = T_e = 6000$ K. In both cases the computed departure coefficients were (for all levels) equal to unity with accuracy better then seven decimal places.

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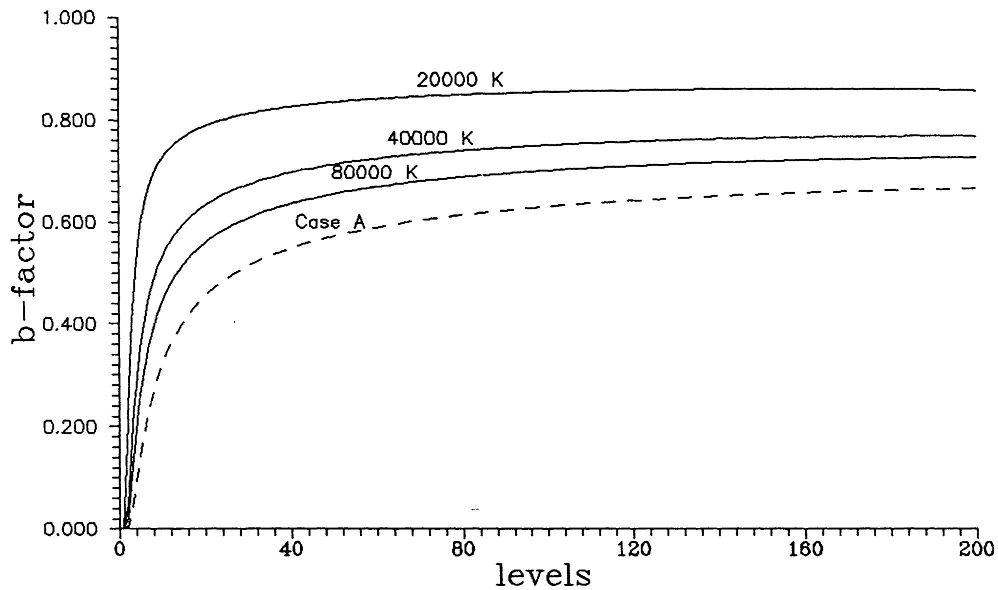
Figure 1: Level populations for various temperatures of incident radiation



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4.2 Non-LTE Models

The non-LTE model of a typical H_{II} region has been computed for the Planckian temperature of incident radiation equal to 20 000 K, 40 000 K and 80 000 K, and the electron temperature 10 000 K. The dilution factor was $W = 10^{-14}$ and the electron density $N_e = 10^4$ cm^{-3}.

Results of calculations are shown in Figure 1 together with results of our calculations simulating Seaton's cascade-matrix solution of case A. Our method was modified for this case by neglecting all photoexcitations and photoionizations from all levels with the exception of the first one.

It follows that the departure coefficients for all radiation temperatures are higher than those for Seaton's method (independent of incident radiation). That suggests that the widely employed Seaton's cascade-matrix solution can be accepted as the rough approximation valid for the very high temperatures of incident radiation.

5 Conclusions

The calculations have shown that the influence of the incident radiation on the level populations in the optically thin H_{II} region is not negligible. The Seaton's cascade-matrix solution of such a problem seems to be a rough approximation, the use of which is limited by very high temperatures of radiation. A computer program for the precise calculation of even very high level populations for all radiative transitions in pure hydrogen plasma was designed, but for more detailed discussion and a comparison with observations the collisional rates and the energy balance equation must be included in future.

Acknowledgements. I should like to thank Dr. P. Hadrava for many helpful discussions concerning the method of level grouping.

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