PREDICTION OF MONTHLY MEAN VALUES OF SOLAR ACTIVITY INDEX F_{2800} USING A MULTIPLICATIVE AUTOREGRESSIVE MODEL

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ABSTRACT. A new autoregressive method is proposed for predicting monthly mean values of the solar radio emission on 2800 MHz, F_{2800}. A prediction one month ahead is carried out using a high-order model. Prediction filter parameters are determined with the aid of the Burg algorithm which id used for estimating the power spectrum density by a maximum entropy method. The optimum--filter length is determined from the condition for variation coefficient minimum. Considering several examples of retrospective prediction, a study is made of the main properties of the prediction model.

PROGNÓZA STREDNÝCH MESAČNÝCH HODNÔT INDEXU F_{2800} SLNEČNEJ AKTIVITY, POČÍTANÁ MNOHONÁSOBNÝM AUTOREGRESÍVNYM MODELOM: Nová autoregresívna metóda umožňuje predpovedať stredné mesačné hodnoty rádiovej emisie Slnka na frekvencii 2800 MHz (F_{2800}). Prognóza je počítaná jeden mesiac dopredu, pomocou modelu vyšších rádov. Burgovým algoritmom sú určované prognózne filtrové parametre a hustota výkonového spektra metódou maximálnej entropie. Z podmienky pre minimálnu variáciu parametrov je vypočítaná optimálna dĺžka filtra. Niekoľko prípadov retrospektívnej prognózy umožnilo určiť hlavné vlastnosti predpovedného modelu.

Monthly mean values of the solar radio emission on 2800 MHz, like most of the other solar activity indices, form a nonstationary time series. Jakimiec (1976) has shown that the time series of values of the F_{2800} index is a multiplicative one. Let us represent the original process F_t, according to the multiplicative model, as

$$F_t = F_t^s(1 + F_s^f) ,$$

where F_t^s is a function which describes the smoothed-out variation of the index over a solar cycle, and the fluctuation addition $F_t^s F_t^f$ is proportional to the mean level of the index at a given instant of time and involves the factor F_t^f which takes fluctuations proper into account. The transformation

$$Y_t = \ln F_t \tag{1}$$

allows us to switch over from the nonstationary time series F_t to the time series Y_t which is characterized by an about constant variance with respect to the mean level of the index. For purposes of global indices predicting of complex systems, autoregressive methods are often successfully employed (Lukashin, 1979). We will be seeking a model that is adequate to the time series Y_t in a class of autoregressive processes

$$Y_i = \sum_{l=1}^{L} W_l \, Y_{i-l} + \mathcal{E}_i, \quad i = 1,2,\dots, N, \tag{2}$$

where Y_i is a centered value of the discrete time series Y_t at time $t = t_i$, L is the order of the model, and $\mathcal{E}_i$ is white noise with zero mathematical expectation. The filter parameters W_1, W_2, ..., and W_L are obtainable using the Burg procedure which is applied for spectral evaluation by the maximum entropy method (see, e.g. Kiselov, 1980).

The i-th value of the time series Y_i is predicted with the aid of the prediction filter

$$\tilde{Y}_i = \sum_{l=1}^{L} W_l \, Y_{i-l}$$

Prediction errors $\mathcal{E}_i = Y_i - \tilde{Y}_i$ form a time series of prediction errors. Optimum values of the filter coefficients W_1, W_2, ..., and W_L are determined from the condition for the minimal sum of the prediction errors squared

$$\frac{\partial}{\partial W_k} \left[\sum_{i=1}^{L} \mathcal{E}_i^2 \right] = 0 , \qquad k = 1, 2, \dots , L.$$

The procedure of estimating the prediction filter parameters is based on recurrent estimations by sequentially increasing the order of the model. At the $(m + 1)$-th step, the parameters $W_{1,m+1}$, $W_{2,m+1}$, ..., and $W_{m+1,m+1}$ are related to the previous-step parameters $W_{1,m}$, $W_{2,m}$, ..., and $W_{m,m}$ by

$$W_{1,m+1} = W_{1,m} - W_{m+1,m+1} \cdot W_{m,m} ,$$

$$W_{2,m+1} = W_{2,m} - W_{m+1,m+1} \cdot W_{m-1,m} ,$$

$$W_{m,m+1} = W_{m,m} - W_{m+1,m+1} \cdot W_{1,m} ,$$

$$P_{m+2} = P_{m+1}(1 - W_{m+1,m+1}^2) ,$$

where $P_{m+1} = \frac{1}{m} \sum_{i-1}^{L-m} \varepsilon_i^2$ is the power of the series of prediction errors. The parameters $W_{m+1,m+1}$ are calculated from the formulae

$$W_{m+1,m+1} = \frac{2 \cdot \sum_{j=1}^{L-m} \varepsilon^f_{j+m+1,m} \varepsilon^b_{j,m}}{\sum_{j=1}^{L-m} \left\{ \left[{}^{f}_{j+m+1,m} \right]^2 + \left[\varepsilon^b_{j,m} \right]^2 \right\}}$$

where

$$\varepsilon^f_{j,m} = Y_j - W_{1,m} Y_{j-1} - \dots - W_{m,m} Y_{j-m} ,$$

$$\varepsilon^b_{j,m} = Y_j - W_{1,m} Y_{j+1} - \dots - W_{m,m} Y_{j+m} ,$$

are the prediction errors in forward and inverse directions. The final values of the filter parameters $W_{1,L}$, $W_{2,L}$, ..., and $W_{L,L}$ are determined for $m + 1 =$ $= L$.

In order to determine the order of the model, L, let us consider a partial autocorrelation function (PAF). By PAF of the process Y_t we mean the quantity $W_{L,L}$ considered to be a function of L. For a stationary autoregressive p order process the PAF fluctuates abot zero when L p + 1 (Lukashin, 1979).

A criterion for breaking off the PAF is

$$\sigma(W_{L,L}) \simeq 1/\sqrt{N} \tag{3}$$

where $\sigma(W_{L,L})$ is the standard error of the thus estimated partial autocorrelation. Figure 1 shows a PAF of the process Y_t that was constructed with the use of data covering a time interval from 1948 to June 1985 ($N = 450$). The PAF decreases rapidly to zero already for $L = 4$; for larger lags the PAF fluctuates about zero. The fluctuation amplitude decreases to reach a value close to $1/\sqrt{N} \simeq 0.05$ after $L = 20$. This leads us to conclude that L 20. The criterion (3) has an approximate character. In order to choose the optimal length of the prediction filter and to illustrate the capabilities inherent to different models, let us consider the dependence of prediction errors on the order of the model.

A time series of monthly mean values of the F_{2800} index, reduced to 1 AU, for the period 1948 - June 1985 was taken as initial data on F_t. According to the model in (1) and (2), the prediction one month ahead was performed using the formula

$$\tilde{F}_i = \exp \left(\sum_{l=1}^{L} W_l Y_{i-l} + \bar{Y} \right) ,$$

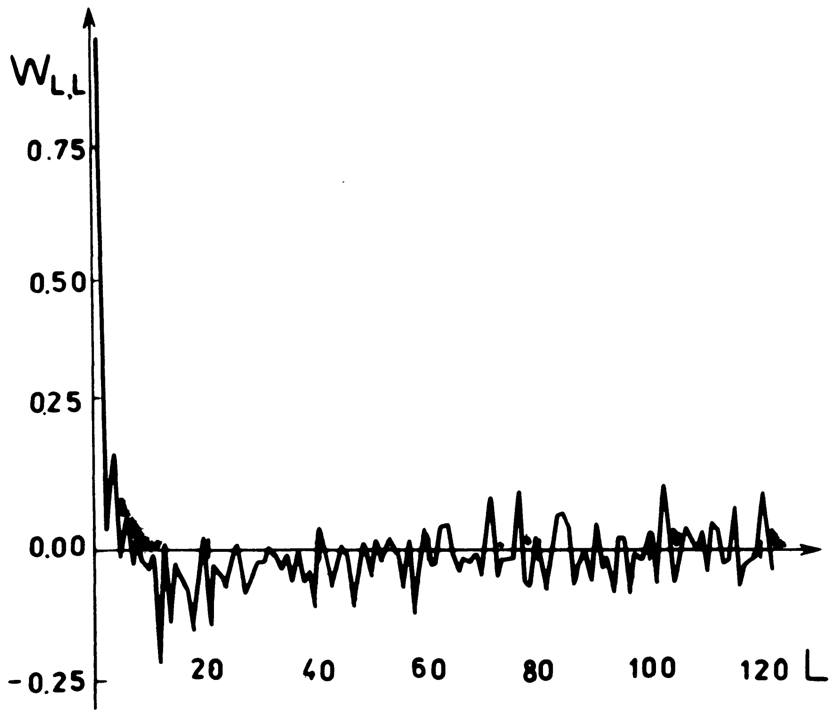

Fig. 1: The partial autocorrelation function of the process Y_t, as constructed using F_{2800} data for the time interval 1948 - June 1985.

where $\bar{Y}$ is an estimate of mathematical expectation Y_t. For different i and L, a retrospective prediction was performed and the prediction accuracy estimated. The prediction quality is characterized by the variation coefficient

$$V = \frac{n\sqrt{\sum_{i=1}^{n}\left(F_i - \tilde{F}_i\right)^2}}{\sqrt{n-1}\ \sum_{i=1}^{n} F_i} \cdot 100\ \%$$

where F_i and $\tilde{F}_i$ are the observed and the predicted values of the series F_t, and n is the number of predictions done. Figure 2 presents the dependence of V on the order of the model L. The predictions have been performed for the last five years, with n = 60. As is apparent from the V(L)-plot, for L = 5, 6 there is a slight decrease of the variation coefficient; afterwards, the prediction error increases. A global minimum of V occurs when L = 18, 19. At larger L, the variation coefficient increases. Such a decline in the prediction quality is, seemingly, due to the excess sensitivity and the instability of multi-parameter models.

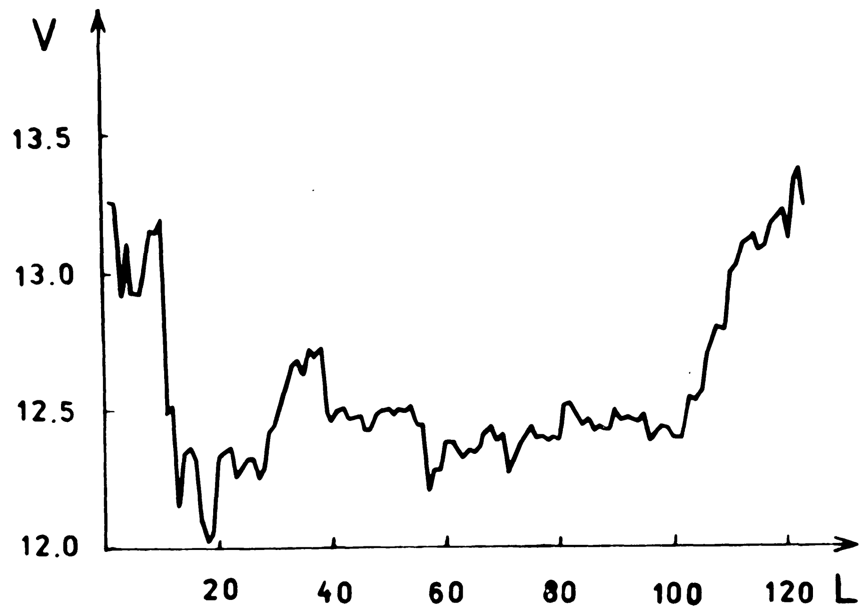

Fig. 2: The dependence of the variation coefficient V on the order of the model L.

Predictions using the procedure we elaborated showed that both the optimal order of the model and the values of the parameters W_l vary in different time intervals. For the optimal logarithmic autoregressive model of the order L=18, to be referred to as LAR (18), the prediction filter coefficients, in ascending order of ℓ, are: 0.8968, -0.1885, 0.2422, -0.0970, 0.0999, -0.0296, 0.0293, 0.0917, -0.0664, 0.2399, -0.2463, 0.1526, -0.1122, 0.0392, 0.0040, 0.0452, -0.0833, and -0.0690; $\bar{Y}$ = 4.795. Values of the parameters W_l are determined from those of F_{2800} from 1948 to 1985 and bear an averaged character. Highest - quality predictions are obtained with the use of an adaptive version of the method proposed in which the prediction filter parameters are determined for a relatively short time interval (of a few years) immediately before performing a prediction. The adaptation mechanism implies that for the time interval under consideration, an optimal predicstion model is always constructed from the variation coefficient minimum condition. In so doing, a feedback is realized when the altered characteristics of the process analyzed introduce a reconstruction in the prediction model. Such a self-adjusting model is capable of representing new tendencies in the development of the process. The adaptive prediction model often includes lags corresponding to two years, which is accounted for by the presence of quasi-biennial variations of solar activity (Apostolov, 1985).

For the sake of comparison, we want to point out that the Mayaud method, often used in prediction practice, for L = 4 (Vitinsky, 1973) for the period 1982-1985 yields V = 16.6 % whereas in the LAR (18) model the variation coefficient is appreciably smaller, V = 12.1 %. For another time interval from March 1980 to February 1982, the LAR (20) model yields V = 11.4 % rather than 13,6 % for the modernized Mayaud method (Vitinsky et al., 1982). The high quality of the LAR prediction model is also evidenced by the absence of significant autocorrelations of ε_i remainders. Errors ε_i are uncorrelated noise whose power P_{L+1} decreases with increasing order of the model. A rapid decrease in the power of noise occurs up to L = 20; afterwards, there occurs a relatively slow decrease of P which is, to some extent, of formal character because there is actually no improvement of the prediction.

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