

## LAGRANGE FORMALISM FOR NONLINEAR WHISTLER ENVELOPES

G. Mann  
Zentralinstitut für Astrophysik der AdW der DDR  
Observatorium für solare Radioastronomie  
DDR - 1501 Tremsdorf

**ABSTRACT.** A Lagrange formalism is developed for nonlinear whistler envelopes originated by the nonlinear interaction of the whistler waves with the background plasma. This problem is analogous to a mechanical movement in a double minimum potential. Both periodic and solarity whistler envelopes are described by this formalism. The existence conditions of the nonlinear whistler envelopes and their relations to the conserving quantities, the energy density and the phase integral, are discussed.

**МЕТОД ЛАГРАНЖА ДЛЯ НЕЛИНЕЙНОГО СВИСТОВОГО ПАКЕТА ВОЛН:** Метод Лагранжа развит для нелинейного свистового пакета волн, который возникает вследствие нелинейного взаимодействия между свистовыми волнами и плазмой. Поведение системы аналогично механическому движению в потенциале с двумя минимумами. Периодические и солитонные нелинейные свистовые волны описываются с помощью этого метода. Условия существования нелинейных свистовых волн и их отношение к величинам сохранения, как плотность энергии и фазовый интеграл, обсуждаются.

**LAGRANGEOVA METÓDA PRE NELINEÁRNE VLNOVÉ OBÁLKY TYPU HVIZD:** Lagrangeova metóda bola rozvinutá pre nelineárne vlnové obálky typu hvizd, ktoré vznikajú nelineárnou interakciou vln typu hvizd s okolitou plazmou. Tento problém je formálne analogický s mechanickým pohybom telesa v poli, ktorého potenciál má dvojité minimum. Uvedenou metódou je možné skúmať tak vlnové obálky periodických ako aj solitónových vln typu hvizd. V článku sú diskutované podmienky existencie nelineárnych obálok typu hvizd a ich vzťah k veličinám, ktorých hodnota sa zachováva (napr. hustota energie a fázový integrál).

The study of nonlinear whistler wave propagation is interesting to interpret various phenomena in the magnetospheric and the coronal plasma.

The nonlinear whistler wave propagation arises from the interaction of the envelope of the high-frequency whistler wave with the ponderomotive-induced plasma motion. This is described by a set of coupled differential equations, which consist of the nonlinear Schrodinger equation and the linearized ideal magnetohydrodynamic equations supplemented by the ponderomotive force (Karpman 1977, Karpman and Washimi 1977, Spatschek et al. 1979). Here we restrict ourselves to the case that all varying quantities only depend on the space variable  $z$  and the time  $t$ , where the external magnetic field is directed along the  $z$ -axis. In searching for stationary envelopes in the wave reference system ( $\xi = z - v_G t$ ,  $v_G$  - whistler group velocity), the set of coupled differential equations reduces to a single nonlinear differential equation generating by the Lagrangian

$$L = \frac{-c\omega_{pe}v'_G}{16\pi\omega_{ce}^2v_G} \frac{1}{\sqrt{x(1-x)}} \left[ \left( \frac{da}{d\xi} \right)^2 + Aa^4 - 2Ba^2 \right] \quad (1)$$

with  $A = \frac{\omega_{pe}^2}{16\pi\varrho_0\omega_{ce}v'_G(v_G^2 - c_s^2)}$  and  $B = \Delta\omega_0/v'_G$

( $c$  - velocity of light,  $\omega_{pe}$  - electron plasma frequency,  $\omega_{ce}$  - electron cyclotron frequency,  $\varrho_0$  - mass density,  $v'_G$  - group dispersion,  $c_s$  - sound velocity,  $x = \omega_W/\omega_{ce}$ ,  $\omega_W$  - whistler frequency,  $\Delta\omega_0$  - nonlinear frequency shift,  $a$  - envelope amplitude).

Under the conditions in the magnetospheric and the coronal plasma we find a large supersonic group velocity.

The Lagrangian (1) corresponds to a mechanical movement in the potential  $V \sim (2Ba^2 - Aa^4)$ , if "a" denotes the particle coordinate. In such potential both periodic and nonperiodic (solitary) movements can occur, if the potential has two minima at  $a = \pm (B/A)^{1/2}$  for  $\Delta v'_G > 0$ . These periodic and solitary movements are stable for  $\Delta < 0$  fulfilled for  $x > 0.25$  in the case of supersonic group velocities. Analogously to the framework of the classical mechanics the occurrence of the periodic or solitary movement depends on the conserving quantities and the parameters of the system ( $\omega_{pe}$ ,  $\omega_{ce}$ , and  $x$ ) (Schmutzer 1973). In this case we have two conserving quantities, the Hamiltonian (energy density) and the phase integral, defined by

$$H = \frac{\partial L}{\partial \left( \frac{da}{d\xi} \right)} \frac{da}{d\xi} - L \quad (2)$$

and

$$I = \frac{1}{2\pi} \oint \frac{\partial L}{\partial \left( \frac{da}{d\xi} \right)} da \quad (3)$$

(Schmutzer 1973).

Thus, the individual form of the whistler envelope is determined by the special values of the Hamiltonian (energy density), the phase integral, the parameters of the background plasma, and the whistler frequency (Mann 1986).

#### REFERENCES

- Karpman, V.I.: 1977, *Nichtlineare Wellen*, Akademie Verlag, Berlin.
- Karpman, V.I., Washimi, H.: 1977, *J. Plasma Phys.* 18, 173.
- Mann, G.: 1986, *J. Plasma Phys.* in press.
- Schmutze, E.: 1973, *Grundprinzipien der klassischen Mechanik und der klassischen Feldtheorie*, VEB Deutscher Verlag der Wissenschaften, Berlin.
- Spatschek, K.H., Shukla, P.K., Yu, M.Y., Karpman, V.I.: 1979, *Phys. Fluids* 22, 576.