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ON THE DISTRIBUTION OF COMETARY ORBITS OF EXTREMELY SHORT PERIHELION DISTANCES

Abstract: The space distribution of perihelia and orbital planes of 105 long-periodic comets with perihelion distances $q < 0.5$ A. U. is investigated. The adequateness of the ellipsoidal analysis for this purpose is briefly discussed.

The perihelia show a marked concentration in a narrow zone near $\lambda = 270-280°$ on both sides of the ecliptic, not far from the galactic centre and the vertex. There is neither a concentration of the aphelia near the apex, as postulated by von Niessl for an interstellar origin of comets, nor any crowding of the lines of apses along the hypothetical accretion axes, as required by the Lyttleton's theory. The unequal number of observers on the northern and southern hemisphere introduces a slight prevalence of comets with positive latitudes of perihelia. There is good agreement with the results obtained previously by Oppenheim and Witkowski but it is questionable whether this may be interpreted as direct evidence of an association with the galactic structure.

The most interesting feature of the distribution of orbital planes is a relative abundance of lower inclinations. The discussion of the discovery conditions leads to the conclusion that the probable reason of this phenomenon consists in the relation between the heliocentric latitude and brightness of comets rather than in the perturbation effects of the major planets or in another effect of selection.

1. *Introduction*

The space distribution of the orbits of long-periodic comets has hitherto been investigated by a number of authors, owing to its significance for the theories of cometary origin and evolution. However, there is no definitive agreement as to the interpretation of the results. The indicated departures from the randomness, as far as they are admitted at all, are attributed by one group of the authors to the interstellar origin of comets or at least to the influence of the gravitational field of the Galaxy, or to the preferential motions of the stars encountered by the solar system. The other group tries to find the source of the non-uniformity in the effects of selection, such as the seasonal effects bringing about the variability of the discovery conditions with the orbital elements.

As early as in the second half of the preceding century Hoek [1] and Schiaparelli [2] drew attention to the fact that the lines of apses of known cometary orbits are not distributed in the space in a completely random fashion. Hoek found several groups of comets showing approximative coincidence in the direction of the aphelia and inferred the possibility of a common origin of the members of these groups; further, he found a zone of avoidance („Hoek's zone") situated just north of the ecliptic between the longitudes 95° and 243° where practically no aphelia occurred. Schiaparelli demonstrated a crowding of the aphelia of comets with extremely short perihelion distances in the region about $\alpha = 72°$, $\delta \doteq +48°$. Similarly, Mohn [3] examined the orientation of the orbital planes and found a preferential direction of the poles towards $\lambda = 2°$, $\beta = +10°$ and $\lambda = 182°$, $\beta = -10°$.

More recently, the problem was mostly treated by the ellipsoidal method of analysis, and in the orientation of the main axes of the distribution elipsoids the reflections of the fundamental planes or directions, such as the ecliptic, galactic plane, apex, or vertices, were sought. Svedstrup [4], Oppenheim [5], and Witkowski and Hurnik [6], using successively more and more extensive data, found a relative abundance of the lines of apses in the direction of the vertices, which favours the opinion of a direct association of cometary orbits with the star streaming. On the basis of this result Witkowski attempts to find a suitable process of the capture of comets (or of particles from which they are additionally built by aggregation) by the solar system from the interstellar space, and to remove the two chief arguments of Strömgren and Schiaparelli against the interstellar origin of comets: the absence of hyperbolic orbits and the condition of zero relative velocity. Merton [7] suggests another explanation of the phenomenon, consisting in the perturbing action of the neighbouring stars on the swarm of comets situated at the boundary of the solar system, in conformity with Oort's hypothesis [8]. In this conception the preferential motion of the stars encountered by the solar system may have impressed some features of the star streaming on the orbits of comets without the necessity of their interstellar origin.

On the contrary, Bourgeois and Cox [9] have obtained a distribution of perihelion points apparently associated not with the galactic plane but with the plane of the ecliptic. This is sometimes quoted in the literature as a direct contradiction of the views of the above-mentioned authors. However, it must be emphasized that the problem treated by Bourgeois and Cox essentially differs from that investigated by Svedstrup, Oppenheim, and Witkowski: instead of dealing with the distribution of the perihelia over the sphere they investigate the correlation between the orientation of the lines of apses and the perihelion distances. Consequently, there is no disagreement in the numerical results or in the methods of elaboration but merely in putting the problem. To Bourgeois and Cox the merit is due of pointing out the importance of the discovery conditions which exercise influence on the resulting statistical distribution. This effect of selection in its different forms must not be overlooked or underestimated.

The significance of the problem of orientation of cometary orbits has newly arisen in connexion with the two new theories of the origin of comets proposed during the last decade by Oort [8] and Lyttleton [10]. According to Lyttleton's theory, the orbits of „new" comets ought to be associated in some way with the accretion axes along which they have been generated. The absence of such arrangement, which fits well to the evolutionary process suggested by Oort, represents at the same time a serious argument against the accretion theory of Lyttleton. However, the validity of this argument is disputable under certain conditions, as was shown by McCrea [11].

The present paper deals with the space distribution of the orbits of long-periodic comets with extremely short perihelion distances, $q < 0.5$ A. U. This restriction of the data has been introduced for the following reasons:

(1) In a given heliocentric distance the aphelion velocity is the lower the shorter the perihelion distance. If we assume, in agreement with Oort, that the comets move primarily in less eccentric orbits in the outer part of the solar system and enter the region of the planets only after the tangential component of their velocity has been substantially reduced by the perturbations, then the limitation of q introduces a selection of just those comets which have been mostly affected by such perturbations. On the other hand, from the standpoint of the accretion theory (considered by the writer as much less probable) the comets of shortest perihelion distances are those which have acquired least angular momentum by the action of the major planets and have consequently remained closest to the accretion axes. In each case, the selection concerns the extreme orbits.

(2) The limitation of the perihelion distances may also partially remove the effects of selection entering the statistics of orbits as a consequence of the dependence of the discovery conditions upon their shape and orientation. For instance it may help to separate the phenomenon found by Bourgeois and Cox from that one associated by Svedstrup, Oppenheim and Witkowski with the star streaming. The results of this separation may appear just in comparison with the results of Witkowski which are based on the statistics of all long-periodic orbits, complete almost up to date, irrespective of q.

It must be pointed out that the numerical value of the upper limit, $q = 0.5$, is entirely convetional. It has been endeavoured to keep the limit as low as possible without seriously restraining the statistical importance of the available data. For the very same reason no discrimination according to the periods of revolution was attempted except the omission of the short-periodic comets with $T < 200$ years. As a matter of fact, for a great proportion of the comets in question only parabolical orbits are available and the actual periods are indeterminate. Although it would be desirable to distinguish at least between the „old" and „new" comets, it is generally possible neither on the basis of the orbital elements nor of the photometric characteristics.

2. *The Principal Data*

The general catalogue of cometary orbits by Baldet and Obaldia [12] includes the elements of 111 different comets with $q < 0.5$; one comet more (1954 II) was discovered since the publication of the catalogue. Leaving out five short-periodic comets (Encke, Brorsen—Metcalf, 1766 II, 1833 II, and 1917 I) and two ancient comets for which the elements are known with limited accuracy or ambiguously (those of A. D. 240 and 539) a collection of 105 orbits was obtained as a basis for the statistical investigation. There are 21 comets with $0.0 < q < 0.1$, 15 comets with $0.1 < q < 0.2$, 17 comets with $0.2 < q < 0.3$, 26 comets with $0.3 < q < 0.4$, and 26 comets with $0.4 < q < 0.5$. Five or six bodies of the first named group are members of the Kreutz group of sun-grazing comets.

As the ordinary orbital elements are not suitable enough for a direct statistical treatment they have been converted into the heliocentric ecliptical co-ordinates and direction cosines of the perihelia and poles. These are mutually connected by the following relations:

$$\begin{aligned} \cos\lambda\cos\beta &= \ \ l = \cos\omega\cos\Omega - \sin\omega\sin\Omega\cos i \\ \sin\lambda\cos\beta &= m = \cos\omega\sin\Omega + \sin\omega\cos\Omega\cos i \\ \sin\beta &= \ n = \sin\omega\sin i \qquad (1) \\ \cos\lambda'\cos\beta' &= \ \ l' = \sin\Omega\sin i \\ \sin\lambda'\cos\beta' &= m' = -\cos\Omega\sin i \qquad (2) \\ \sin\beta' &= \ n' = \cos i \end{aligned}$$

where the letters without suffices relate to the perihelia and those with suffices to the poles from which the comets are seen to revolve in the positive direction[1]. Obviously, the elements have to be referred to a common equinox and ecliptic; in our case 1950.0 was chosen. The results of the reductions are summarized in Table I (elements and ecliptical co-ordinates) and Table II (direction cosines). A great deal of the necessary computations was carried out by Mr. A. Aldor whose kind assistance is gratefully acknowledged.

3. *The Method of Analysis*

There are several different methods which may be applied to express numerically the irregularities of a distribution of directions like that of the lines of apses or orbital planes. In our case the number of orbits concerned is too low to favour a group method, such as the statistics in the Charlier's areas which was applied to a similar problem (however, with the use of more extensive data) by Bourgeois and Cox [9]. Of the individual methods the construction of the distribution ellipsoid is mostly common. However, it happens that this treatment is sometimes used without desirable precautions, i. e. without considering whether the ellipsoidal distribution represents a satisfactory approximation to the actual state at all. For instance if the comets were associated into groups with roughly coinciding lines of apses, as pointed out by Hoek [1] and postulated by McCrea [11], the results obtained by means of the ellipsoidal analysis would be quite illusory. Still another feature of the distribution seems to have been frequently overlooked or underestimated. In the method usually adopted the position of the plane of extreme concentration is investigated irrespective of the distribution to both sides of the plane, thus making the linear terms in the ellipsoid equation vanish. This leads to the omission of the assymetry of the distribution with respect to the sun and the results, although mathematically correct, need not represent the actual state truly enough.

The determination of the anisotropy of the distribution is based on Lagrange's method of undetermined multipliers for finding the constrained extremals. The spherical distance ϑ of a point having the direction cosines ξ, η, ζ from the perihelion of a given orbit is defined by the relation

$$\cos\vartheta = l\xi + m\eta + n\zeta \qquad (3)$$

The general task is to find the extremal of the sum of a certain function $F(\vartheta)$, extending over all orbits concerned, under the constraint

$$\xi^2 + \eta^2 + \zeta^2 = 1 \qquad (4)$$

The condition that the sum of the squares of the distances from a plane passing through the origin should be maximum,

$$F(\vartheta) = \cos^2\vartheta, \qquad (5)$$

leads to the distribution ellipsoid having the equation

$$a_{11}x^2 + a_{22}y^2 + a_{33}z^2 + 2a_{23}yz + 2a_{13}xz + 2a_{12}xy = 1 \qquad (6)$$

where

$$\begin{aligned} a_{11} &= \frac{1}{N}\Sigma\, l^2, & a_{23} &= \frac{1}{N}\Sigma\, mn, \\ a_{22} &= \frac{1}{N}\Sigma\, m^2, & a_{13} &= \frac{1}{N}\Sigma\, ln, \qquad (7) \\ a_{33} &= \frac{1}{N}\Sigma\, n^2, & a_{12} &= \frac{1}{N}\Sigma\, lm, \end{aligned}$$

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[1] The signs of l' and m' are reversed in the quoted paper by Oppenheim [5].

Table I

No.	B.—O.	Comet	ω	Ω	i	q	λ	β	λ′	β′
1	360	1880 I	86°.2	7°.1	144°.7	0.005	281°.7	+35°.2	277°.1	−54°.7
2	218	1843 I	82.6	2.8	144.3	0.006	281.9	+35.3	272.8	−54.4
3	702	1945 VII	50.9	321.7	137.0	0.006	279.7	+32.0	231.7	−47.0
4	375	1882 II	69.6	347.0	142.0	0.008	282.2	+35.2	257.0	−52.0
5	396	1887 I	58.3	325.5	128.5	0.010	280.2	+41.8	235.5	−38.5
6	84	1680	350.6	275.9	60.7	0.006	271.3	−8.2	186.0	+29.3
7	308	1865 I	111.7	253.4	92.5	0.026	79.7	+68.1	163.4	−2.5
8	196	1826 V	279.6	236.9	90.6	0.027	240.6	−80.4	146.9	−0.6
9	90	1695	59.1	285.3	93.6	0.042	279.4	+58.9	195.3	−3.6
10	236	1847 I	254.4	23.1	48.7	0.043	270.2	−46.3	293.1	+41.4
11	338	1874 I	269.5	31.4	58.9	0.045	300.4	−58.9	301.4	+31.1
12	172	1816	304.3	325.1	43.1	0.048	278.2	−34.4	235.1	+46.9
13	374	1882 I	209.0	205.9	73.8	0.061	34.7	−27.7	115.9	+16.2
14	330	1872	63.4	48.1	148.4	0.064	348.5	+27.9	318.0	−58.5
15	89	1689	78.1	283.1	63.2	0.064	348.1	+60.9	193.1	+26.8
16	80	1668	109.8	2.5	144.4	0.067	248.6	+33.2	272.5	−54.4
17	—	1954 II	94.1	114.6	13.6	0.072	208.8	+13.5	24.7	+76.4
18	620	1931 IV	168.2	101,8	169.3	0.074	293.3	+2.2	11.7	−79.3
19	70	1593	12.1	169.2	87.9	0.089	169.7	+11.8	79.2	+2.2
20	180	1821	169.2	50.5	106.5	0.092	233.6	+10.4	320.5	−16.5
21	129	1780 I	237.1	126.1	125.6	0.096	264.1	−43.1	36.1	−35.6
22	79	1665	156.1	232.0	103.9	0.106	58.1	+23.1	142.0	−13.9
23	720	1947 XII	196.2	336.6	138.5	0.110	144.4	−10.6	246.7	−48.5
24	121	1769	329.1	177.6	40.8	0.123	153.2	−19.6	87.6	+49.2
25	202	1830 II	26.9	339.6	135.3	0.126	319.7	+18.6	249.6	−45.3
26	508	1910 I	320.9	89.3	138.8	0.129	120.8	−24.6	359.3	−48.8
27	732	1948 XI	107.3	210.4	23.1	0.135	319.0	+22.0	120.4	+66.9
28	199	1827 III	258.7	151.4	125.9	0.138	260.2	−52.6	61.4	−35.9
29	252	1851 IV	294.4	45.7	74.0	0.142	14.5	−61.1	315.7	+16.0
30	67	1582	57.0	18.3	34.6	0.169	70.0	+28.4	288.3	+55.4
31	260	1853 IV	277.8	221.4	119.0	0.173	295.6	−60.1	131.6	−29.0
32	601	1927 IX	47.2	77.5	85.1	0.176	82.8	+46.9	347.5	+4.9
33	65	1577	255.7	30.5	104.9	0.178	165.4	−69.5	300.6	−14.9
34	194	1826 III	4.8	42.3	174.7	0.188	37.6	+0.4	312.3	−84.8
35	441	1895 IV	272.7	321.3	141.6	0.192	47.9	−38.3	231.3	−51.6
36	701	1945 VI	216.7	325.5	49.5	0.194	171.3	−27.0	235.5	+40.5
37	285	1859	282.0	358.6	95.5	0.201	22.8	−76.8	268.6	−5.5
38	725	1948 IV	317.1	203.2	23.2	0.208	162.6	−15.6	113.2	+66.8
39	112	1758	36.8	233.5	68.3	0.215	248.9	+33.8	143.5	+21.7
40	489	1906 I	199.3	92.7	43.7	0.215	286.9	−13.1	2.7	+46.4
41	106	1744	151.5	48.6	47.1	0.222	208.3	+20.5	318.6	+42.9
42	100	1737 I	99.5	229.4	18.3	0.223	329.4	+18.1	139.4	+71.7
43	185	1823	28.5	304.8	103.8	0.227	297.4	+27.6	214.8	−13.8
44	470	1901 I	203.0	110.3	131.1	0.245	274.7	−17.2	20.3	−41.1
45	223	1844 III	177.7	119.8	45.6	0.252	298.2	+1.6	29.8	+44.4
46	156	1801	219.8	44.6	159.3	0.256	186.6	−13.1	314.6	−69.3
47	412	1890 I	199.9	9.3	56.7	0.270	200.5	−16.5	279.3	+33.3
48	391	1886 V	201.3	193.5	87.7	0.270	14.4	−21.3	103.5	+2.3
49	262	1854 II	101.6	316.8	97.5	0.277	169.1	+76.2	226.8	−7.5
50	82	1677	99.2	240.6	100.9	0.281	110.2	+75.7	150.6	−10.9
51	64	1558	119.6	340.5	110.9	0.281	192.7	+54.3	250.5	−20.9
52	288	1860 III	76.9	85.9	79.3	0.293	124.4	+73.1	355.9	+10.7
53	558	1920 I	276.6	316.0	123.2	0.298	34.1	−56.3	226.0	−33.2
54	516	1911 IV	71.7	89.2	96.5	0.303	70.4	+70.6	359.2	−6.5
55	259	1853 III	170.4	141.9	61.5	0.307	317.3	+8.4	51.9	+28.5
56	40	1299	103.9	116.2	111.0	0.318	351.6	+65.0	26.2	−21.0
57	242	1848 I	261.0	213.0	95.6	0.320	1.5	−79.4	123.0	−5.6
58	588	1926 III	354.8	282.8	123.0	0.323	285.6	−4.3	192.8	−33.0
59	462	1899 I	8.7	25.7	146.3	0.327	18.4	+4.8	295.7	−56.3
60	62	1533	278.3	305.2	28.3	0.327	224.6	−28.0	215.2	+61.7
61	50	1449	356.7	268.1	155.7	0.327	271.1	−1.4	178.1	−65.7
62	241	1847 VI	276.6	192.3	108.1	0.329	261.8	−70.7	102.3	−18.1
63	478	1903 IV	127.3	294.2	85.0	0.330	107.7	+52.4	204.2	+5.0
64	314	1867 III	148.6	66.1	96.6	0.330	250.1	+31.1	336.2	−6.6
65	88	1686	81.9	357.7	35.0	0.336	77.9	+34.6	267.7	+55.0
66	111	1757	268.7	217.0	12.8	0.337	125.6	−12.8	127.0	+77.2
67	177	1819 II	13.4	275.5	80.8	0.342	277.7	+13.3	185.5	+9.2
68	139	1787	99.2	109.1	131.7	0.349	5.5	+47.5	19.1	−41.7
69	362	1880 III	323.1	46.3	141.9	0.355	76.8	−21.8	316.3	−51.9
70	272	1857 III	134.1	25.0	121.0	0.368	233.0	+38.0	295.0	−31.0
71	670	1941 I	199.6	295.9	49.9	0.368	128.8	−14.9	205.9	+40.1
72	48	1402	91.1	124.6	55.0	0.380	216.5	+54.9	34.6	+35.0

Continuation of Table I

No.	B.—O.	Comet	ω	Ω	i	q	λ	β	λ'	β'
73	335	1873 V	233.7	177.8	121.5	0.385	322.3	−43.4	87.8	−31.5
74	59	1506	242.3	139.1	134.9	0.386	265.7	−38.8	49.1	−44.9
75	364	1880 V	73.5	258.6	129.2	0.387	193.7	+48.2	168.6	−39.0
76	324	1870 IV	90.6	95.9	147.3	0.389	5.2	+32.7	5.9	−57.3
77	74	1618 II	287.4	80.3	37.2	0.390	11.8	−35.2	350.3	+52.8
78	163	1808 I	253.7	324.9	134.3	0.390	77.6	−43.4	235.0	−44.3
79	419	1891 I	178.8	194.8	120.4	0.398	15.4	+1.1	104.8	−30.4
80	474	1902 III	153.0	50.0	156.4	0.401	255.1	+10.5	320.1	−66.3
81	226	1845 III	75.8	339.3	131.1	0.401	270.3	+46.9	249.3	−41.1
82	147	1793 I	239.8	110.7	119.6	0.403	250.3	−48.7	20.7	−29.7
83	572	1924 II	66.6	80.4	120.1	0.406	31.2	+52.5	350.4	−30.1
84	525	1913 I	279.3	303.4	80.4	0.407	257.9	−76.6	213.4	+9.6
85	615	1930 VII	62.8	229.6	4.2	0.408	292.3	+3.7	139.6	+85.8
86	475	1903 I	133.7	3.0	30.9	0.411	141.1	+21.8	273.0	+59.1
87	138	1786 II	325.0	196.7	50.9	0.411	172.9	−26.5	106.8	+39.1
88	460	1898 IX	162.4	35.6	28.9	0.420	200.0	+8.4	305.6	+61.2
89	95	1706	59.4	16.6	55.3	0.427	60.6	+45.0	286.6	+34.7
90	136	1785 II	127.2	67.0	92.6	0.427	250.4	+52.7	336.8	−2.6
91	77	1661	33.4	85.9	33.0	0.443	114.9	+17.5	355.9	+57.0
92	540	1915 IV	118.8	78.2	53.5	0.443	211.0	+44.8	348.2	+36.5
93	371	1881 VI	6.3	275.1	112.8	0.449	272.7	+5.8	185.1	−22.8
94	472	1902 I	228.4	52.9	66.5	0.451	257.1	−43.3	322.9	+23.5
95	206	1833	260.8	325.2	7.3	0.464	225.9	−7.2	235.2	+82.7
96	44	1362	9.9	245.1	148.0	0.470	236.7	+5.2	155.1	−5.8
97	388	1886 II	119.6	69.2	84.4	0.479	239.5	+59.9	339.2	+5.6
98	611	1930 III	47.0	90.5	67.1	0.482	113.1	+42.3	0.5	+22.9
99	152	1798 I	343.0	124.2	43.8	0.485	111.8	−11.7	34.2	+46.2
100	55	1472	245.8	292.2	170.9	0.486	46.7	−8.3	202.2	−80.9
101	517	1911 V	153.0	293.5	33.8	0.489	90.6	+14.6	203.5	+56.2
102	63	1556	100.9	180.7	32.4	0.491	283.6	+31.7	90.7	+57.6
103	49	1433	189.4	103.6	104.0	0.493	281.3	−9.1	13.6	−14.0
104	117	1763	88.6	359.0	72.6	0.498	84.3	+72.5	269.0	+17.5
105	477	1903 III	184.9	213.8	66.5	0.499	35.8	−4.5	123.8	+23.5

Table II

No.	B.—O.	Comet	l	m	n	l'	m'	n'
1	360	1880 I	+0.1653	−0.7998	+0.5771	+0.0713	−0.5739	−0.8158
2	218	1843 I	+0.1679	−0.7986	+0.5780	+0.0288	−0.5821	−0.8126
3	702	1945 VII	+0.1423	−0.8364	+0.5293	−0.4227	−0.5349	−0.7316
4	375	1882 II	+0.1730	−0.7982	+0.5770	−0.1390	−0.5998	−0.7880
5	396	1887 I	+0.1324	−0.7336	+0.6666	−0.4435	−0.6453	−0.6221
6	84	1680	+0.0229	−0.9895	−0.1421	−0.8672	−0.0904	+0.4896
7	308	1865 I	+0.0667	+0.3662	+0.9281	−0.9575	+0.2848	−0.0436
8	196	1826 V	−0.0819	−0.1454	−0.9859	−0.8371	+0.5467	−0.0111
9	90	1695	+0.0839	−0.5092	+0.8565	−0.9626	−0.2637	−0.0625
10	236	1847 I	+0.0018	−0.6910	−0.7228	+0.2949	−0.6903	+0.6607
11	338	1874 I	+0.2616	−0.4458	−0.8561	+0.4456	−0.7310	+0.5168
12	172	1816	+0.1180	−0.8172	−0.5643	−0.3906	−0.5606	+0.7302
13	374	1882 I	+0.7279	+0.5037	−0.4654	−0.4195	+0.8639	+0.2790
14	330	1872	+0.8659	−0.1767	+0.4679	+0.3891	−0.3498	−0.8522
15	89	1689	+0.4763	−0.1005	+0.8735	−0.8695	−0.2018	+0.4509
16	80	1668	−0.3051	−0.7788	+0.5481	+0.0256	−0.5820	−0.8128
17	—	1954 II	−0.8516	−0.4691	+0.2340	+0.2132	+0.0979	+0.9721
18	620	1931 IV	+0.3952	−0.9177	+0.0378	+0.1818	+0.0378	−0.9826
19	70	1593	−0.9621	+0.1749	+0.2091	+0.1867	+0.9817	+0.0378
20	180	1821	−0.5841	−0.7915	+0.1797	+0.7398	−0.6103	−0.2832
21	129	1780 I	−0.0755	−0.7266	−0.6828	+0.6572	+0.4787	−0.5821
22	79	1665	+0.4860	+0.7807	+0.3930	−0.7650	+0.5973	−0.2405
23	720	1947 XII	−0.7988	+0.5724	−0.1848	−0.2625	−0.6081	−0.7491
24	121	1769	−0.8413	+0.4242	−0.3350	+0.0274	+0.6524	+0.7574
25	202	1830 II	+0.7235	−0.6125	+0.3183	−0.2460	−0.6596	−0.7102
26	508	1910 I	−0.4654	+0.7814	−0.4156	+0.6588	−0.0076	−0.7522
27	732	1948 XI	+0.6999	−0.6079	+0.3749	−0.1984	+0.3383	+0.9197
28	199	1827 III	−0.1038	−0.5988	−0.7941	+0.3881	+0.7108	−0.5867

Continuation of Table II

No.	B.−O.	Comet	l	m	n	l'	m'	n'
29	252	1851 IV	+0.4686	+0.1209	−0.8751	+0.6883	−0.6709	+0.2759
30	67	1582	+0.3002	+0.8264	+0.4763	+0.1787	−0.5392	+0.8230
31	260	1853 IV	+0.2158	−0.4502	−0.8665	−0.5790	+0.6555	−0.4848
32	601	1927 IX	+0.0854	+0.6772	+0.7308	+0.9729	−0.2148	+0.0854
33	65	1577	−0.3397	+0.0885	−0.9364	+0.4913	−0.8323	−0.2569
34	194	1826 III	+0.7928	+0.6095	+0.0077	+0.0618	−0.0678	−0.9958
35	441	1895 IV	+0.5261	+0.5818	−0.6202	−0.3884	−0.4844	−0.7839
36	701	1945 VI	−0.8806	+0.1344	−0.4545	−0.4310	−0.6262	+0.6497
37	285	1859	+0.2101	+0.0885	−0.9736	−0.0240	−0.9951	−0.0956
38	725	1948 IV	−0.9194	+0.2881	−0.2681	−0.1548	+0.3617	+0.9194
39	112	1758	−0.2982	−0.7755	+0.5563	−0.7470	+0.5524	+0.3697
40	489	1906 I	+0.2827	−0.9318	−0.2278	+0.6895	+0.0323	+0.7236
41	106	1744	−0.8247	−0.4443	+0.3499	+0.5497	−0.4847	+0.6803
42	100	1737 I	+0.8181	−0.4841	+0.3102	−0.2388	+0.2047	+0.9492
43	185	1823	+0.4082	−0.7866	+0.4634	−0.7973	−0.5545	−0.2388
44	470	1901 I	+0.0784	−0.9523	−0.2950	+0.7069	+0.2617	−0.6572
45	223	1844 III	+0.4723	−0.8810	+0.0283	+0.6205	+0.3551	+0.6992
46	156	1801	−0.9675	−0.1126	−0.2268	+0.2486	−0.2520	−0.9353
47	412	1890 I	−0.8979	−0.3361	−0.2842	+0.1354	−0.8253	+0.5483
48	391	1886 V	+0.9024	+0.2321	−0.3630	−0.2335	+0.9715	+0.0407
49	262	1854 II	−0.2343	+0.0450	+0.9712	−0.6787	−0.7228	−0.1302
50	82	1677	−0.0850	+0.2307	+0.9692	−0.8555	+0.4818	−0.1897
51	64	1558	−0.5692	−0.1281	+0.8121	−0.3115	−0.8805	−0.3573
52	288	1860 III	−0.1640	+0.2394	+0.9570	+0.9802	−0.0697	+0.1854
53	558	1920 I	+0.4598	+0.3115	−0.8316	−0.5813	−0.6023	−0.5471
54	516	1911 IV	+0.1113	+0.3125	+0.9433	+0.9935	−0.0139	−0.1126
55	259	1853 III	+0.7267	−0.6713	+0.1461	+0.5427	+0.6913	+0.4772
56	40	1299	+0.4182	−0.0617	+0.9062	+0.8377	+0.4119	−0.3586
57	242	1848 I	+0.1844	+0.0047	−0.9829	−0.5413	+0.8351	−0.0976
58	588	1926 III	+0.2681	−0.9605	−0.0753	−0.8177	−0.1856	−0.5449
59	462	1899 I	+0.9453	+0.3153	+0.0840	+0.2408	−0.5004	−0.8316
60	62	1533	−0.6292	−0.6201	−0.4687	−0.3869	−0.2731	+0.8808
61	50	1449	+0.0199	−0.9995	−0.0237	−0.4118	+0.0134	−0.9112
62	241	1847 VI	−0.0469	−0.3265	−0.9439	−0.2019	+0.9286	−0.3112
63	478	1903 IV	−0.1852	+0.5813	+0.7923	−0.9086	−0.4083	+0.0872
64	314	1867 III	−0.2908	−0.8051	+0.5171	+0.9086	−0.4016	−0.1144
65	88	1686	+0.1724	+0.8052	+0.5674	−0.0225	−0.5726	+0.8195
66	111	1757	−0.5680	+0.7924	−0.2220	−0.1335	+0.1775	+0.9750
67	177	1819 II	+0.1309	−0.9646	+0.2290	−0.9824	−0.0951	+0.1607
68	139	1787	+0.6731	+0.0650	+0.7368	+0.7052	+0.2444	−0.6657
69	362	1880 III	+0.2114	+0.9044	−0.3707	+0.4466	−0.4268	−0.7864
70	272	1857 III	−0.4738	−0.6296	+0.6157	+0.3621	−0.7766	−0.5155
71	670	1941 I	−0.6057	+0.7534	−0.2563	−0.6878	−0.3337	+0.6446
72	48	1402	−0.4619	−0.3412	+0.8186	+0.6739	+0.4649	+0.5741
73	335	1873 V	+0.5747	−0.4436	−0.6877	+0.0327	+0.8522	−0.5223
74	59	1506	−0.0580	−0.7772	−0.6266	+0.4638	+0.5348	−0.7063
75	364	1880 V	−0.6482	−0.1579	+0.7450	−0.7614	+0.1540	−0.6298
76	324	1870 IV	+0.8378	+0.0755	+0.5406	+0.5378	+0.0554	−0.8412
77	74	1618 II	+0.7996	+0.1672	−0.5769	+0.5960	−0.1017	+0.7965
78	163	1808 I	+0.1560	+0.7096	−0.6871	−0.4110	−0.5859	−0.6984
79	419	1891 I	+0.9640	+0.2652	+0.0188	−0.2197	+0.8345	−0.5053
80	474	1902 III	−0.2530	−0.9500	+0.1823	+0.3074	−0.2576	−0.9160
81	226	1845 III	+0.0040	−0.6828	+0.7305	−0.2665	−0.7048	−0.6574
82	147	1793 I	−0.2221	−0.6217	−0.7512	+0.8130	+0.3072	−0.4947
83	572	1924 II	+0.5204	+0.3151	+0.7936	+0.8528	−0.1440	−0.5020
84	525	1913 I	−0.0486	−0.2261	−0.9729	−0.8229	−0.5429	+0.1673
85	615	1930 VII	+0.3793	−0.9231	+0.0646	−0.0554	+0.0471	+0.9974
86	475	1903 I	−0.7221	+0.5836	+0.3715	+0.0265	−0.5133	+0.8578
87	138	1786 II	−0.8884	+0.1109	−0.4457	−0.2236	+0.7431	+0.6307
88	460	1898 IX	−0.9293	−0.3390	+0.1461	+0.2809	−0.3923	+0.8759
89	95	1706	+0.3474	+0.6154	+0.7075	+0.2348	−0.7875	+0.5698
90	136	1785 II	−0.2027	−0.5708	+0.7956	+0.9194	−0.3906	−0.0459
91	77	1661	−0,4009	+0.8655	+0.3002	+0.5435	−0.0391	+0.8385
92	540	1915 IV	−0.6081	−0.3657	+0.7046	+0,7874	−0.1645	+0.5941
93	371	1881 VI	+0.0463	−0.9938	+0.1011	−0.9181	−0.0822	−0.3878
94	472	1902 I	−0.1625	−0.7094	−0.6857	+0.7318	−0.5528	+0.3985
95	206	1833	−0.6903	−0.7126	−0.1258	−0.0728	−0.1046	+0.9919
96	44	1362	−0.5468	−0.8323	+0.0909	−0,4810	+0.2235	−0.8477
97	388	1886 II	−0.2542	−0.4321	+0.8653	+0.9305	−0.3531	+0.0970
98	611	1930 III	−0.2901	+0.6800	+0.6734	+0.9215	+0.0086	+0.3883
99	152	1798 I	−0.3637	+0.9094	−0.2021	+0.5721	+0.3893	+0.7220
100	55	1472	+0.6783	+0.7204	−0.1447	−0.1469	−0.0600	−0.9873

Continuation of Table II

No.	B.—O.	Comet	l	m	n	l'	m'	n'
101	517	1911 V	−0.0095	+0.9675	+0.2524	−0.5102	−0.2218	+0.8310
102	63	1556	+0.1994	−0.8268	+0.5260	−0.0069	+0.5355	+0.8445
103	49	1433	+0.1932	−0.9683	−0.1582	+0.9433	+0.2277	−0.2416
104	117	1763	+0.0299	+0.2994	+0.9537	−0.0164	−0.9539	+0.2999
105	477	1903 III	+0.8089	+0.5827	−0.0791	−0.5099	+0.7620	+0.3990

and N denotes the number of orbits over which the summation extends. This is the treatment applied to the distribution of cometary perihelia by Oppenheim and Witkowski. Introducing the linear terms,

$$a_{14} = \frac{1}{N} \Sigma\, l, \qquad a_{24} = \frac{1}{N} \Sigma\, m, \qquad a_{34} = \frac{1}{N} \Sigma\, n, \tag{8}$$

a similar ellipsoid in a displaced position but the same orientation may be obtained by changing the function $F(\vartheta)$. Putting

$$F(\vartheta) = (1 + \cos \vartheta)^2 \tag{9}$$

we have:

$$a_{11}x^2 + a_{22}y^2 + a_{33}z^2 + 2a_{23}yz + 2a_{13}xz + 2a_{12}xy + {} + 2a_{14}x + 2a_{24}y + 2a_{34}z = 1 \tag{10}$$

In this form the shift of the centre of the ellipsoid to a point outside the sun permits the assymetry of the distribution to be expressed. Evaluating the determinants associated with the quadratic equations (6) and (10), respectively, the three semi-axes ϱ_i, their direction cosines ξ_i, η_i, ζ_i, and the heliocentric ecliptical co-ordinates λ_i, β_i ($i = 1, 2, 3$) are obtained in the usual manner. If making use of the formula (10), the polar co-ordinates of the centre of the ellipsoid ϱ_0, λ_0, β_0 may be determined in addition to the above quantities which remain unaltered.

However, there is another, far simpler method which in the present case does not seem inferior to those just mentioned. Putting

$$F(\vartheta) = \cos \vartheta \tag{11}$$

or

$$\Sigma \cos \vartheta = \Sigma\,(l\,\xi + m\eta + n\,\zeta) = \text{Max.} \tag{12}$$

we have with regard to (4) and (8):

$$\xi_M = \frac{a_{14}}{\left|\sqrt{a_{14}^2 + a_{24}^2 + a_{34}^2}\right|}, \qquad \eta_M = \frac{a_{24}}{\left|\sqrt{a_{14}^2 + a_{24}^2 + a_{34}^2}\right|}, \qquad \zeta_M = \frac{a_{34}}{\left|\sqrt{a_{14}^2 + a_{24}^2 + a_{34}^2}\right|} \tag{13}$$

Now ξ_M, η_M, ζ_M are the direction cosines of the point towards which the crowding of the perihelia takes place. For a uniform distribution

$$N(\vartheta) \operatorname{cosec} \vartheta = \text{const.} \tag{14}$$

so that the departures from the randomness must appear in a non-uniform distribution of the quantity $\cos \vartheta$.

It is recommendable to check the results obtained by the above mentioned methods by means of a graphical representation of the actual distribution in an equivalent drawing. Only such representation makes it possible to distinguish the local irregularities (like that produced by the Kreutz group of sun-grazing comets) from a general tendency, and to verify the adequateness of an ellipsoidal distribution.

Formulae for investigating the distribution of the orbital planes are quite analogous to (3)—(14), except that the co-ordinates of the perihelion points are replaced by those of the poles. In the adjoined tables all quantities relating to the poles are denoted by suffices.

4. The Numerical Results

The results of the analysis are summarized in Tables III—XIV. The comets have been distributed into two groups according to the perihelion distances ($0.00 < q < 0.25$; $0.25 < q < 0.50$), the former consisting of 44 members, the latter of 61 members. As the Kreutz group of sun-grazing comets has very likely originated from a single body, two separate solutions have been worked out, the five comets 1843 I, 1880 I, 1882 II, 1887 I, and 1945 VII being omitted in the 2nd solution.

Tables III and IV include the coefficients of the ellipsoid equation for the distribution of perihelia and poles respectively, computed according to (7) and (8). From these coefficients the principal semi-axes ϱ_i of the ellipsoid and their directions in the ecliptical system λ_i, β_i may be calculated which, again, may be transformed into equatorial and galactic co-ordinates α_i, δ_i, L_i, B_i ($i = 1, 2, 3$).

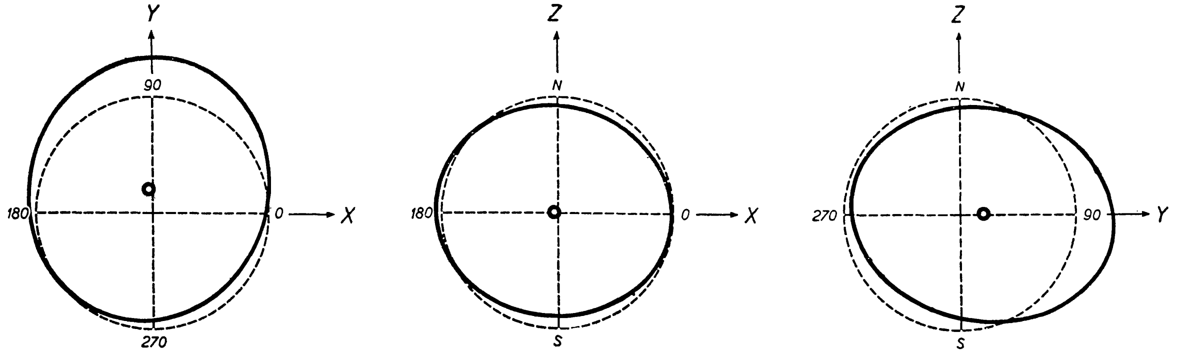

Figure 1. The distribution ellipsoid of the perihelia cut by the principal planes of the orthogonal ecliptical system. N, S denote the poles of the ecliptic, the ciphers the ecliptical longitudes. The dashed circle represents a uniform distribution.

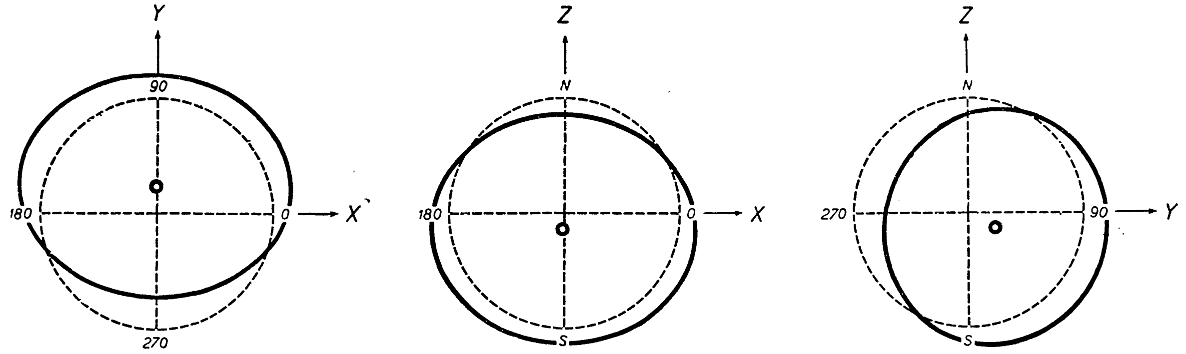

Figure 2. The distribution ellipsoid of the poles cut by the principal planes of the orthogonal ecliptical system. Plotted analogously to Figure 1.

In (6) the centre of the ellipsoid coincides with the Sun, in (10) it is displaced by an amount ϱ_0 in the direction λ_0, $\beta_0 \equiv \alpha_0$, $\delta_0 \equiv L_0$, B_0 (see Table V and VI). The points of intersection of the ellipsoid's surface with the orthogonal axes of the ecliptical system X, Y, Z according to (10) are given in Tables VII and VIII. For the total of 105 comets (solution I, $0.00 < q < 0.50$) the bisections of the principal planes with the ellipsoid are indicated in Figures 1 and 2.

The definition of the point of the highest concentration according to (11)—(13) yields the direction cosines ξ_M, η_M, ζ_M which are transformed into spherical co-ordinates $\lambda_M, \beta_M, \alpha_M, \delta_M, L_M, B_M$ in Tables IX and X. The observed numbers of the perihelia and poles in six equidistant zones around the point of the highest concentration (taken from solution I, $0.00 < q < 0.50$ throughout) are given in Tables XI and XII compared with the expected numbers for a random distribution; an analogous distribution with respect to the poles of the ecliptic is demonstrated in Tables XIII and XIV. In Tables XI—XIV the difference between solution I and II is indicated by denoting with asterisks the zones in which the comets of the Kreutz group are present.

Figures 3 and 4 show the distribution of the individual perihelia and poles, respectively. They are constructed in the ecliptical system of co-ordinates but also the equator, galactic equator, and their poles are indicated.

5. *The Distribution of the Perihelia*

The discussion of the results shows the following characteristic features of the orientation of the perihelia:

(A) A pronounced maximum not far from the ecliptic in the longitude 270°—280°. For the comets with the shortest perihelion distances ($q < 0.25$) the maximum is situated farther southwards, for those with perihelion distances $q > 0.25$ northwards (Figure 1, Table IX).

(B) A predominance of the northern hemisphere over the southern one. For the comets with the shortest perihelion distances the numerical

Table III

	I			II		
q	0.00−0.25	0.25−0.50	0.00−0.50	0.00−0.25	0.25−0.50	0.00−0.50
a_{11}	+0.2623	+0.2628	+0.2626	+0.2928	+0.2628	+0.2745
a_{22}	+0.3973	+0.3813	+0.3880	+0.3674	+0.3813	+0.3759
a_{33}	+0.3404	+0.3559	+0.3494	+0.3398	+0.3559	+0.3496
$a_{23} = a_{32}$	−0.0383	+0.0131	−0.0084	+0.0162	+0.0131	+0.0143
$a_{13} = a_{31}$	+0.0361	−0.0212	+0.0028	+0.0291	−0.0212	−0.0016
$a_{12} = a_{21}$	−0.0336	+0.0342	+0.0058	−0.0220	+0.0342	+0.0123
a_{14}	+0.0356	−0.0204	+0.0031	+0.0201	−0.0204	−0.0046
a_{24}	−0.2534	−0.0969	−0.1625	−0.1842	−0.0969	−0.1310
a_{34}	−0.0096	+0.1388	+0.0766	−0.0859	+0.1388	+0.0511

Table IV

	I			II		
q	0.00−0.25	0.25−0.50	0.00−0.50	0.00−0.25	0.25−0.50	0.00−0.50
a_{11}	+0.2858	+0.3572	+0.3272	+0.3121	+0.3572	+0.3396
a_{22}	+0.2992	+0.2543	+0.2732	+0.2932	+0.2543	+0.2695
a_{33}	+0.4150	+0.3885	+0.3996	+0.3947	+0.3885	+0.3909
$a_{23} = a_{32}$	+0.0630	−0.0032	+0.0245	+0.0145	−0.0032	+0.0037
$a_{13} = a_{31}$	−0,0185	+0.0247	+0.0066	−0.0366	+0.0247	+0.0008
$a_{12} = a_{21}$	−0.0355	+0.0049	−0.0120	−0.0538	+0.0049	−0.0180
a_{14}	−0.0567	+0.0822	+0.0240	−0.0407	+0.0822	+0.0342
a_{24}	−0.1387	−0.0499	−0.0871	−0.0812	−0.0499	−0.0621
a_{34}	−0.0605	+0.0164	−0.0158	+0.0284	+0.0164	+0.0211

Table V

i	0	1		2		3	
ϱ_i	0.46	1.60		1.70		1.95	
λ_i	93°	268°	88°	77°	257°	177°	357°
β_i	−27°	+12°	−12°	+78°	−78°	+2°	−2°
α_i	92°	268°	88°	282°	102°	178°	358°
δ_i	−3°	−12°	+12°	+77°	−77°	+3°	−3°
L_i	179°	344°	164°	76°	256°	244°	64°
B_i	−8°	+5°	−5°	+27°	−27°	+63°	−63°

Table VI

i	0	1		2		3	
ϱ'_i	0.32	1.57		1.74		1.94	
λ'_i	101°	72°	252°	168°	348°	258°	78°
β'_i	+4°	+79°	−79°	+1°	−1°	+11	−11°
α'_i	103°	284°	104°	169°	349°	258°	78°
δ'_i	+27°	+76°	−76°	+6°	−6°	−12°	+12°
L'_i	157°	75°	255°	224°	44°	338°	158°
B'_i	+14°	+26°	−26°	+60°	−60°	+13°	−13°

Table VII

	I			II			Co-ordinates
q	0.00−0.25	0.25−0.50	0.00−0.50	0.00−0.25	0.25−0.50	0.00−0.50	
Z_+	1.74	1.33	1.49	1.99	1.33	1.55	$\beta = +90°$
X_+	1.82	2.03	1.94	1.78	2.03	1.93	$\lambda = 0°, \beta = 0°$
Y_+	2.35	1.89	2.08	2.23	1.89	2.02	$\lambda = 90°, \beta = 0°$
X_-	2.09	1.87	1.96	1.92	1.87	1.89	$\lambda = 180°, \beta = 0°$
Y_-	1.07	1.39	1.24	1.22	1.39	1.32	$\lambda = 270°, \beta = 0°$
Z_-	1.69	2.11	1.93	1.48	2.11	1.84	$\beta = -90°$

Table VIII

	I			II			Co-ordinates
q	0.00–0.25	0.25–0.50	0.00–0.50	0.00–0.25	0.25–0.50	0.00–0.50	
Z'_+	1.70	1.56	1.62	1.52	1.56	1.55	$\beta' = +90°$
X'_+	2.08	1.46	1.68	1.93	1.46	1.62	$\lambda' = 0°, \beta' = 0°$
Y'_+	2.35	2.19	2.26	2.14	2.19	2.17	$\lambda' = 90°, \beta' = 0°$
X'_-	1.68	1.92	1.82	1.66	1.92	1.82	$\lambda' = 180°, \beta' = 0°$
Y'_-	1.42	1.80	1.62	1.59	1.80	1.71	$\lambda' = 270°, \beta' = 0°$
Z'_-	1.41	1.65	1.54	1.67	1.65	1.65	$\beta' = -90°$

Table IX

	I			II		
q	0.00–0.25	0.25–0.50	0.00–0.50	0.00–0.25	0.25–0.50	0.00–0.50
λ_M	278°	258°	271°	276°	258°	268°
β_M	−2°	+54°	+25°	−25°	+54°	+21°
α_M	279°	262°	271°	278°	262°	268°
δ_M	−25°	+31°	+2°	−48°	+31°	−2°
L_M	338°	22°	357°	315°	22°	352°
B_M	−11°	+29°	+9°	−20°	+29°	+9°

Table X

	I			II		
q	0.00–0.25	0.25–0.50	0.00–0.50	0.25–0.50	0.00–0.25	0.00–0.50
λ'_M	248°	329°	285°	243°	329°	299°
β'_M	−22°	+10°	−10°	+17	+10°	+17°
α'_M	241°	328°	288°	245°	328°	298°
δ'_M	−43°	−3°	−32°	−4°	−3°	−4°
L'_M	305°	24°	333°	338°	24°	5°
B'_M	+5°	−42°	−21°	+29°	−42°	−17°

Table XI

	$0.00<q<0.25$		$0.25<q<0.50$		$0.00<q<0.50$	
	O	C	O	C	O	C
$0° < \vartheta < 30°$	8*	3	8	4	16*	7
$30° < \vartheta < 60°$	12	8	11	11	23	19
$60° < \vartheta < 90°$	8	11	18	15	26	26
$90° < \vartheta < 120°$	8	11	14	15	22	26
$120° < \vartheta < 150°$	7	8	7	11	14	19
$150° < \vartheta < 180°$	1	3	3	4	4	7

Table XII

	$0.00<q<0.25$		$0.25<q<0.50$		$0.00<q<0.50$	
	O	C	O	C	O	C
$0° < \vartheta' < 30°$	2	3	1	4	3	7
$30° < \vartheta' < 60°$	13*	8	14	11	27*	19
$60° < \vartheta' < 90°$	13	11	21	15	34	26
$90° < \vartheta' < 120°$	6	11	15	15	21	26
$120° < \vartheta' < 150°$	8	8	5	11	13	19
$150° < \vartheta' < 180°$	2	3	5	4	7	7

Table XIII

	$0.00 < < q < 0.25$		$0.25 < < q < 0.50$		$0.00 < < q < 0.50$	
	O	C	O	C	O	C
$+60° < \beta < +90°$	2	3	6	4	8	7
$+30° < \beta < +60°$	9*	8	17	11	26*	19
$0° < \beta < +30°$	13	11	13	15	26	26
$-30° < \beta < 0°$	9	11	15	15	24	26
$-60° < \beta < -30°$	6	8	7	11	13	19
$-90° < \beta < -60°$	5	3	3	4	8	7

Table XIV

	$0.00 < < q < 0.25$		$0.25 < < q < 0.50$		$0.00 < < q < 0.50$	
	O	C	O	C	O	C
$+60° < \beta' < +90°$	4	3	5	4	9	7
$+30° < \beta' < +60°$	8	8	14	11	22	19
$0° < \beta' < +30°$	7	11	11	15	18	26
$-30° < \beta' < 0°$	9	11	13	15	22	26
$-60° < \beta' < -30°$	14*	8	14	11	28*	19
$-90° < \beta' < -60°$	2	3	4	4	6	7

superiority of the „northern“ perihelia is moderate and may be wholly attributed to the Kreutz group of sun-grazing comets (β near $+35°$). However, among the comets with $q >$ > 0.25 the northern hemisphere is distinctly preferred, the ratio being 36 : 25 (Figure 1, Table XIII).

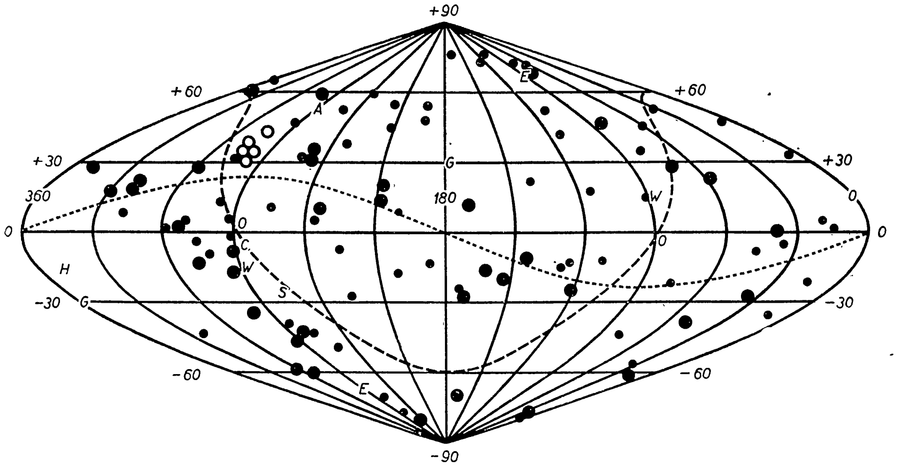

Figure 3. The distribution of individual perihelia in the ecliptical system. Open circles — Kreutz's group, filled circles — other comets with $q < 0.25$, black dots — comets with $q > 0.25$, dotted line — celestial equator, dashed line — galactic equator, E — celestial poles, G — galactic poles, C — galactic centre, A — solar apex, H — centre of the Hoek zone of avoidance, S — point of maximum concentration according to Schiaparelli, O — direction of the shortest axis of the distribution ellipsoid of Oppenheim, W— direction of the shortest axis of the distribution ellipsoid of Witkowski.

(C) A non-uniformity of the distribution. Apart from the comets of the Kreutz group the perihelia of which obviously lie close together, there seem to exist some irregularities beyond the random fluctuation. The Hoek zone [1] is not outstanding at all: there are other similar zones of avoidance, e. g. that in the region about $\lambda = 160°, \beta = -50°$. On the other hand the perihelia do not appear to be associated into dense groups, as it might be expected on the basis of Lyttleton's theory; rather they show a tendency to crowding in long chains, such as the arc between the South pole of the ecliptic and $\lambda = 330°$, $\beta = +30°$ (Figure 3). In general, the representation of the distribution by means of an ellipsoid does not seem to be quite adequate.

It is not difficult to separate the effects of selection connected with the observing conditions from the features of the actual distribution of the orbits. First of all, the prevalence of the observatories on the northern hemisphere prefers the discoveries of those comets which, attaining the most favourable observing conditions, are situated northwards of the ecliptic. The conditions depend partly on the instantaneous configuration Sun-Earth-Comet, partly on the brightness of the comet itself which is maximum near the perihelion passage. It follows that there are better suited condiions for the discovery of comets with positive latitudes of the perihelia. However, with decreasing perihelion distance the ratio of geocentric latitude: heliocentric latitude at perihelion becomes smaller and at the same time the orbital arc between the nodes, including the perihelion, becomes shorter. Thus a prevalence of perihelia on the northern hemisphere is expected to occur down to certain limiting size of the perihelion distance (low compared with the distance Sun-Earth) and to change in favour of the southern hemisphere afterwards. This is just what is found in the statement (B). The discrepancy between the comets of greater and smaller perihelion distances contained in the statement (A) may be interpreted in the very same way.

The other substantial feature of the distribution — the crowding of perihelia near $\lambda = 270°$ to $280°$ — is much more difficult to explain. The concetration takes place within a relatively narrow zone which does not agree with the presumption of a seasonal effect. A seasonal effect would appear most pronouncedly in the distribution of the times of perihelion passages and would have rather a smoothed progress. On the other hand, the general conditions of visibility should be correlated to the angular distance $\lambda - E$, where λ is the heliocentric longitude of the perihelion and E the heliocentric longitude of the earth in the moment of the comet's perihelion passage. This is not the actual case as is evident from Table XV—XVII showing a comparison of the distribution of comets according to λ, E, and $\lambda - E$.

Table XV

q λ	0.00—0.25	0.25—0.50	0.00—0.50
0—30°	2	7	9
30—60°	4	4	8
60—90°	3	6	9
90—120°	0	6	6
120—150°	2	4	6
150—180°	5	2	7
180—210°	2	5	7
210—240°	1	7	8
240—270°	4	8	12
270—300°	14*	9	23*
300—330°	4	2	6
330—360°	2	1	3

Table XVI

q E	0.00—0.25	0.25—0.50	0.00—0.50
0—30°	5	5	10
30—60°	3	8	11
60—90°	8	4	12
90—120°	5	2	7
120—150°	4	5	9
150—180°	5	4	9
180—210°	2	6	8
210—240°	5	7	12
240—270°	3	5	8
270—300°	1	3	4
300—330°	0	1	1
330—360°	3	11	14

Table XVII

q $\lambda - E$	0.00—0.25	0.25—0.50	0.00—0.50
0—30°	2	5	7
30—60°	2	7	9
60—90°	7	4	11
90—120°	1	3	4
120—150°	6	5	11
150—180°	4	4	8
180—210°	6	8	14
210—240°	3	7	10
240—270°	3	3	6
270—300°	5	7	12
300—330°	3	5	8
330—360°	2	3	5

The times of the perihelion passages exhibit a more conspicuous departure from a random distribution, the winter months being distinctly preferred among the sun-grazing comets. It is striking that of the eight comets with $q < 0.03$ six passed the perihelion during the winter months (December till February), two during the autumn months (September till November), and none in the remaining half of the year! The reason of this irregularity is obscure, as we can hardly assume that such bright comets as those of the Kreutz group may have escaped observation only on account of the shortness of the summer nights at the northern hemisphere. Possibly it is a matter of chance only.

From all that was said above we are driven to conclude that there exist some preferred directions of the lines of apses which nearly coincide with the direction to the galactic centre or with the direction of the vertices. This conclusion reached previously by Schiaparelli [2] agrees well with the results of Svedstrup, Oppenheim, and Witkowski although the data on which it is founded are different due to the selection according to the perihelion distances. The distribution of perihelia is distinctly assymetrical and cannot be satisfactorilly approximated by an ellipsoidal distribution. It rather looks as if there were two groups of comets: one with the lines of apses orientated in a completely random fashion and the other, less numerous, whose members approach the Sun predominantly from an elongated region situated not far from the galactic anticentre.

Nevertheless, it would be premature to take this coincidence for an evidence of the interstellar origin of comets. There are other principal directions (such as the direction apex—antapex or the direction of the spiral arms) to which relation of aphelia is possible. As far as there is no reliable theoretical foundation as to why the aphelia are associated just with the direction to the anticentre, one must be very cautious in interpreting the phenomenon. In fact, an entirely different distribution—say, a crowding of aphelia to the apex mentioned by von Niessl [13]—might be attributed to the interstellar origin with the very same justice. According to the writer's opinion the phenomenon may be explained more probably in the way suggested by Merton [7], in which case the zone of concentration could perhaps be considered as related the path of a star's approach to the vicinity of the Sun. This interpretation, however, is entirely tentative and a thorough dynamical discussion of the consequences of such an approach are necessary for judging of its acceptability.

6. *The Distribution of the Orbital Planes*

To avoid an ambiguity in discussing the data concerning the distribution of orbital planes, only those poles were considered from which the comets are seen to revolve directly. The following characteristic features were found:

(A) A moderate concentration of the poles towards the poles of the ecliptic, both among the comets with $q < 0.25$ and with $q > 0.25$ (Table XIV). This is in accord with the results of Oppenheim [5].

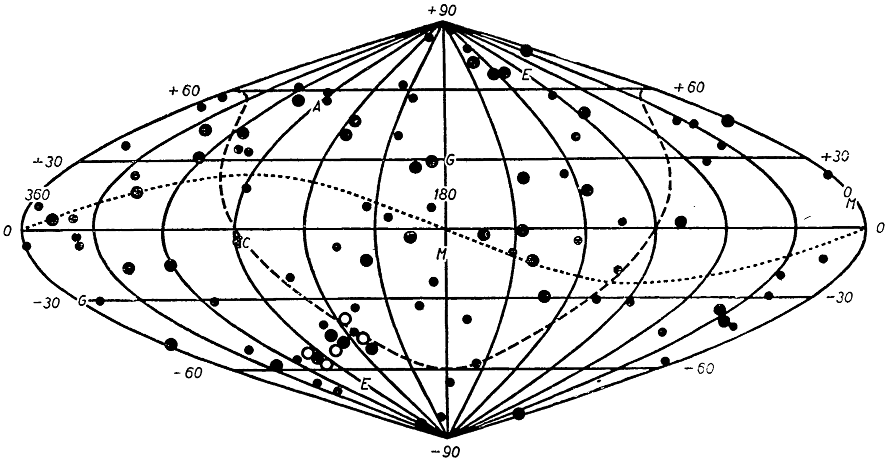

Figure 4. The distribution of individual poles in the ecliptical system. M — the points od maximum concentration according to Mohn; for other symbol consult the explanation under Figure 3.

(B) A crowding of the poles in the region around $\lambda = 260°$, $\beta = -55°$, not far from the south celestial pole. The comets of the Kreutz group participate in this crowding; nevertheless, also poles of other comets appear here in an enhanced number.

(C) A minimum of occurrence near $\lambda = 100°$, not far from the ecliptic. This minimum, although appearing pronounced in the ellipsoidal analysis (Figure 2), is much less striking in the direct representation (Figure 4) and is probably due to the combination of the primary effects (A) and (B). Also here the ellipsoidal method appears inadequate.

The prevalence of high ecliptical latitudes of the poles—both northern and southern—may be accounted to three entirely different reasons, viz.:

(1) The orbits may actually be slightly concentrated to the plane of the ecliptic due to the perturbing action of the major planets. In spite of the fact that the effect is much less conspicuous than for short-periodic comets, this interpretation appears rather improbable since the periods of a great majority of the comets in question are too long. Nevertheless, this interpretation cannot be absolutely rejected.

(2) The concentration may be only fictitious, produced by the effect of selection which has been investigated by Bourgeois and Cox [9]. Comets in highly inclined orbits rarely happen to pass close to the Earth and the approach cannot endure long; hence they may generally more easily escape observation than the comets moving in orbits of low inclination.

(3) A fictitious enhancement may be also due to the dependence of the comets' brightness on their heliocentric latitude. Beyer [14] demonstrated on several examples that this dependence is a pronounced one especially when the comet reaches the latitude of more than ±70°. As a consequence, the brightness of the comets in highly inclined orbits may be on average lower (except in the vicinity of the nodes) and a greater proportion of them may remain undiscovered.

In order to discriminate between the two latter possibilities the following procedure was employed. The comets were distributed into two groups according to the heliocentric latitude of the perihelion point ($\beta < 30°$ and $\beta > 30°$, respectively) and their positions in the orbits for the moments of discovery were computed from the discovery dates quoted in Baldet's list [15]. These positions are plotted in Figure 5 where all orbits are transponed into a common plane so as to make their lines of apses coincide. The figure affords an instructive insight into the discovery conditions with regard to the eventual effects of selection. First it is shown that a great majority of comets with $q < 0.5$ was discovered before the perihelion passage: about $^{1}/_{3}$ before having approached the Sun winthin 1 A. U., $^{1}/_{3}$ between 1 A. U. and the perihelion passage, and $^{1}/_{3}$ afterwards. Only five comets out of 105 have been discovered after having receded again beyond the distance of the Earth.

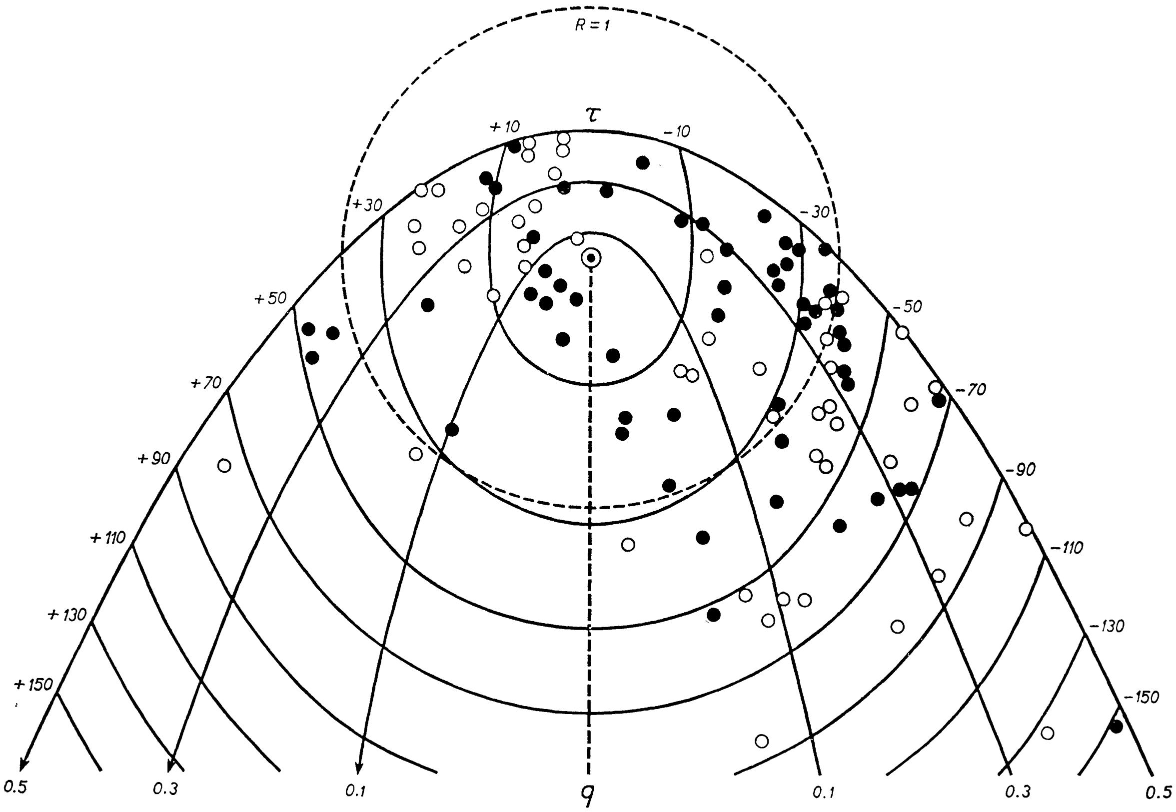

Figure 5. Heliocentric positions of all comets with $q < 0.5$ at the moments of their discovery. Open circles — comets with the latitudes of the perihelia $\beta < 30°$, filled circles — those with $\beta > 30°$, dashed line — the common line of apses, τ — the time interval between the discovery and perihelion passage in days. The sphere centred at the Sun which cuts the plane in the dashed circle represents the loci of the Earth's orbit, i. e. of the observer's position.

It is interesting that as many as $^{1}/_{4}$ of the comets with $q < 0.25$ were discovered during the single next week after the perihelion passage.

At the first glance we see a difference between the comets the perihelia of which lie in higher and lower ecliptical latitudes, respectively. The comets which pass the perihelion at high latitudes ($|\beta| > 30°$; black dots) are mostly discovered about one month before reaching the perihelion, at the time when they approach the Sun to a distance $\simeq$ 1 A. U. Only seldom the discovery is made earlier or till in the other half of the orbit. The comets passing the perihelion close to the ecliptic ($|\beta| < 30°$; open circles) are, on the contrary, much more frequently discovered shortly after the perihelion passage than shortly before it; also the discoveries at considerable heliocentric distances are here less scarce. This substantial difference between the two groups of comets is irreconcilable with the conception (1) or (2) but readily explainable on the basis of the conception (3):

The main distinction between the two effects of selection appearing in (2) and (3) is that the former is associated with the geocentric position and the latter with the heliocentric position of the comet. If the heliocentric distance of the comet is small compared with that of the Earth as in all cases which we consider (comets of $q < 0.5$ at perihelion) then the geocentric latitude is always lower than the heliocentric latitude, and the comet remains anyway in the vicinity of the Sun. Hence the geometrical conditions of visibility are relatively unsensitive to the variation of the latitude of the perihelion point.

The circumstances become altered if we assume a variability of the comet's brightness with the heliocentric latitude. A comet passing the perihelion near to the ecliptic may exhibit more violent changes of brightness which increase the probability of the discovery about the perihelion passage. It must be emphasized that the heliocentric latitude generally varies much more rapidly near the perihelia of comets with extremely short perihelion distances than in other cases and that, as a consequence, this sort of comets is especially suitable for an investigation like that of Beyer [14]. The peculiar phase effect pointed out by Richter [16] on the basis of his laboratory experiments, may also introduce an effect of selection, being in relation to the orientation of the orbit with respect to the Earth; however, the correlation between the heliocentric latitude of the perihelion point and the discovery conditions is hardly explainable in this way.

The evidence that the comets of low latitudes of the perihelia are unexpectedly often discovered only several days after the perihelion passage favours the opinion that the changes of the comets' brightness are connected with some anisotropical source in the solar radiation, and somewhat delayed against it. This is in contradiction to the results of Beyer who on the basis of the direct measurements of bringhtness concludes that there is no perceptible delay at all and that therefore the due radiation cannot be corpuscular. It is curious that the delay should be appreciable just in the comets of low heliocentric distances where the path to be travelled by the corpuscles is short. Although it is seen from Figure 5 that the effect appears mainly at $q > 0.25$, the disagreement with Beyer's results and the too low velocity of the corpuscular radiation required (about 100 to 200 km/s) casts doubts on the interpretation of the delay on account of the radiation velocity. Perhaps there simultaneously operate two different factors: the solar radiation, more effective near the plane of the Sun's equator, and the time lag in the heat transfer by the cometary nucleus. As the time lag may be expected to appear more pronouncedly in the comets of short perihelion distances, this interpretation is reconcilable with Beyer's results. In each case the data on the discovery of comets with short perihelion distances strongly support the view that their course of brightness is assymetric, the maximum being attained until after the perihelion passage. Nevertheless this conclusion must also be taken for tentative only, since the available data on this subject are not extensive enough.

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ЛЮБОР КРЕСАК

РАСПРЕДЕЛЕНИЕ КОМЕТНЫХ ОРБИТ С МАЛЫМ ПЕРИГЕЛИЙНЫМ РАССТОЯНИЕМ

Пространственное распределение плоскостей орбит и перигелий долгопериодических комет из-за узкой связи с вопросом о происхождении комет было исследовано уже целым рядом авторов (1—6,9). Но в трактовке результатов нет единства мнений. Обнаружение отклонения от равномерного распределения некоторые авторы приводят в форме доказательства междузвездного происхождения комет, или в крайнем случае как результат видимого влияния поля тяготения Галактики или соседних звезд на орбиты комет на границах солнечной системы. Другие объясняют эти отклонения эффектом селекции при открытии комет, переменностью наблюдательных условий в течение года и их зависимостью на расположении орбиты. Важность этого вопроса, о котором уже десятилетия ведутся споры, сказалась снова при появлении двух новых теорий происхождения комет, предлагаемых Оортом (Oort) [8] и Литлтоном (Lyttleton) [10].

В настоящей работе приводится расположение направлений перигелий и полюсов орбит долгопериодических комет с абнормально малым перигелийным расстоянием. Ограничение перигелийных расстояний позволяет с одной стороны обсудить действие некоторых эффектов селекции, с другой стороны избирает крайние случаи из всех комет, как для теории Оорта (Oort) (самое сильное возмущающее действие околостоящих звезд), так для теории Литлтона (Lyttleton) (самый маленький момент вращения, приданный планетами комете, движущейся по оси акреции к солнцу). В качестве основного статистического материала были взяты элементы 105-ти комет с периодом большим чем 200 лет и перигелийным расстоянием не превышающим 0,5 астрономических единиц, по каталогу Бальде (Baldet) [12]. Эклиптикальные координаты и косинусы направления перигелий ($\pi, \beta; l, m, n$) и полюсов ($X', B'; l', m', n'$) всех этих комет приведенные к среднему равноденствию 1950,0 даны в таблицах I и II и изображены на рис. 3 и 4.

Статистический материал обработывался при помощи нескольких независимых методов, причем кометы были собраны в зависимости от перигелийных расстояний в две группы: $q < 25$ (44 случаев) и $0{,}25 < q < 0{,}5$ (61 случаев). Так как очень правдоподобно, что кометы группы Крейца возникли распадением единого тела, производились по два вычисления, причем одно из них вычислялось с исключением 5-ти определенных членов этой группы. Численные результаты полученные при помощи отдельных методов приведены в таблицах III—XVI.

Разбор результатов показывает, что обычно применяемый элипсоидальный метод для этой цели не пригоден, так как не уделяет достаточного внимания асимметрии и локальным неправильностям распределения. Также преобразование метода Лагранжа (Lagrange), именно изменение выбора условий минима не является вовсе удовлетворительным. Хоть и статистический материал до сих пор не достаточно широк, кажется, что перевес определенных направлений перигелий и полюсов проявляется скорее в виде творения отдельных групп чем в плавном распределении, которое бы могло быть заменено с желаемой точностью элипсоидальным распределением. Наблюдательные условия, в нашем случае условия открытия, влекут за собой различные эффекты селекции, которые впрочем можно сравнительно легко определить и учесть.

Распределение перигелий содержит следующие характеристические знаки:

1. Космического происхождения кажется группа перигелий расположенная около эклиптикальной долготы 270—280°, в узкой полосе по обеих сторонах эклиптики (рис. 3). Здесь речь идет о кометах приходящих из области галактического антицентра и вертекса; однако это сходство трудно считать достаточным доказательством зависимости системы комет со строением Галактики или их междузвездного происхождения. Центр зоны максимальной концентрации перигелий хорошо сходится с направлением самой малой оси элипсоидов Опенгейма (Oppenheim) [5] и Витковского (Witkowski) [6], выведеных для совокупности всех долгопериодических комет, так что видимо дело не заключается в особенном свойстве орбит с малыми перигелийными расстояниями. Совершенно не проявляется повышение числа комет появляющихся из направления апекса Солнца, которые требует Нисль (Niessl) для доказательства междузвездного происхождения комет [13], ни существование групп комет с общей прямой апсид, которая бы соответствовала теории Литлтона (Lyttleton) о происхождении комет акрецией [11]. Более подробный разбор показывает, что также дело и не в эффекте селекции, который связан с условиями наблюдения, хотя на этом месте интересно заметить, что несоответственно большой процент наблюдаемых комет с самыми малыми перигелийными расстояниями прошло через перигелий зимой.

2. Эффект селекции приводит перевес перигелий на северном полушарии против южному в отношении 4 : 3. Этот перевес получается в следствие неравномерного размещения наблюдателей на Земле и сказывается, что надо было ожидать, яснее в группе комет с перигелийным расстоянием $0{,}25 < q < 0{,}50$, так как в этом случае дуга орбиты между узлами, на которой находится перигелий, является более длинной (главным образом в смысле времени).

В распределении полюсов орбит, хотя меньше, но все-таки можно наблюдать некоторую неравномерность. Для устранения двухсмыслия в определении движения рассматривался всякий раз только тот полюс, из которого движение кометы представляется в прямом направлении.

1. Сравнительно бросающийся в глаза максимум полюсов находится в зоне $\lambda = 260°$, $\beta = -55°$ (рис. 4). Но густота распределения полюсов в этих местах не исключает возможность их случайной группировки, если взять во внимание влияние, которое оказывает 5—6-членная группа комет Крейца (Kreutz).

2. Весьма интересной является концентрация полюсов орбит у полюсов эклиптики, которая может быть следствием трех следующих причин: а) инстинное соотношение плоскостей орбит и основной плоскости солнечной системы вызванное направляющим возмущающим действием планет, б) эффект селекции, вызванный неодинаковыми условиями открытия кометы при разных положениях относительно к Земле и Солнцу, в) эффект селекции вызванный изменением яркости комет в зависимости от их расположения относительно солнечного экватора. Третье объяснение показалось самым правдоподобным и поддерживает мнение Бейера (Beyer) о зависимости яркости кометы от эклиптикальной широты [14]. Теорию подтверждает прежде всего интересная зависимость эпохи открытия кометы и гелиоцентрической широты перигелия. Несмотря к сравнительно неширокому использованному материалу показывается, что ход яркости исследованных комет должен быть асимметричным с максимумом после перехода перигелием. Не исключается возможность, что изменения яркости вызываемы корпускулярным излучением; Бейер (Beyer) напротив, на основании непосредственных измерений яркости эту возможность отрицает.

ĽUBOR KRESÁK

ROZDELENIE KOMETÁRNYCH DRÁH S MALOU VZDIALENOSŤOU PERIHÉLIA

Priestorové rozloženie rovín obehu a perihélií dlhoperiodických komét pre jeho úzku súvislosť s pôvodom komét skúmal už celý rad autorov (Hoek, Schiaparelli, Mohn, Svedstrup, Oppenheim, Witkowski, Bourgeois, Cox). Vo výklade výsledkov však názory rozchádzajú. Odchýlky od rovnomerného rozdelenia pokladajú niektorí autori za dôkaz interstelárneho pôvodu komét, prípadne aspoň za prejav viditeľného pôsobenia gravitačného poľa Galaxie alebo susedných hviezd na dráhy komét na hraniciach slnečnej sústavy. Iní ich pripisujú výberovému efektu pri objavovaní komét, premenlivosti pozorovacích podmienok v priebehu roku a ich závislosti od polohy dráhy. Dôležitosť tohto problému, o ktorý sa vedú spory už niekoľko desaťročí, znova sa ukázala v súvislosti s dvoma novými teóriami vzniku komét, ktoré v poslednom čase formulovali Oort a Lyttleton.

Práca sa zaoberá smerovým rozložením perihélií a pólov dráh dlhoperiodických komét s abnormálne malou perihéliovou vzdialenosťou. Obmedzenie perihéliových vzdialeností dovoľuje jednak posúdiť pôsobenie niektorých výberových efektov, jednak vyberá z celého systému komét prípady extrémne, či už pre teóriu Oortovu (najsilnejšie poruchové pôsobenie okolných hviezd) alebo Lyttletonovu (najmenší rotačný moment, dodaný planétami kométe pohybujúcej sa po akrečnej osi k Slnku). Ako základný štatistický materiál slúžili dráhové elementy 105 komét s obežnou dobou dlhšou ako 200 rokov a perihéliovou vzdialenosťou menšou ako 0,5 astronomickej jednotky. Ekliptikálne súradnice a smerové kosínusy perihélií a pólov dráh všetkých týchto komét, prevedené na spoločné ekvinokcium 1950,0, sú uvedené v tabuľkách I a II a zakreslené na obrázkoch 3 a 4.

Materiál bol spracovaný niekoľkými neodvislými spôsobmi, pričom kométy sa podľa perihéliových vzdialeností delili do dvoch skupín: $0{,}00 < q < 0{,}25$ (44 prípadov) a $0{,}25 < q < 0{,}50$ (61 prípadov). Pretože je veľmi pravdepodobné, že kométy Kreutzovej skupiny vznikli rozpadom jediného telesa, boli vyčíslené vždy po dve riešenia, jedno z nich s vylúčením 5 istých členov tejto skupiny.

Rozbor výsledkov ukazuje, že bežne používaná elipsoidálna metóda nie je pre náš problém najvhodnejšia pre nedostatočný ohľad na asymetriu a miestne nepravidelnosti rozdelenia. Ani úprava Lagrangeovej metódy pozmenenou voľbou podmienok minima nie je celkom uspokojivá. Hoci štatistický materiál nie je dosiaľ dostatočne bohatý, zdá sa, že prevaha určitých smerov pólov a najmä perihélií sa prejavuje skôr v tvorení skupín ako v plynulom zhusťovaní, ktoré by sa s dostatočným priblížením dalo nahradiť elipsoidálnym rozdelením alebo výpočtom bodov maximálnej koncentrácie. Pozorovacie podmienky, v našom prípade podmienky objavu, prinášajú so sebou rôzne výberové efekty, ktoré sa však pomerne ľahko dajú rozpoznať a oddeliť.

Rozloženie perihélií má tieto hlavné znaky:

1. Kozmického pôvodu sa zdá byť zhustenie perihélií okolo ekliptikálnej dĺžky 270—280°, v úzkom oblúku po oboch stranách ekliptiky (obr. 3). Ide o kométy prichádzajúce približne z oblasti galaktického anticentra; ťažko však možno pokladať túto zhodu za dostatočný dôkaz súvislosti systému komét so stavbou Galaxie alebo

ich interstelárneho pôvodu. Stred oblasti najväčšej koncentrácie perihélií dobre súhlasí so smerom najkratšej osi elipsoidov Oppenheima a Witkowského, odvodených pre súbor všetkých dlhoperiodických komét, takže zrejme nejde o osobitnú vlastnosť dráh s malými perihéliovými vzdialenosťami. Vôbec sa neprejavuje zvýšenie počtu komét prichádzajúcich zo smeru slnečného apexu, ako to von Niessl požaduje pre potvrdenie interstelárneho pôvodu komét, ani existencia skupín komét so spoločnou priamkou apsíd, ktorá by zodpovedala Lyttletonovej teórii vzniku komét akréciou. Bližší rozbor ukazuje, že nemôže ísť ani o výberový efekt, závislý od pozorovacích podmienok, hoci tu je zaujímavá okolnosť, že z pozorovaných komét s najkratšími perihéliovými vzdialenosťami (rádove niekoľko polomerov Slnka) neúmerne vysoké percento prešlo perihéliom v zime.

2. Výberový efekt spôsobuje prevahu perihélií na severnej pologuli proti južnej v pomere asi 4 : 3. Prevahu zapríčiňuje nerovnomerné rozmiestenie pozorovateľov na Zemi a je, ako sa dalo očakávať, zreteľnejšia v skupine komét s perihéliovou vzdialenosťou $0{,}25 < q < 0{,}50$, kde oblúk dráhy medzi uzlami, na ktorom leží perihélium, je v priemere podstatne dlhší (najmä časove).

Rozdelenie rovín obehu jeoniečo rovnomernejšie, ale i tu sa prejavujú určité nepravidelnosti. Pre odstránenie dvojznačnosti v zmysle pohybu sa bral do úvahy vždy iba ten pól, pri pohľade z ktorého kométa obieha okolo Slnka priamym smerom.

1. Dosť výrazné maximum pólov leží v oblasti okolo $\lambda = 260°$, $\beta = -55°$ (obr. 4). Hustota pólov v týchto miestach nevylučuje však celkom ani možnosť náhodného zoskupenia, najmä ak uvážime, že k nej prispieva i päťčlenná až šesťčlenná Kreutzova skupina komét.

2. Veľmi zaujímavá je koncentrácia pólov dráh k pólom ekliptiky, ktorá môže mať tri rôzne príčiny: *a)* skutočný vzťah rovín dráh k základnej rovine slnečnej sústavy, vyvolaný usmerňujúcim poruchovým pôsobením planét, *b)* výberový efekt, vyvolaný odlišnými podmienkami objavu pri rôznych polohách kométy voči Zemi a Slnku, *c)* výberový efekt, vyvolaný závislosťou jasnosti kométy od jej polohy voči slnečnému rovníku. Tretie vysvetlenie sa ukázalo ako najprijateľnejšie a podporuje Beyerovu domnienku o vzťahu medzi ekliptikálnou šírkou a jasnosťou komét, vyplývajúcom z anizotropie slnečného žiarenia. V prospech tohto vysvetlenia hovorí predovšetkým štatistická závislosť medzi dobou objavu kométy a heliocentrickou šírkou jej perihélia (obr. 5). Napriek pomerne malému materiálu sa ukazuje, že priebeh jasnosti skúmaných komét by mal byť asymetrický, s maximom až po prechode perihéliom; nie je teda vylúčená ani možnosť, že zmeny jasnosti by budilo korpuskulárne žiarenie, čo Beyer na základe priamych meraní jasnosti popiera.