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ABSTRACT. Minnaert's diagram can be used to construct a "diagram of observational conditions" of sunspot groups with the aid of which a number of problems of statistics of sunspot on the rotating Sun can be solved by means of computer modelling or the graphical method. This diagram was used to demonstrate the effect of the visibility function and of the shape of the sunspot area development curve on the frequency distribution of sunspot groups according to their observed lifetimes, on the E-W asymmetry of sunspot generation and on other statistical parameters. The results indicate that a correct interpretation of the results of the statistical processing of sunspot group observations is complicated.

ДИАГРАММА УСЛОВИЙ НАБЛЮДЕНИЙ ГРУПП СОЛНЕЧНЫХ ПЯТЕН И ЕЁ ИСПОЛЬЗОВАНИЕ. На основе диаграммы Миннаерта можно построить "диаграмму условий наблюдений" групп пятен позволяющую решить задачи статистики пятен на вращающемся Солнце путем моделирования на ЭВМ или графическим методом. С помощю этой диаграммы продемонстрировано влияния функции видимости и формы кривой розвития площади групп пятен на частотное распределение групп пятен по их наблюдаемой продолжительности жизни, на восточно-западную асимметрию появления групп пятен и на другие статистические параметры. Результаты показывают сложность правильной интерпретации результатов статистической обработки наблюдений групп солнечных пятен.

DIAGRAM PODMÍNEK POZOROVATELNOSTI SKUPIN SKVRN A JEHO POUŽITÍ. Na základě Minnaertova diagramu je možno zkonstruovat "diagram podmínek pozorovatelnosti" skupin skvrn umožňující řešit řadu úloh statistiky skvrn na rotujícím Slunci modelováním pomocí počítače nebo grafickou metodou. Použitím tohoto diagramu je ukázán vliv funkce viditelnosti a tvaru křivky vývoje plochy skupin skvrn na četnostní rozdělení skupin skvrn podle jejich pozorované životní doby, na E-W asymetrii vznikání skupin skvrn a na další statistické parametry. Výsledky ukazují na složitost správné interpretace výsledků statistic

kého zpracování pozorovacích materiálů skupin slunečních skvrn.

Minnaert's diagram /Minnaert 1939/ provides an illustrative idea of the effect of the visibility function, the rotation of the Sun and the shape of the sunspot area development curve on various statistical laws and statistical parameters of sunspot groups. It can be solved mathematically only under particular simplifying assumptions /Link 1951, Kopecký 1956/, which enables certain qualitative conclusions to be drawn.

However, Minnaert's diagram can be used to construct the "diagram of observational conditions" /hereinafter OC diagram/ of sunspot groups which enables problems of statistics of sunspot on the rotating Sun to be modelled /Kopecký et al. 1985/.

If the angular distance of the place where the sunspot group actually originated from the CM is denoted  $\lambda_i$ , the distance to the place where it actually went extinct  $\lambda_i$ , and the angular distance from the CM of the place where the group was first observed on the solar disk  $\lambda_i$  and the distance of the place where the group was observed on the solar disk last  $\lambda_i$ , the OC diagram represents the dependence of  $\lambda_i$  and  $\lambda_i$  on  $\lambda_i$  for the given sunspot visibility curve  $S_i\phi(\lambda)$  and the given sunspot area development curve S(t,T).

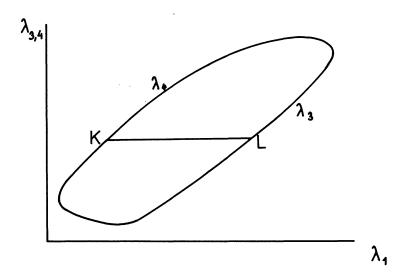


Fig. 1

The OC diagram is shown schematically in Fig. 1 for the case of uninterrupted sunspot group observations. If a line is drawn in this diagram parallel with the vertical axis, the length of the line segment MN is proportional to the observed lifetime  $T = (\lambda_4 - \lambda_3)/\omega$  of the group, where  $\omega$  is the angular velocity of the Sun's rotation. If a line is drawn in the OC diagram parallel with the  $\lambda_4$ -axis, the length of the line segment KL is proportional to the number of groups observed at distance  $\lambda = \lambda_{1,4}$  from the CM.

In reality, sunspot groups are not observed continually, without inter-

ruption, but only once in 24 hours. To adapt the OC diagram to this fact, we shall proceed as follows: we shall displace curve  $\lambda_3(\lambda_1)$  upwards by  $\omega$ ,  $2\omega$ ,  $3\omega$ , etc., and curve  $\lambda_{\mu}(\lambda_1)$  downwards by  $\omega$ ,  $2\omega$ ,  $3\omega$ , etc. /see Fig. 2/.

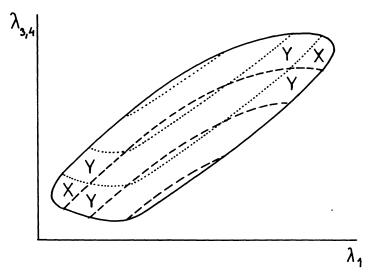
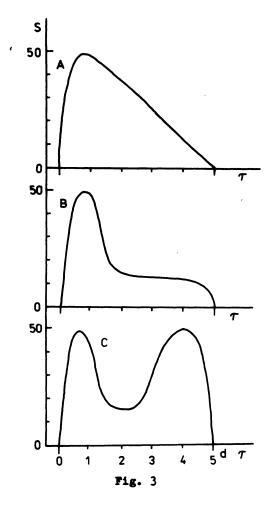


Fig. 2

The groups observed for the first time are located in the area between curves  $\lambda_3(\lambda_l)$  and  $\lambda_3(\lambda_l)+\omega$  , the groups observed for the second time are located in the area between curves  $\lambda_3(\lambda_l)+\omega$  and  $\lambda_3(\lambda_l)+\omega$  etc. Similarly, the groups observed for the last time are located in the area betwenn curves  $\lambda_4(\lambda_l)$  and  $\lambda_4(\lambda_l)-\omega$ . The system of these curves then creates areas which have the following significance: The area marked X contain the groups which, if observations are made once in 24 hours, are observed for the first time and, simultaneously, for the last time; their observed lifetime is thus 1 day and the groups will be referred to as one-day groups. The area of X is also proportional to the number of these apparently one-day sunspot groups. Areas Y contain the groups which, if observations are made once in 24 hours, are observed on 2 consecutive days and which will be referred to as two-day groups, etc.

A more detailed description of the OC diagram and its practical uses can be found in the papers by Kopecký et al. /1985/ and Kopecký /1986/. In this paper, we shall now concentrate on some of the most substantial results achieved with the aid of the OC diagram so far.

In modelling with the aid of the OC diagram, we adopted two shapes of the visibility curve  $S^* = S_o\phi(\lambda)$ , i.e.  $S^* = 2\sec\lambda$  and  $S^* = 5\sec^2\lambda/S^*$  is the minimum sunspot group area visible at distance  $\lambda$  from the CM;  $S_o$  is the minimum sunspot group area visible in the centre of the solar disk;  $\phi(\lambda)$  is the visibility function; the areas in this paper are expressed in millionths of the surface area of the solar hemisphere/. The basic shapes of the sunspot area development curve S(T,t) used are given in Fig. 3/S is the area of the group, T its lifetime in days, t the time lapsed from the actual origination of the group, i.e. the group age/. For some of the modelling, the A-type of curve

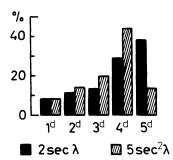


S(T,t) has been expressed analytically /Kopecký et al. 1985/.

The OC diagram enables us, in the first place, to determine the frequency distribution of the group lifetimes  $\tilde{T}$ , i.e. the percentage of cases in which the sunspot groups with lifetime T will be observed as an apparent one-day, two-day, three-day group, etc., provided this sunspot group has the given shape of curve S(T,t) and the visibility curve is  $S_{\phi}\phi(\lambda)$ .

To what extent the shape of  $S \phi(\lambda)$  affects the frequency distribution of the observed lifetime T of the sunspot groups with the A-type development curve S(T,t) is illustrated in Fig. 4 for groups with actual lifetimes T=5 days and T=9 days /Kopecký et al. 1985/.

To what extent the shape of curve S(T,t) affects the frequency distribution in terms of lifetime T can be seen from Fig. 5, in which the frequency distribution in terms of T is given for sunspot groups with different types (A,B,C) of curve S(T,t) and T=5 days with a maximum area  $S_M=50$  and  $S_O\phi(\lambda)=5$  sec<sup>2</sup>  $\lambda$ /Kopecký 1986/.



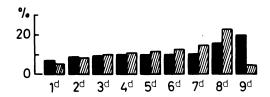


Fig. 4

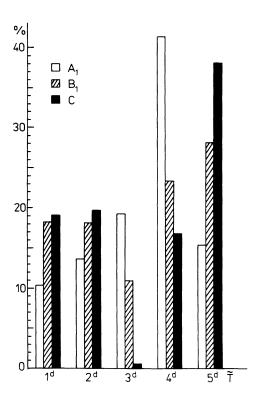


Fig. 5

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Figures 4 and 5 clearly show that the majority of the sunspot groups are observed over a shorter interval than their actual lifetime, even as regards relatively short-lived sunspot groups /e.g. five-day/ which are, however, the most frequent on the Sun. Simultaneously, Figs 4 and 5 imply that the frequency distribution of the short-lived groups in terms of observed lifetimes  $\tilde{T}$  is strongly affected by the shape of  $S_0 \phi(\lambda)$  and of S(T,t).

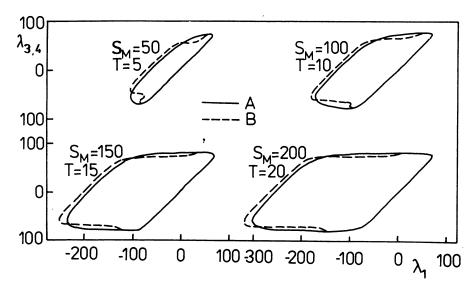


Fig. 6

However, if the actual lifetime T and the maximum sunspot group area  $S_M$  increase simultaneously, the effect of the shape of curve S(T,t) decreases. This can be seen from Fig. 6 which shows the OC diagrams for the A-type of S(T,t) (solid line) and B-type of S(T,t) /dashed line/, for T=5 and  $S_M=50$ , T=10 and  $S_M=100$ , T=15 and  $S_M=150$ , T=20 and  $S_M=200$ . Although there is a certain difference between the OC diagrams for the A- and B-type curves (S(T,t)) and largest values of T and  $S_M$ , the difference is not as large for T=5 and  $S_M=50$  and, therefore, the frequency distributions in terms of the observed lifetime T will not differ as markedly in the A- and B-type curves S(T,t) for the large sunspot groups as for the small groups.

Let us now deal with the east-west asymmetry of sunspot group generation. Pajdušáková /1967/ has proved that the predominance in the number of groups generated in the eastern half of the solar disk does not apply to one-day groups, that their numbers in the eastern and western hemispheres of the disk are roughly the same, in some cases even larger in the west, particularly in its limb parts. She also proved that this was due to the apparent one-day sunspot groups.

Proof of her conclusion can be seen in Fig. 2, where the areas which correspond to the number of one-day groups /marked X/ are larger in the western part of the OC diagram than in the eastern. But in Fig. 2 we can see that not only areas X, but also Y are substantially larger in the west than in the east. This means that more short-lived groups are apparently generated in the west than in the east, and that these apparently short-lived groups should display

distinct maximum in the Western half of the solar disk.

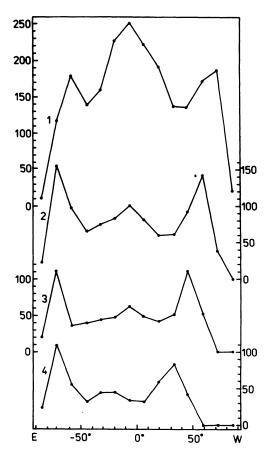


Fig. 7

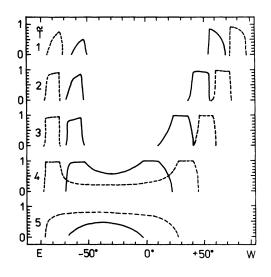
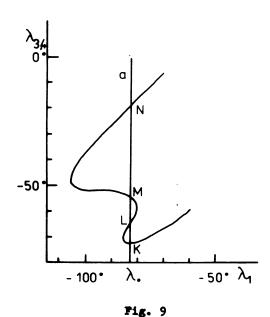


Fig. 8

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That this really so can be seen in Fig. 7 which shows the dependence of the number of generated sunspot groups with observed lifetimes of 1, 2, 3 and 4 days on the distance from the CM as obtained from the Greenwich Photoheliographic Results /Kopecký 1985/. For comparison Fig. 8 shows the dependence of the number of generated groups with observed lifetimes of 1, 2, 3, 4 and 5 days when the actual lifetime T = 5 days, curve S(T,t) is of the A-type and  $S^* = 5 \sec^2 \lambda$  /solid line/ and  $S^* = 2 \sec \lambda$  /dashed line/ as obtained with the aid of the OC diagram.



Let us now briefly mention the so-called interrupted sunspot groups, i.e. groups which become extinct soon after being generated and re-appear one or several days later. As implied by the OC diagram for B- and C-type area development curves S(T,t) with T=5,  $S_M=50$  and  $S^*=5$  sec  $^2\lambda$ , some of these interrupted groups are interrupted only apparently /Kopecký 1986/. Figure 9 shows the eastern part of the OC diagram for the B-type of curve S(T,t) /the whole diagram is shown in Fig. 6/. Line a represents the development of the group which actually originated at  $\lambda_0=-83^\circ$  from the CM, first became visible at point K at a distance of  $-72^\circ$  from the CM, stopped being visible at point L at a distance of  $-65^\circ$  from the CM, became visible again at point M at a distance of  $-54^\circ$  from the CM, and definitely stopped being visible at point N at a distance of  $-19^\circ$  from the CM.

From the point of view of observation, this was a typical interrupted group, although it in fact continued to exist the whole time and only became interrupted apparently due to the combination of effects of the shape of the area development curve of the group, of the visibility function and solar rotation on its visibility.

We have mentioned just a few examples of how the OC diagram can be used to solve some of the problems of statistics of sunspots on the rotating Sun

by means of modelling. These alone indicate how complicated the correct interpretation of the results of statistical processing of sunspot group observations can be. These problems are treated in greater detail in a series of papers which have been and will be published in the Bull. Astron. Inst. Czechosl. under the common title "The visibility function and its effect on the observed characteristics of sunspot groups."

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