CURVE OF GROWTH FOR TiII LINES OF CH CYGNI DURING THE ACTIVE PHASE 1981-1982

A. Skopal
Astronomical Institute, Slovak Academy of Sciences
059 60 Tatranská Lomnica, Czechoslovakia

Recevied 10 September 1984

ABSTRACT. The curve of growth of TiII absorption lines is constructed for high-dispersion spectrograms of CH Cygni, obtained in July to October 1981 and in June to August 1982. The excitation temperatures and turbulent velocities are determined.

КРИВАЯ РОСТА ДЛЯ ЛИНИЙ TiII CH ЛЕБЕДЯ В ТЕЧЕНИЕ ФАЗЫ АКТИВНОСТИ 1981-1982. Для высокодисперсионных спектрограмм CH Лебедя, снятых с июля до октября 1981 И с июня до августа 1982, была построена кривая роста для линий поглощения TiII . Были определены температуты возбуждения и турбулентные скорости.

KŘIVKA RŮSTU PRO ČÁRY TiII CH CYGNI BĚHEM AKTIVNÍ FÁZE 1981-1982. Pro vysokodisperzní spektrogramy CH Cygni, získané v červenci až říjnu 1981 a v červnu až srpnu 1982, je zkonstruována křivka růstu pro absorpční čáry TiII. Jsou určeny excitační teploty a turbulentní rychlosti.

1. INTRODUCTION

The spectral analysis of the light of stars is the only way the basic parameters (chemical composition, temperature and electron pressure) of stellar atmospheres can be determined. Discrete spectral lines are observed in the continuous radiation of the stellar spectrum, absorption as well as emission. The spectral line is characterized in full by its profile which represents the observed energy distribution as a function of frequency. The line profile contains information about the physical conditions and chemical composition of the stellar atmosphere. Since the resolution of the spectrograph is roughly the same as the width of weak and medium spectral lines, only the profiles of strong spectral lines can be determined. Since it is frequently difficult to define the line profile, an integral quantity is used, the so-called equivalent line width which corresponds to the energy of the continuum, absorbed by the spectral line, expressed in angströms (in the case of an absorption line). Also the equivalent line width depends on the physical conditions in the atmosphere, its chemical composition and parameters, characterizing the atom and the transition corresponding to the line involved. However, the equivalent width of weak lines can also be determined, as opposed to their profiles which are given by the instrumental profile of the spectrograph. If the mechanism of line generation is known (line dispersion or absorption), the curve of growth, which in principle expresses of the equivalent line width W_λ on the number of atoms taking place in generating the line, can be computed theoretically. By comparing this curve with the observed W_λ, it is possible to determine the chemical composition of the atmosphere, the excitation temperature, the turbulent velocity, etc.

In constructing the theoretical curve of growth, to which we shall refer herein, the following simplifying assumptions have been adopted: 1. The line generation mechanism is absorption proper under LTE. 2. The spectral lines are generated in a layer for which the temperature and electron pressure can be given uniquely (so- called rough analysis). 3. The ratio of the absorption coefficient in the line and in the continuum is constant (Milne-Eddington´s model). 4. The profile of all lines is determined by the same formula and all lines with the same equivalent width have the same profile.

2. OBSERVATIONAL MATERIAL AND APPLICATION

The high-dispersion spectrograms of CH Cygni (8.5A/mm) were obtained in the coudé focus of the 2-m telescope of the Astronomical Institute of the Czechoslovak Academy of Sciences in Ondřejov and at the National Observatory of the Bulgarian Academy of Sciences in Rozhen in 1981 and 1982 (Tab. 1). The rough analysis was carried out for the absorption spectral lines Ti II which are more abundant in all spectrograms. The method of quantitative analysis was not applied to spectrogram 4134 in which the Ti II absorption lines are split into two components and, therefore, the above assumptions (in particular 2 and 4) cannot be satisfied. As regards the other 1981 spectrograms, the error in determining the equivalent widths is about 50%. On spectrograms 4090 and 4102, the intensities of the absorption and emission components in the inverse P Cygni profile are roughly equivalent and weak on the whole. On 4119, the Ti II absorption lines in the red wings are deformed by motions in the absorbing layer which leads to the overestimation of the actual equivalent widths. More accurate values were obtained from the 1982 spectrograms. The intensities of the absorption components are dominant and, consequently, Assumption 4 in particular

Table 1
The spectrograms

Plate No.	Date	JD 2444	Dispersion (A/mm)	Wavelength Region (A)
4090	1981 July 5	790.5	8.5	3660-5020
4102	1981 July 12	797.5	8.5	3670-5020
4119	1981 August 5	821.5	8.5	3660-5020
4134	1981 August 7	823.5	8.5	3680-5020
R0757	1981 October 8	885.5	9	3500-4800
4539	1982 June 25	1145.5	8.5	3700-5020
4543	1982 July 3	1153.5	8.5	3660-4930
4600	1982 August 19	1200.5	8.5	3630- 5020

will be satisfied. If the region of spectral line generation low ionized plasma, only widening by attentuation and thermal motion of atoms can be considered for metal lines. It has, therefore, been assumed that the profile of the absorption coefficient is given by Voigt´s function,

$$H(a,v) = (a/\pi)\int_{-\infty}^{+\infty} \exp(-y^2)[(v - y)^2 + a^2]^{-1}dy, \tag{1}$$

where $v = \Delta\nu/\Delta\nu_D$, $a = \Gamma/4\pi\Delta\nu_D$ is the constant of radiation damping, $\Delta\nu_D = \nu_0\xi_0/C$ is the Doppler line width, ξ_0 is the most probable velocity of the atoms of the medium, and quantity Γ corresponds to the sum of the inverse lifetimes of the levels between which transition occurs. The line absorption coefficient may then be expressed as $k_\nu = k_0 H(a,v)$. According to Assumption 1 the source function is Planck´s function which is taken to be a linear function of the optical depth τ in the continuum, i.e. $B_\nu/T(\tau)/ = B_0 + B_1\tau$. It is also assumed that parameters a and $\Delta\nu_D$ are given in the region of line generation and, with a view to Assumption 3, the ratio $k_\nu/k_c = (k_0/k_c)H(a,v) \equiv \eta H(a,v) \equiv \eta_\nu$ is independent of depth. Under these assumptions, the equivalent width reads

$$W(a,\eta_0) = 2Ao\Delta\nu_D \int_0^\infty \eta_0 H(a,v)[1 + \eta_0 H(a,v)]^{-1}dv , \tag{2}$$

where $A_0 = /1 + \frac{3}{2}(B_0/B_1)/^{-1}$ is the central depth of a very strong line. The integration in (2) is carried out by means of the 14-point Gauss-Legendre quadra ture formula with an accuracy of 10^{-3}. In computing function H(a,v) it is assumed $a < 0.075$ and this function is taken to be equal to $\sum_{n=0}^{4} a^n H_n(v)$, where $H_0(v) = \exp(-v^2)$, $H_1(v) = (2/\sqrt{\pi})(2vF(v) - 1)$, $H_2(v) = \exp(-v^2)(1 - 2v^2)$, $H_3(v) = -(4/(3\sqrt{\pi}))(1 - v^2 - (3 - 2v^2)vF(v))$, $H_4(v) = (1/6)\exp(-v^2)(3 - 12v^2 + 4v^4)$ and $F(v) = \exp(-v^2)\int_0^v \exp(y^2)dy$. The excitation temperatures and turbulent velocities are computed in the standard way from the mutual displacement of the theoretical and empirical curves of growth.

3. RESULTS

The Ti II spectral lines, used to determine the curve of growth, are given in Tab. 2. Figure 1 shows an example of the theoretical curve of growth for

Table 2

λ	Mult.	$\log(gf\lambda)$	E_{exc} /eV/	$\log(W_\lambda/\lambda) + 6$						
				4090	4102	4119	R0757	4539	4543	4600
4012.372	11	1.87	0.59			1.28	1.63	1.74	1.63	1.82
4025.136		1.51				1.16			1.58	1.67
3813.390	12	1.16				1.11	1.56	1.68	1.45	1.61
3814.580		1.91				1.26	1.83	1.66	1.42	1.94
3759.291	13	3.87		2.34	2.20	2.22	2.20	2.28	2.25	2.53
3761.320		3.76		2.32	2.31	2.15	2.19	2.38	2.23	2.53
3685.192	14	3.67			2.17	2.22	2.07	2.31	2.18	2.57
4395.031	19	3.09	1.08	1.75	1.68	1.94	1.94	2.18	2.01	2.15
4443.802		2.91		1.72	1.48	1.80	1.83	2.04	1.86	2.13
4450.487		2.09		1.31		1.35	1.53	1.66	1.59	1.93
4287.893	20	1.73			1.10	1.12	1.63	1.73	1.30	1.72
4294.101		2.58			1.58	1.71	1.82	1.91	1.98	2.15
4337.916		2.60		1.59	1.45	1.75	1.92	2.06	1.83	2.06
4344.291		1.64			1.34	1.08	1.43	1.46	1.34	1.76
4161.524	21	1.47							1.11	1.38
4468.493	31	3.02	1.12	1.51	1.52	2.09	1.83	1.93	1.93	2.08
4501.270		2.88		1.40	1.31	1.89	1.84	1.98	1.95	2.08
3900.546	34	3.32		1.80	1.82	2.00	2.02	2.30	2.17	2.28
3913.464		3.22		2.06	1.73	1.97	1.91	2.29	2.20	2.18
4417.718	40	2.58	1.17			1.30	1.53	1.74	1.65	1.79
4464.458		2.03		0.80		1.32	1.72	1.55	1.44	1.66
4290.222	41	2.66		1.49	1.10	1.93	1.78	1.91	1.87	1.99
4300.052		3.16		1.61	1.59	2.12	1.94	2.14	2.01	2.17
4301.928		2.35		1.12		1.53	1.77	1.84	1.55	1.93
4307.900		2.51		1.36	1.21	1.54	1.81	1.91	1.79	2,07
4312.861		2.48		1.50	1.34	1.53	1.81	1.87	1.73	1.93
4314.979		2.45		1.31		1.59	1.70	1.84	1.64	2.00
4330.708		1.56					1.15	0.93		1.29
4399.767	51	2.32	1.23			1.37	1.53	1.51	1.67	2.18
4533.966	50	2.96		1.79		1.74	1.81	2.07	1.97	2.17
4563.761		2.80		1.45		1.72	1.74	1.98	1.86	2.11
4589.961		1.94				1.20	1.44	1.38	1.44	1.85
3624.826	52	2.81					1.78	2.24	2.02	2.29
3641.330		2.98					2.07	2.33	2.18	2.31
3741.633	72	3.32	1.57	1.89	1.67	2.06	2.01	2.25	2.09	2.37

Table 2.(continued)

3757.684	72	2.60	1.57	1.75	1.65	1.65	1.84	1.87	1.83	2.08
3776.062		1.83					1.59			
3659.765	75	3.03			1.50	1.49	2.02	2.02	2.06	2.18
3662.237		2.97			1.65	1.89	1.92	2.01	1.91	2.17
4571.971	82	3.34		1.86	1.40	1.83	1.93	2.03	1.96	2.24
4549.622		3.44				1.84	2.14	1.97	1.88	2.22
4529.465		1.89							1.31	
4367.657	104	2.96	2.59			1.14	1.45		1.31	1.67
4386.858		2.85					1.40		1.26	1.67
4163.644	105	3.47		1.24		1.57	1.86		1.79	1.91
4171.897		3.29		1.30		1.49	1.74		1.78	1.81

a = 5.10^{-3}, 0.1 and A_0 = 0.5 together with the plotted points of the empirical Ti II curve on spectrogram 4600. The points of the empirical curve have been displaced to render the dispersion variance relative to the theoretical curve minimum. The displacement of the group of lines with the same excitation potential E_{exc} then defines the excitation temperature. Figure 2 shows an example of the dependence of logX = $\log\eta_0 - \log(gf\lambda)$ on E_{exc} for spectra 4090 and 4600. The higher dispersion in spectrum 4090 reflects the larger error in measuring the equivalent widths on this spectrogram and a less rigorous satisfaction of the above assumptions, in particular of Assumption 2. The curve was fitted by the least-squares method. The displacements of the empirical and theoretical

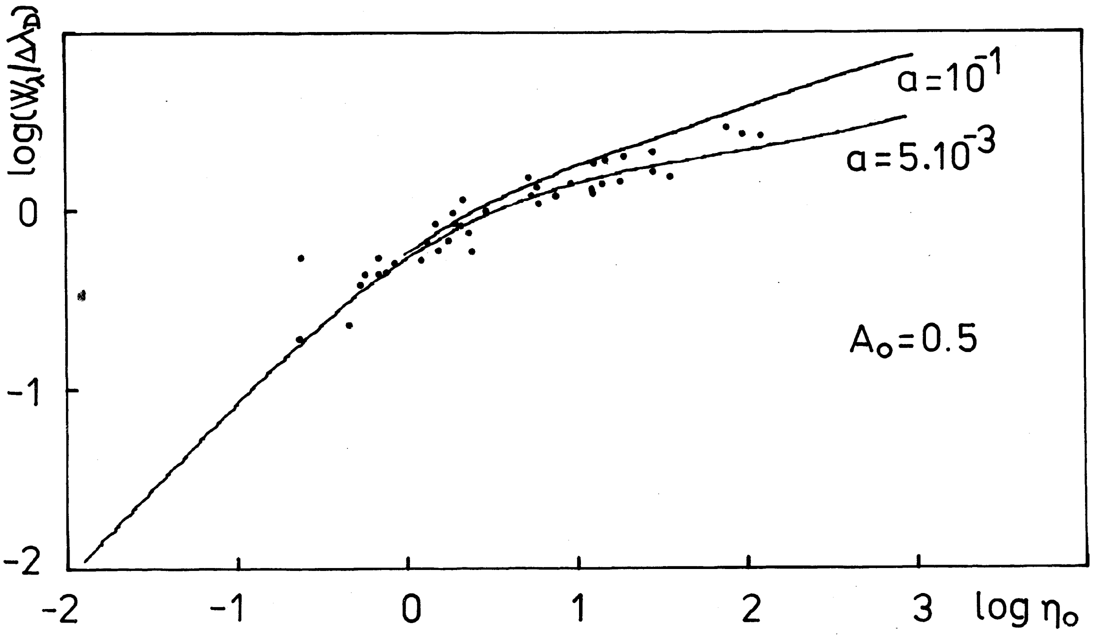

Fig. 1. Empirical curve of growth for line Ti II on spectrogram 4600.

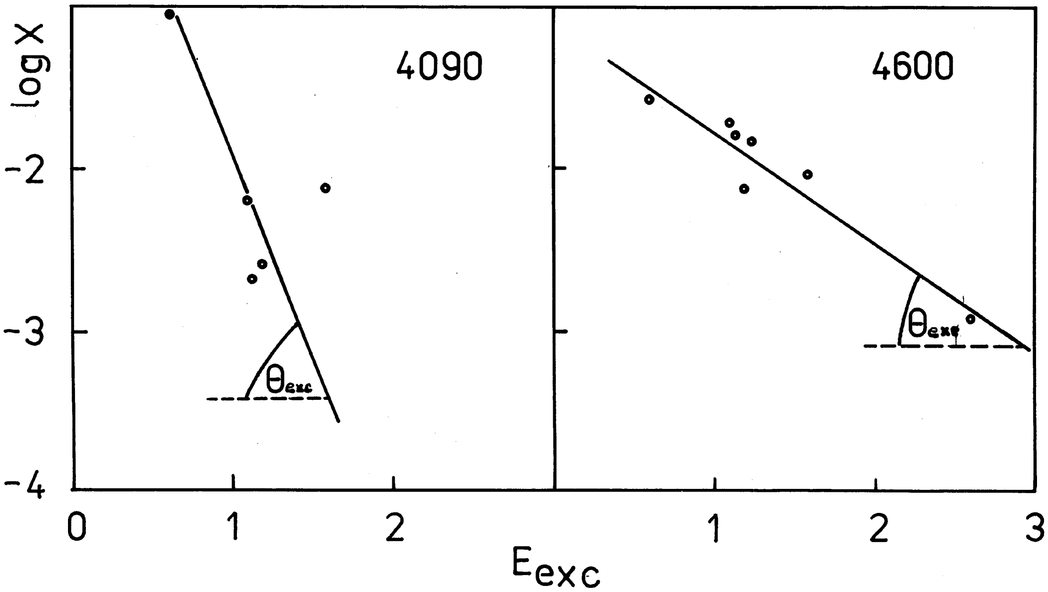

Fig. 2. Displacement logX = logη_0 - log(gfλ) of the empirical and theoretical curve of growth on excitation temperature θ_{exc} (θ_{ex}= 5040/T_{exc}) for spectrograms 4090 and 4600.

curves of growth along the Y-axis are used to determine the turbulent velocity ξ_T (T_{exc} is put equal to $T_{thermal}$) from the following relation,

$$\xi_0 = [(2k\,T_{exc}/Am_H) + \xi_T^2]^{\frac{1}{2}} , \tag{3}$$

where A is the atomic weight of the element. The central depth of the line was determined from the strongest H and Ca II absorption lines. The damping coefficient a could not be determined in any of the spectrograms. Its value can only be estimated within limits of about 10^{-2} to 10^{-3}. The values of A_o, T_{exc}, ξ_T are summarized in Tab. 3 and in Fig. 3.

Table 3

Spectrogram	A_o	T_{exc} /K/	/kms^{-1}/
4090	0.8	2000	8.4
4102	0.9	3600	5.1
4119	0.6	7400	18.3
R0757	0.7	6200	12.9
4539	0.9	6100	17.1
4543	0.8	6500	13.2
4600	0.8	7300	17.7

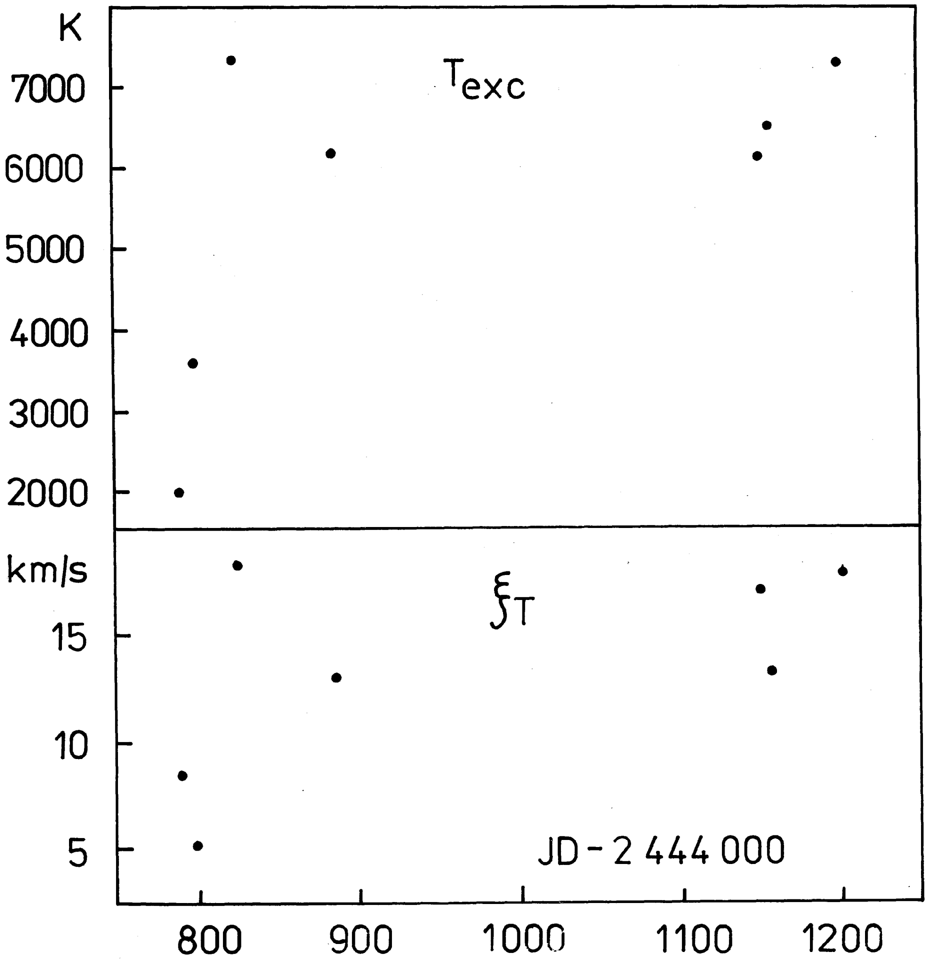

Fig. 3. Excitation temperatures T_{exc} and turbulent velocities ξ_T for the layer of generation of spectral lines Ti II of CH Cygni during the 1981-1982 outburst.

REFERENCES

Allen, C.: 1973, Astrophysical Quantities 3rd ed., London, Athole Press.
Faraggiana,R., Hack, M.: 1971, Astron. Astrophys. 15, 55.
Kurucz, R. L., Peytremann, E.: 1975, Table of Semiempirical gf Values, SAO Special Report No. 362, Cambridge, Massachusetts.
Kříž, S.: 1966, Thesis, Astron. Inst. Czechosl. Acad. of Sci., Ondřejov, in Czech.
Mihalas, D.: 1970, Stellar Atmospheres, San Francisco, W.H. Freeman and Company.