# A CLASSIFICATION OF CYCLE PAIRS ACCORDING TO THE "CYCLE HEIGHT-ASCENDING BRANCH LENGTH" RELATIONSHIP TYPE AND RECONSTRUCTION OF THE SOLAR ACTIVITY LEVEL VARIATION IN THE 16TH AND 17TH CENTURIES

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ABSTRACT. It is found that there exist 4 variants of dependence of the height of a maximum of 11-year cycles on the length of their ascending branch (2 for even and 2 for odd cycles). This permits the pairs of "even-odd" cycles to be classified according to the character of this dependence for the members of a pair. A solving rule has been constructed which permits the determination of the pair type from ascending branch lengths of the members of a pair. From a knowledge of cycle extrema epoch, the cycle heights are derivable on the basis of the regularities obtained. In such a way, the behaviour of the solar activity level during the 16th and 17th centuries has been reconstructed. The realization obtained of a duration of nearly 480 years suggests that the "solar cycle" dynamical system is either a strange attractor or a spirally-chaotic attractor.

**КЛАССИФИКАЦИЯ ПАР ЦИКЛОВ ПО ТИПУ СВЯЗИ "ВЫСОТА ЦИКЛА - ДЛИНА ВЕТВИ РОСТА" И РЕКОНСТРУКЦИЯ ИЗМЕНЕНИЙ УРОВНЯ СОЛНЕЧНОЙ АКТИВНОСТИ В I6 И I7 ВЕКАХ.** Обнаружено, что существует 4 варианта зависимостей высоты максимума II - летних циклов от продолжительности их ветви роста (2 для четных и 2 для нечетных циклов). Это позволяет классифицировать пары циклов "четный-нечетный" в соответствии с характером указанной зависимости у членов пары. Построено решающее правило, позволяющее определить тип пары по длинам ветвей роста членов пары. Зная эпохи экстремумов циклов, на основе найденных закономерностей можно найти высоты циклов. Таким путем восстановлен ход уровня солнечной активности в I6 и I7 веках. Построенная реализация продолжительностью почти 480 лет дает возможность считать, что динамическая система "солнечный цикл" является либо странным аттрактором, либо спирально-хаотическим аттрактором.

KLASIFIKACE PÁRŮ CYKLŮ PODLE TYPU ZÁVISLOSTI "VÝŠKA CYKLU-DÉLKA VĚTVE RŮSTU" A REKONSTRUKCE ZMĚN ÚROVNĚ SLUNEČNÍ ČINNOSTI V 16. A 17. STOLETÍ. Bylo zjištěno, že existují 4 varianty závislosti výšek maxima 11-letých cyklů na délce větve růstu (2 pro sudé a 2 pro liché cykly). To umožňuje klasifikovat páry cyklů (sudý-lichý) podle charakteru této závislosti u členů páru. Je odvozeno základní pravidlo, které umožňuje určit typ páru podle délky větví růstu členů páru. Jestliže známe extrémy cyklů, můžeme najít výšku cyklů na základě těchto zákonitostí. Tímto způsobem byl rekonstruován průběh úrovně sluneční činnosti v období 16. a 17. století. Obdržená realizace v délce okolo 480 let ukazuje, že dynamický systém "sluneční cyklus" je buď divný nebo spirálně-chaotický atraktor.

The fundamental property of the 11-year solar activity cycles is the inverse relation between the cycle height $W_M^N$ and the duration of its ascending branch $T_A^N$: the higher the cycle, the shorter its ascending branch. Waldmeier (1935) found an empirical formula describing this relation and ascertained that coefficients of this formula are different for even and odd cycles. Chistyakov (1959) discovered that each of these relations divides into two: the correlation between $W_M^N$ and $T_A^N$ for even and odd cycles may be represented in each case by two linear expressions (two almost parallel lines on the plot) but this conclusion was not proved sufficiently because the data sample were small.

We were able to show (Kuklin, 1985a) that there really exist 4 variants of the relation

$$W_M^N = \alpha - \beta\, T_A^N$$

which corresponds to cycles of types A,B,C and D we introduced, and that the reliability of this conclusion exceeds 0.90 (Fig.1).
In particular,

$$\begin{array}{llll}
\alpha_A = 239.4, & \alpha_B = 201.5, & \alpha_C = 331.1 & \alpha_D = 292.0, \\
\beta_A = 27.79, & \beta_B = 25.87, & \beta_C = 40.11, & \beta_D = 46.10, \\
r_A = -0.993, & r_B = -0.995, & r_C = -0.987, & r_D = -0.986.
\end{array}$$

Here r is the linear correlation coefficient. The source data are given in Table 1 where the cycle types are also shown.

If we form cycle pairs of the "even-odd" kind, we find that our sample contains 4 pairs of the AC type, no pairs of the AD type, 1 pair of the BC type and 5 pairs of the BD type. Let us designate the duration of the ascending branch of the preceding even cycle in each pair as $T_A'$ and of the following odd one as $T_A''$. In Fig. 2, which shows 10 points representing all pairs in the mentioned coordinates, it is not difficult to see that there is a dividing line for the pairs with preceding cycles of the A and B types, as well as with following cycles of the C and D types. The discrimination functions which enab-

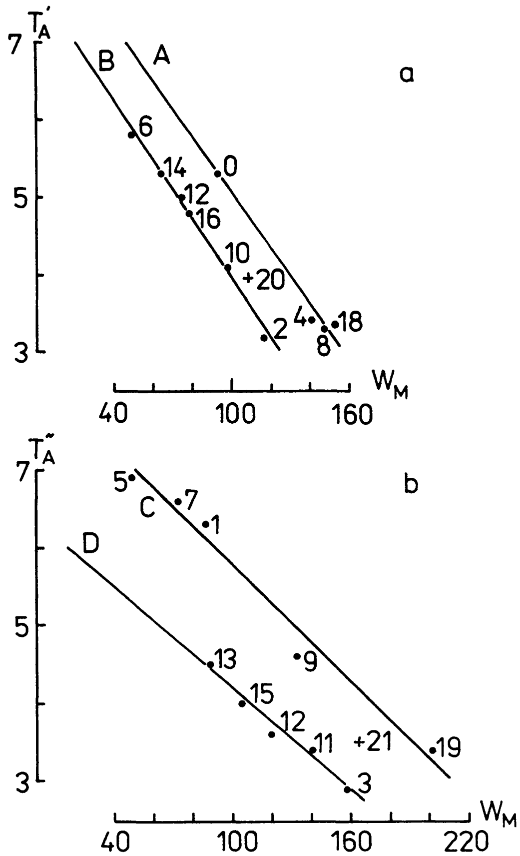


Fig. 1 The relationships between the cycle height $W_M$ and ascending branch duration $T_A$ for even(a) and odd(b) cycles.

le the cycle pairs to be classified by the values $T'_A$ and $T''_A$ have the following forms (Kuklin, 1985b)

$$Q' = 0.821\, T'_A - 0.571\, T''_A - 0.869,$$
$$Q' = 0.730\, T'_A - 0.685\, T''_A - 0.234.$$

$Q' < 0$ for cycles of the A type, $Q' > 0$ for cycles of the B type, $Q'' < 0$ for cycles of the C type and $Q'' > 0$ for cycles of the D type.

Cycles 20 and 21 do not satisfy the variants of the $W_M(T_A)$ relation. The analysis has shown that this pair rather belongs to the BC type but in this case it is necessary to suppose that systematic errors have been made in estimating accepted values of $W_M$ and $T_M$ for these cycles (Kuklin, 1985b).

Let us designate the cycle pair in which the preceding even cycle is lower than the following odd one as G and the cycle pair the preceding even cycle is higher than the following odd one as F. The analysis of the 2x2 contin-

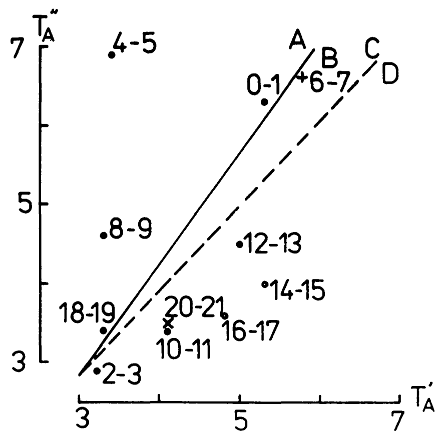


Fig. 2 The classification of the cycle pairs according to the ascending branch durations of the preceding even cycle $T_A'$ and the following odd one $T_A''$. Straight lines correspond to $Q'= 0$ (solid) and $Q''=0$(dashed).

Table 1

| N | $T_m^N$ | $T_M^N$ | $T_A^N$ | $W_M^N$ | | N | $T_m^N$ | $T_M^N$ | $T_A^N$ | $W_M^N$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1745.0 | 1750.3 | 5.3 | 92.6 | A | 11 | 1867.2 | 1870.6 | 3.6 | 140.5 | D |
| 1 | 1755.0 | 1761.5 | 6.3 | 86.5 | C | 12 | 1878,9 | 1883.9 | 5.0 | 74.6 | B |
| 2 | 1766.5 | 1769.7 | 3.2 | 115.8 | B | 13 | 1889.6 | 1894.1 | 4.5 | 87.9 | D |
| 3 | 1775.5 | 1778.4 | 2.9 | 158.5 | D | 14 | 1901.7 | 1907.0 | 5.3 | 64.2 | B |
| 4 | 1784.7 | 1788.1 | 3.4 | 141.2 | A | 15 | 1913.6 | 1917.6 | 4.0 | 105.4 | D |
| 5 | 1798.3 | 1805.2 | 6.9 | 49.2 | C | 16 | 1923.6 | 1928.4 | 4.8 | 78.1 | B |
| 6 | 1810.6 | 1816.4 | 5.8 | 48.7 | B | 17 | 1933.8 | 1937.4 | 3.6 | 119.2 | D |
| 7 | 1823.3 | 1829.9 | 6.6 | 71.7 | C | 18 | 1944.2 | 1947.5 | 3.3 | 151.8 | A |
| 8 | 1833.9 | 1837.2 | 3.3 | 146.9 | A | 19 | 1954.5 | 1957.9 | 3.4 | 201.3 | C |
| 9 | 1843.5 | 1848.1 | 4.6 | 131.6 | C | 20 | 1964.8 | 1968.9 | 4.1 | 110.6 | ? |
| 10 | 1856.0 | 1860.1 | 4.1 | 97.9 | B | 21 | 1976.5 | 1980.0 | 3.5 | 164.5 | ? |

gency tables (Upton, 1982) in order to establish the relations between the pair types and their properties yielded the following results. The combination rule AC/BD is non-stochastic with its reliability equal to 0.978 (the cycle pair 20-21 was considered to be BC). At the same time, the non-stochastic relation AF/BG exists with reliability equal to 0.9928. But the relation with the odd cycle type is expressed more weakly. The reliability of it being non-stochastic is 0.934. It looks as if the character of the pair is determined to a large degree by the preceding even cycle. The alternation rule of pairs with property G/F has been established recently (Kuklin, 1984).

The results obtained above are based on a small data sample, but more sufficiently reliable and homogeneous and also extended data are not available.

Table 2

| N | $T_m^N$ | $T_M^N$ | $T_A^N$ | $W_M^N$ | N | $T_m^N$ | $T_M^N$ | $T_A^N$ | $W_M^N$ |
|---|---|---|---|---|---|---|---|---|---|
| -22 | 1501.5 | 1506.5 | 5.0 B | 72.1 | -11 | 1620.2 | 1625.8 | 5.6 C | 106.5 |
| -21 | 1514.2 | 1517.7 | 3.5 D | 130.6 | -10 | 1633.7 | 1639.3 | 5.6 B | 56.6 |
| -20 | 1524.7 | 1528.2 | 3.5 B | 110.9 | -9 | 1645.5 | 1650.8 | 5.3 D | 47.6 |
| -19 | 1534.3 | 1537.4 | 3.1 D | 149.0 | -8 | 1655.5 | 1661.0 | 5.5 A | 86.6 |
| -18 | 1543.7 | 1547.4 | 3.7 B | 105.8 | -7 | 1666.7 | 1673.5 | 6.8 C | 58.3 |
| -17 | 1554.5 | 1558.3 | 3.8 C | 178.6 | -6 | 1679.5 | 1685.0 | 5.5 B | 59.2 |
| -16 | 1567.7 | 1571.3 | 3.6 B | 108.3 | -5 | 1689.5 | 1694.5 | 5.0 D | 61.4 |
| -15 | 1578.4 | 1581.5 | 3.1 D | 149.0 | -4 | 1699.0 | 1705.7 | 6.5 B | 33.3 |
| -14 | 1587.2 | 1593.8 | 6.6 B | 30.7 | -3 | 1712.5 | 1718.2 | 5.7 D | 29.2 |
| -13 | 1598.8 | 1604.4 | 5.6 D | 33.8 | -2 | 1723.5 | 1727.5 | 4.0 A | 128.3 |
| -12 | 1608.9 | 1614.3 | 5.4 B | 61.8 | -1 | 1734.0 | 1738.7 | 4.7 C | 142.6 |

However, our conclusions are supported by the fact that our consideration is based on the fundamental Hale law of magnetic field polarity alternation according to which the cycles should combine into pairs with undoubted causal relations of a physical nature.

We shall attempt to use the regularities we have found to reconstruct the solar activity changes in the historical past which are necessary if its mechanism is to be studied over long time intervals. This problem is similar to the problem of forecasting solar activity. One finds values of solar activity using the regularities established for relatively short time intervals (in this particular case the Zurich data only cover 280 years) and by extrapolating them into the past or the future. However, the reconstruction, only in rare cases as distinct from the forecast, may be verified with the help of indirect data because direct observations in the past are practically absent.

We have used Schove's data (1979) on the extremum epochs of 11-year cycles in the 16-th and 17-th centuries. After estimating the durations of the ascending branches of the cycles from -22 to -1 and combining the cycles into pairs, we determined the pair types and found the cycle heights using the relation $W_M(T_A)$ and taking into account the determined types of cycles. The results are given in Table 2 and Fig. 3.

Our computations agree with the estimates of Chistyakov (1985) satisfactorily at the qualitative level and in some cases at the quantitative level. This was to be expected in as much as both methods in the end are based on the same regularity but the inner accuracy of our results for cycles 0-19 is much higher. But the comparisons with the radiocarbon method data lead to a worse agreement. However, the very calibration of the radiocarbon method on the Wolf number scale is somewhat vague because of several difficulties connected with systematic errors (due to opposite signs of the galactic and solar cosmis rays effects) and needs a special analysis.

Now when all 22 cycle pairs are considered, the conclusions about the non-stochastic combinations of cycles become more valid: for combination AC/BD

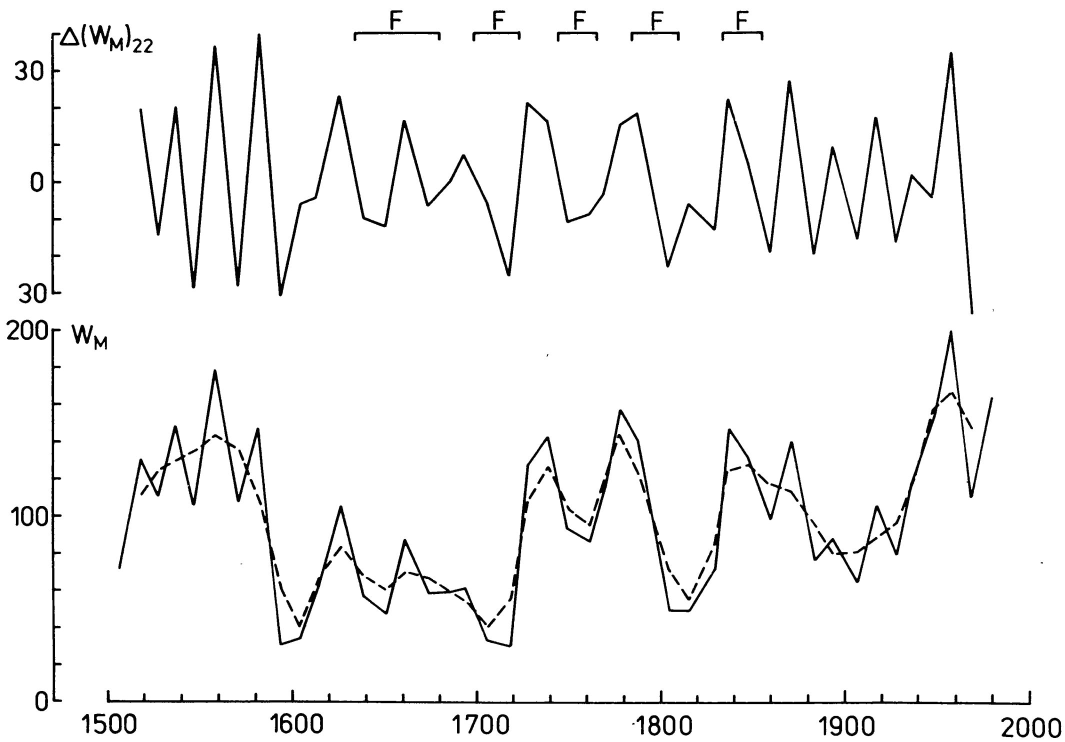


Fig. 3 The lower part - the $W_M$ variation in the years 1500-1980 (solid line) and the result of its smoothing in order to eliminate the pair effect (dashed line). The upper part - the $\Delta(W_M)_{22}$ variation (deviations from the smoothed $W_M$). Labels F correspond to pairs of the F type when the preceding cycle is higher than the following one.

the reliability is equal to 0.9984, for combination AF/BG to 0.9889 and for combination DG/CF to 0.779. These figures may be overestimated since, in computing $W_M^N$, we relied upon the cycle grouping into pairs, nevertheless, the results argue in favour of the reality of the pairs and the principal role of preceding cycles in pairs. This throws some doubt on Rivin's statement (1985) that in periods of low solar activity (the Maunder minimum), the cycles are not organized in pairs. The upper part of Fig. 3 shows the variation $\Delta(W_M)_{22}$ from which it follows that an apparent decrease of the variation amplitude may take place even at a comparatively high level of solar activity.

It is interesting to study the envelope of the 11-year cycle peaks during the time interval covering 480 years (1500-1980). We have approximated this curve by a cubic spline and computed its derivative. Also the phase trajectory was plotted (Fig.4). It is seen clearly that the phase trajectory has at least two foci around which the representative point moves along diverging or converging spirals. From time to time quasi-stochastic jumps occur from one region

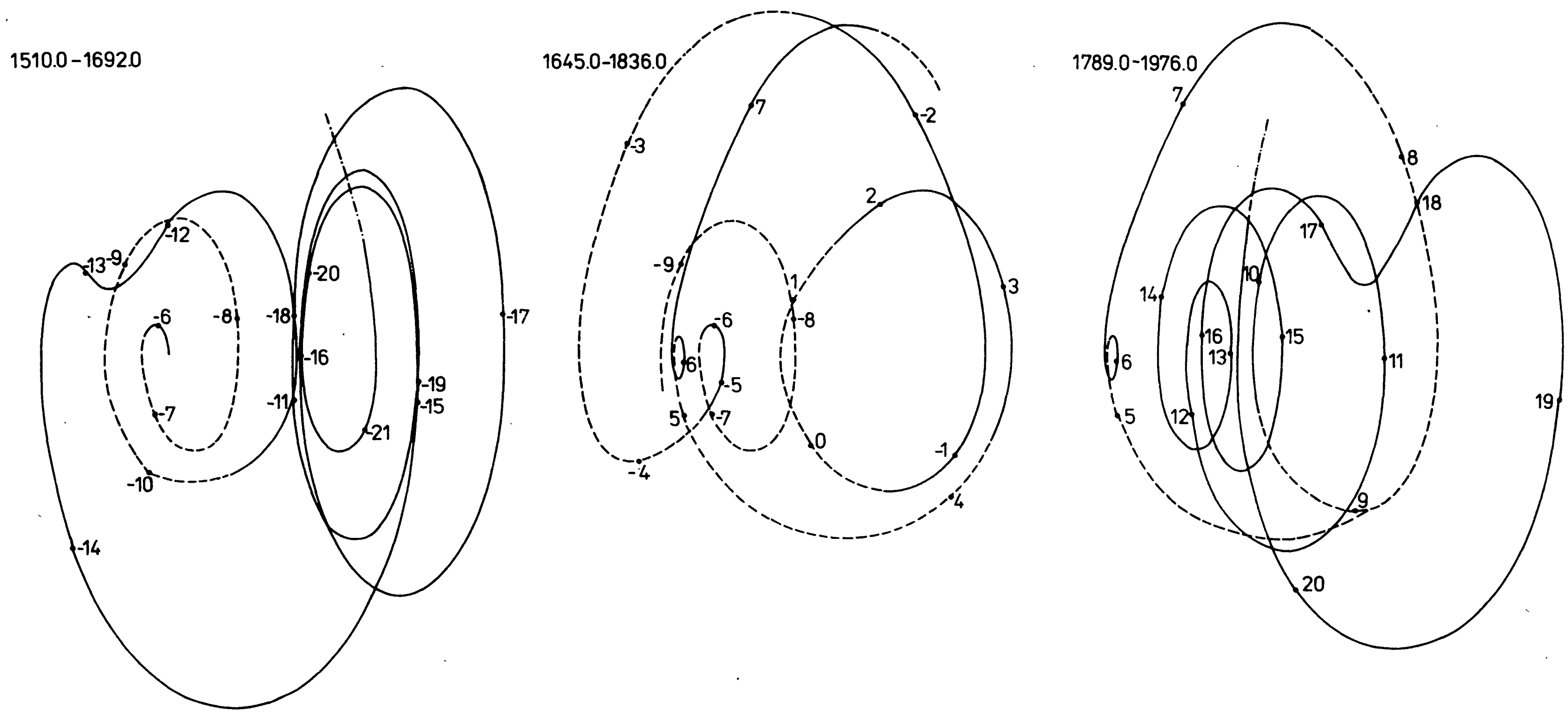


Fig. 4 The phase trajectory of the $W_M$ envelope curve. The numbers mark the corresponding cycle maximum positions. The solid line for pairs of the G type and the dashed one - of the F type.

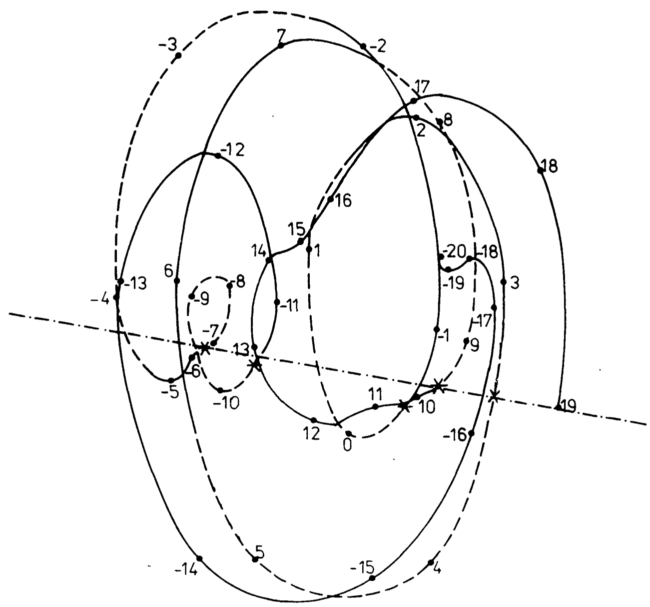


Fig. 5 The modification of the phase trajectory using the smoothed $W_M$ values and the cycle number as an argument. Designations are the same as in Fig.4. Asterisks mark changes of the G/F pair types and they lie on the straight line (dash-dot).

to the other. The whole pattern resembles the behaviour of the phase trajectory of a spirally chaotic or strange attractor (Gilmore, 1984) if the phase space is viewed from a direction which does not coincide with the coordinate axes.

If the cycle number is used instead of real time argument and the cycle-pair effect is eliminated by smoothing, the curve of the phase trajectory type has a similar appearance (Fig.5). In this case it is possible to earmark points corresponding to the changes of the pair types or to the beginnings of phase catostrophes (Kuklin, 1982). It is remarkable that these points practically lie on a straight line, starting from the coordinate origin and interesecting the phase trajectory at the fixed phase after recurrent maximum of the curve smoothing $W_M$. Therefore, the anomalies may be generated at the fixed phase of a quasi-secular or long cycle of solar activity: this is what occurred in 5 of the 6 cases of intersections. The last seventh intersection is on the decreasing branch of cycle 19 and, perhaps, this is what led to a violation of some regularities for cycles 20-21.

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