Algebraic Geometry: a Tool for Resolving the Enigma of Time?

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1. INTRODUCTION

Although time is a concept that attracted and occupied the thoughts of a countless number of thinkers and scholars over centuries, its true nature still remains wrapped in a shroud of mystery. Perhaps the main reason why this is so is that there are too many aspects this notion entails. Out of these, there are two that deserve particular attention for, in our opinion, they characterize best the nature of time's enigma. The first is the notion of physical or objective time. Time, from the physicist's viewpoint, is one of the dimensions of a differentiable, Riemannian manifold. It is a linear variable, structureless and undifferentiated. It is very much like a spatial dimension, the only difference between the two being embodied in the Lorentz signature of the metric tensor. The second is psychological or subjective time; time as we perceive it, experience it. This time dimension exhibits a non-trivial internal structure, consisting of the past (events that have happened) and the future (events that will happen), the two domains being separated from each other by a unique moment of the present, the event that is happening. This is the time that seems to "flow", to proceed at a steady pace from the past into the future - the time with an arrow pointing in one direction.

It is a well-known fact that the fundamental equations of physics are time-reversible, i.e. they do not distinguish between the past and future. Moreover, the very concept of the present, the now, has no proper place in the temporal of physics at all; this holds true whether one is talking of

classical physics, quantum mechanics or relativity theory. This is because physics merely *quantifies* the points of the time dimension, thus stripping the latter of its crucial experiential aspect - its arrow. The physical time is thus a plain, quantitative mathematical abstraction and, as such, it simply cannot account for the fine structure of the subjective time. In order to capture the latter, it is, therefore, necessary to go beyond quantifying and consider a generalized mathematical concept of dimension where a *qualitative* distinction between the individual elements (or, at least, between the groups of the latter) can be implemented.

An illustrative example of such a temporal dimension is provided by a specific linear, single-parametric set (so-called pencil) of conics in the projective plane ^{1,3–8}. This set of conics is found to nicely reproduce the experienced arrow of time when the projective plane is affinized; it simply suffices to postulate that each proper conic of the pencil stands for a single temporal event, and relate three distinct kinds of (proper) affine conic, viz. a hyperbola, a parabola and an ellipse, with the three different kinds of temporal event, viz. the past, present and future, respectively ^{1–3}. The model, albeit very simple, turns out to be very useful and fertile, for in addition to the ordinary arrow, it also gives rise to a number of other, differently structured features of the temporal. These patterns emerge as a combination of two important properties of the projective plane: the *way* in which the plane is affinized and the *character* of its ground field ^{4–8}. And the fine structure of psychological time is, indeed, found much richer and more intricate than outlined above ^{9–136}.

Already in a normal state of health there are, every now and then, aberrations of subjective time such as acceleration or deceleration of the lapse of time. Under severe mental disturbances (like those characterizing mental psychoses, drug-induced states, trances, mystical states as well as other deep "altered" states of consciousness), these anomalies/peculiarities become more pronounced. The flux of time may even cease completely (the sensations usually described as "time standing still", or "suspended", "arrested" time), or expand without limit (the feelings of "everlasting now", "eternity"). In some cases, time's flow may be experienced as discontinuous, fragmented, or even reversing its direction. Finally, in most extreme cases, time as a dimension is transcended, or simply non-existent (an "atemporal" state)

It is remarkable that already our pencil-model is able to grasp, at the semi-quantitative level, many of the above-introduced "distortions" of the subjective experience of time^{4–8}. There are a couple of serious implications this fact conveys to us. On the one hand, we see that although the manifestations of psychological time are so diverse, unusual and failing to conform to any current physical paradigm, they are accommodated by a

definitive algebraic geometrical pattern. On the other hand, we are inevitably led to the murky, bizarre and often contra-intuitive realm of phenomenology, which has always been potentially dangerous to enter. Yet this is a territory whose "topology" is very likely to contain a decisive clue to ultimate unveiling of the nature of time - all the more that the methods and results are in no case less mathematically rigorous than the currently widely-accepted practice of dynamic modeling of biological systems.

There is, however, another important message the above-listed examples of the anomalies/pathologies in time perception try to impart on us. It is the fact that the intrinsic structure of the arrow, an intriguing aspect of which is the pace at which time seems to "flow" is not rigid, but rather flexible and subject to profound changes and dramatic alterations. This means that the subjective time has a strongly *non*-linear character. As a consequence, we have to abandon its pencil-model and consider a more general single-parametric aggregate of conics as its representative.

2. A SIMPLE QUADRATIC GENERALIZATION OF THE PENCIL-MODEL

It is obvious and natural that we demand this non-linear aggregate of conics to be the simplest possible generalization of the pencil of conics employed in our debut model^{1,3}. The pencil in question enjoys three important properties. Firstly, it features linear base (i.e. shared by all the conics) elements*, i.e., its base elements are either points or lines. Secondly, it is self-dual, i.e., it looks the same irrespective of the representation adopted. Finally, it is full, i.e., all its base elements are real, or, in the case of a general ground field, all belonging to the field. Hence, we are to seek for such an aggregate that also meets these constraints.

To begin with, we recall the *duality* principle of the projective plane, i.e. the fact that the projective geometry of the plane exhibits a twofold character. Namely, the set of all the points of the plane and the set of all the lines of the plane are symmetrically related to each other, and to every property of lines in the geometry of points there corresponds a property of points in the geometry of lines. There is thus a perfect correspondence between the algebra of line-geometry and that of point-geometry; we say that a point and a line of the projective plane are dual to each other entities. In other words, instead of viewing the points of the plane as the fundamental entities, and the lines as the ranges (loci) of points, we may equally well take the lines as the primary entities and define points in terms of lines,

^{*} The terminology, symbols, and notation used here are the same as in ref. 3-8.

characterizing a point by the complete set of lines passing through it. The points and lines of the projective plane may thus be regarded as two equivalent representations of the latter; a(ny) projective theorem remains valid if the concepts of point and line are interchanged. A given geometrical object of the plane will, in general, look different in the two representations of the plane; if it looks the same, it is called self-dual. So, if our aggregate of conics is to be self-dual, there must be parity between the number of its base points and that of its base lines.

Next, a proper point-conic Q,

$$Q(c) = \sum_{i,j=1}^{3} c_{ij} \bar{x}_{i} \bar{x}_{j} =$$

$$\equiv c_{11} \bar{x}_{1}^{2} + c_{22} \bar{x}_{2}^{2} + c_{33} \bar{x}_{3}^{2} + 2c_{12} \bar{x}_{1} \bar{x}_{2} + 2c_{13} \bar{x}_{1} \bar{x}_{3} + 2c_{23} \bar{x}_{2} \bar{x}_{3} = 0, \qquad (1)$$

$$\det c_{ij} \neq 0, \qquad (2)$$

is defined by six distinct quantities c_{ij} 's. Of these, however, only five are independent for $\tilde{c}_{ij} = \rho c_{ij}$ ($\rho \neq 0$) clearly represent the same conic. In order to get a single-parametric set of conics, we have thus to impose four different constraints on the values of c_{ij} 's. Making the conic pass through a given point is a simple constraint; dually, requiring the conic to touch a particular line is also such a constraint. Hence, a self-dual, single-parametric aggregate of conics with linear base elements must feature *two* base points and *two* base lines.

In the case of our favorable pencil of conics, the base points (B and B') lie on the base lines (β and β' , respectively) as shown in Fig. 1a ^{1,3}. It turns out that in order to get a simplest non-linear aggregate of conics with the desired symmetry properties, it suffices to relax this incidence relation for one point-line pair - see Fig. 1b.

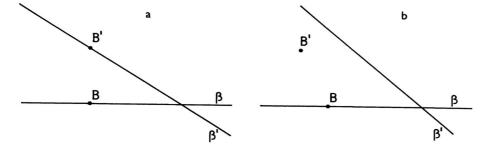


Figure 1. The configuration of the base elements of our favorable pencil of conics (a) and of (one of) its simplest non-linear generalization(s) (b).

Let us find the equation of a generic conic of this aggregate. In order to keep reasoning at the simplest possible level, we choose the coordinate triangle in such a way that the base points B and B' lie on its $\check{x}_3 = 0$ side and coincide, respectively, with its vertices V_2 and V_1 , that is

B:
$$\rho \ddot{x}_i = (0, 1, 0), \rho \neq 0,$$
 (3)

$$B': \rho \check{x}_i = (1, 0, 0), \rho \neq 0,$$
 (4)

and the base lines are selected as follows

$$\beta: \ \breve{x}_1 = 0, \tag{5}$$

and

$$\beta': \ \breve{x}_1 = \kappa \breve{x}_2, \ \kappa \neq 0, \tag{6}$$

as illustrated in Fig. 2.

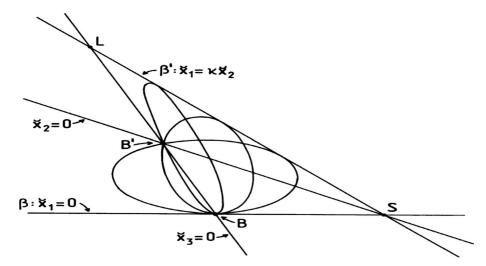


Figure 2. Selection of the coordinate system which facilitates description and examination of the non-linear aggregate of conics with the base configuration as depicted in Fig. 1b. Also shown are three proper conics of the aggregate, belonging to the same family.

Now, a generic conic of the projective plane, Q, is given by eqn. (1). If this conic passes through B and B', then

$$c_{22}=0$$
 (7)

and

$$c_{11}=0,$$
 (8)

respectively. Further, β is a tangent to Q iff

$$c_{23}=0;$$
 (9)

for substituting eqn. (5) into eqn. (1) and taking into account eqn. (7) gives

$$c_{33}\ddot{x}_3^2 + 2c_{23}\ddot{x}_2\ddot{x}_3 = 0, (10)$$

which indeed yields repeated roots (in the variable $x \equiv \tilde{x}_3/\tilde{x}_2$) under eqn. (9). Similarly, inserting eqn. (6) into eqn. (1) and taking into account eqns. (7)-(9) leads to

$$c_{33}\ddot{x}_{3}^{2} + 2\kappa c_{13}\ddot{x}_{2}\ddot{x}_{3} + 2\kappa c_{12}\ddot{x}_{2}^{2} = 0,$$
(11)

from where it follows that β' touches \mathcal{Q} iff

$$c_{12} = \frac{\kappa c_{13}^2}{2c_{33}}. (12)$$

Combining eqns. (7)-(9), (12) with eqn. (1) and putting

$$\vartheta \equiv \frac{c_{13}}{c_{33}} \tag{13}$$

we thus find

$$Q(\vartheta) = \kappa \vartheta^2 \check{x}_1 \check{x}_2 + 2 \vartheta \check{x}_1 \check{x}_3 + \check{x}_3^2 = 0.$$
 (14)

We see that the aggregate is indeed a non-linear one, for the last equation depends *quadratically* on the variable parameter ϑ . It contains two degenerate conics,

$$Q(\vartheta = 0) = \check{x}_3^2 = 0 \tag{15}$$

and

$$Q(\vartheta = \pm \infty) = \breve{x}_1 \breve{x}_2 = 0, \tag{16}$$

which separate the set of the proper conics into two distinct families, viz. $-\infty < v < 0$ and $0 < v < +\infty$.

3. VARIETY AND STRUCTURE OF THE AGGREGATE-BORNE TIME DIMENSIONS

Our principal task is to find all the possible types of temporal arrow borne by this aggregate of conics and examine in detail the intrinsic structure of each of them. The strategy to be followed is fully paralleled that pursued in refs. 1-8. Namely, either family of aggregate (14) is taken to represent a single time dimension, every proper conic being viewed as a particular event. We thus have a pair of time dimensions, which can be regarded as twin dimensions. They are both originally homogeneous/structureless, because from the projective point of view all proper conics stand on the equal footing. To 'de-homogenize' them, we pick up one line of the projective plane (henceforth referred to as the d-line - broken line in Figs. 3 and 4) to play a special role, and examine intersection properties of this line with the conics of the aggregate. The (proper) conics that are cut by this line in two distinct points (solid curves in Figs. 3 and 4) are then postulated to stand for the past events, those having no points in common with it (dotted) are regarded to represent the future events, and the unique conics featuring this line as a tangent (dot-dashed) are identified with the moments of the present.

Due to a quadratic nature of the aggregate, there are, in general, *two* distinct conics which are touched by the *d*-line. They can be found to belong to different families (Figs. 3a, 4a; 3g, 4g), or lie in the same family (Figs. 3c, e). This can easily be substantiated quantitatively. We define *d*-line by the equation

$$\ddot{x}_2 = a\ddot{x}_1 + b\ddot{x}_3,\tag{17}$$

a and b being fixed real parameters, and combine this equation with eqn. (14) to obtain

$$x^{2} + \vartheta(b\kappa\vartheta + 2)x + a\kappa\vartheta^{2} = 0, \quad x \equiv \breve{x}_{3}/\breve{x}_{1}. \tag{18}$$

Obviously, *d*-line is a tangent to those conics of (14) whose \mathcal{S} s lead to coinciding (repeated) roots of eqn. (18),

$$\vartheta^2 \left| (b \kappa \vartheta + 2)^2 - 4a \kappa \right| = 0. \tag{19}$$

The last equation is satisfied by $\vartheta=0$, which is not a proper conic (see eqn. (15)), as well as by

$$\vartheta_{\pm} = \frac{-2 \pm 2\sqrt{a\kappa}}{b\kappa} \,. \tag{20}$$

Hence, if $a\kappa$ is positive and less than one, ϑ_+ and ϑ_- are of the same sign, i.e., the conics are in the same family; on the other hand, for $a\kappa$ exceeding one ϑ_+ and ϑ_- have the opposite signs and, hence, the two conics belong to different families. If $a\kappa=1$, there is only one proper conic enjoying this property (that corresponding to ϑ_{-} ; the other conic, $\vartheta_{+}=0$, is degenerate), as it is also the case when $a \kappa = 0$ ($\vartheta_+ = \vartheta_- = -2/b \kappa$). Finally, for negative κ there are no such conics. Rephrased in terms of physics/psychology, there are two present moments for positive $a\kappa$, which can be either situated in the same time dimension $(a \times 1)$, or shared by the two temporal coordinates $(a \times 1)$, a single-now time dimension for a = 0, 1, and the presentless temporal for $a\kappa < 0$. As it is only the present, the 'now', that is directly perceived, it is natural to further stipulate that out of the two potential dimensions only that is observable which contains a present moment; the one lacking this particular moment is viewed as latent, unobservable. From what has just been said it follows that if the position of the d-line with respect to the aggregate varies, the corresponding time configurations will differ not only in their internal structure, but also in their dimensionality. This is the most crucial aspect of our theory and we shall, therefore, have a detailed qualitative look at it.

We shall begin with the case when both the families exhibit a *perfect arrow*, Figs. 3a, 4a. In order to see the link between different time patterns most explicitly, we shall move the d-line in such a way that it remains all the time tangent to one and the same conic of a given family - the family shown in Fig. 3, the conic in question drawn dot-dashed. The shift is done in a manner that the point of contact moves on the conic counter-clockwise. Starting, as mentioned above, from a perfect-arrow pattern, this structure is qualitatively preserved till d-line gets identical with β' - Figs. 3b, 4b. In this case the line is a tangent to all the proper conics of both families, and so both time dimensions comprise solely the present moments; in what follows we shall call this type of the temporal the *pure present of the first kind*.

Shifting d-line further (Figs. 3c, 4c), we arrive at a mode where there is a profound distinction in the structure of the two dimensions. While one dimension (Fig. 4c) becomes homogeneous (and, hence, unobservable), built exclusively of the past events, the other (Fig. 3c) is structured in a highly

non-trivial way: namely, it features two distinct domains of the past that are separated by two different moments of the present from a single region of the future. This can be viewed as a union of two ordinary arrows of time that have the domain of the future in common. Although the two arrows are perfectly equivalent on their own, this symmetry is broken when this structure (Fig. 3c) is related/compared to that depicted in Fig. 3a. For from this comparison it follows that one of the arrows, viz. the one that shares with the arrow of Fig. 3a the moment of the present, can be regarded as principal (illustrated in Fig. 3c by the double arrow); the other, seen located in what would normally be the region of the future of the principal arrow, as subsidiary (single arrow). We note that the principal arrow is opposite to the 'reference' arrow of Fig. 3a, whereas the subsidiary one points in the same direction as the latter. In what follows we shall call this configuration an arrow within arrow.

This structure remains qualitatively intact until d-line is found to pass through the base point B', Fig. 3d. In this case, the domain of the future completely disappears and the time dimension is endowed with two regions of the past, separated from each other by a single moment of the present: this pattern will be referred to as a *past-present-past* mode. Concerning the twin time dimension, it still keeps its latency as the corresponding family of conics preserves its homogeneity (Fig. 4d).

A further slide of *d*-line brings us again to an arrow-within-arrow configuration, Fig. 3e. This a-w-a pattern is, however, related differently to the original arrow than that represented by Fig. 3c. For it is now the domain of the future of the reference arrow (Fig. 3a) that is transformed into a subsidiary arrow, and it is the principal arrow which shares the polarity of the reference arrow. In order to distinguish between the two a-w-a configurations, we shall call the former (Fig. 3c) the *past*-related, the latter (Fig. 3e) the *future*-related one. The corresponding second temporal dimension is again hidden, as it still features past events only (Fig. 4e).

The a-w-a pattern of Fig. 3e vanishes when d-line hits the point L, and a perfect arrow is re-established - Fig. 3f. This pattern is seen all the way through to the point when d-line becomes identical with the other base line, β - Fig. 3h. At this mode, all the conics have d-line for a tangent and we again have a time dimension consisting exclusively of the 'nows', the present; as it will be seen later on, this mode is fundamentally different from that represented by Fig. 3b, and to make this distinction evident we shall call it the *pure present of the 2nd kind*. This is the last pattern in the sequence, for a further displacement of d-line leads to the perfect arrow we began with (Fig. 3a, 4a).

Summing up, we arrive at six different patterns of the intrinsic structure of the time dimension borne by one family of the quadratic set of conics

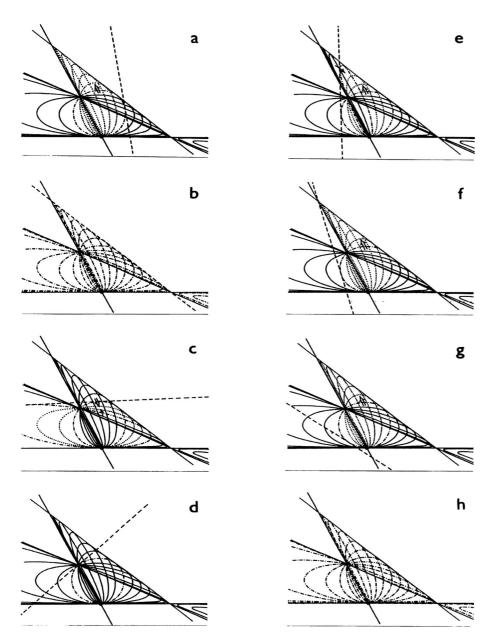


Figure 3. A schematic sketch of the varieties of the internal structure of time dimension generated by a given family of conics of the aggregate defined by eqn. (14): a - a perfect (ordinary) arrow; b - a pure present of the 1st kind; c - a past-related arrow within arrow; d - a past-present-past pattern; e - a future-related arrow-within-arrow mode; f, g - perfect arrows, and b - a pure present of the 2nd kind. For more details, see the text.

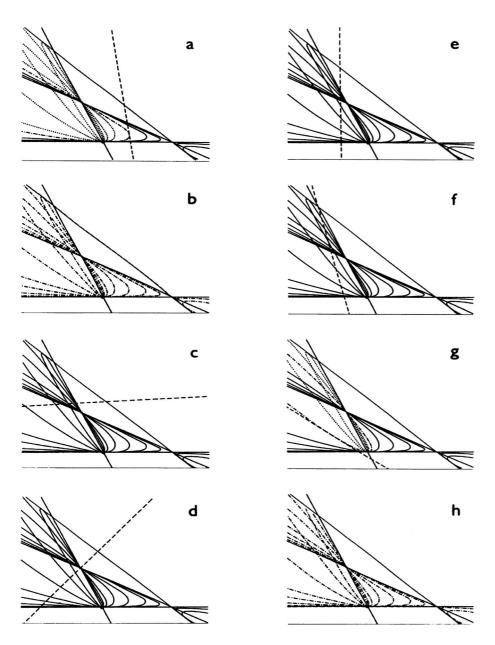


Figure 4. The same as in the previous figure for the time dimension generated by the other family of conics of aggregate (14). The position of d-line on the corresponding figures is identical.

defined by eqn. (14): the perfect arrow (Figs. 3a, f, g), a couple of pure-present configurations (Figs. 3b, h), two arrow-within-arrow modes (Figs. 3c, e), and a past-present-past structure (Fig. 3d). The perfect arrow of time is a structure which each of us is familiar with; but what about the remaining five? They are, indeed, quite peculiar and, at first sight, they may seem to belong only to the realm of pure academic interest. In this case, however, appearances and reality differ perhaps more greatly than in any other area of human experience, for the theory can be linked with the phenomenological world in a way that has had absolutely no previous existence of any kind, of which science has not allowed us to form even the remotest conception. What the term 'phenomenological' world means here is nothing but the already-mentioned (Sec. 1) realm of psychopathological time dimensions characterizing altered states of consciousness^{9–136}. It is a world where there is so much unsettled and uncertain that seemingly small matters can take an outsize significance and where, thus, the strength of our formalism can be shown in full.

4. ALTERED STATES OF CONSCIOUSNESS AS A VALUABLE TEST BED OF THE MODEL

4.1 Pure Present of the First Kind

So, our forthcoming task is to try to fill up the above-examined 'anomalous' temporal patterns with a concrete experiential content. We shall start our reasoning with the (relatively) least complicated structures. These are obviously the pure-present modes, Figs. 3b, h. We shall focus first on a 1st kind mode, Fig. 3b. This mode represents a smooth transition between the perfect arrow, Fig. 3a, and the past-related a-w-a configuration, Fig. 3c. As the flux of time characterizing the perfect arrow is opposite to that of the principal arrow of the a-w-a structure in question, it follows that the 1st kind pure present must be characterized by a complete cessation of time's flow. And the psychopathologic literature is found to be quite abundant in experiences like 'time standing still', 'arrested', or 'suspended' time^{10–26}, 28–30,36,38,41,45–52,60–66,68,70–75,77,84,88–93,102,106–108,113,118–129,132

A quadruple of descriptions given below were drawn from different sources, and they should give the reader a sufficiently complex picture of what a generic experience of this type of disturbed temporal awareness looks like. The first excerpt was taken from a beautiful paper by Muscatello and Giovanardi Rossi (ref. 123, p. 784)*:

"Il tempo per me è fermo completamente, mi sembra. Forse sono soltanto attimi che io sto così male. Guardo l'orologio e mi sembra, se lo riguardo, che sia trascorsa una distanza di tempo enorme, come se fossero trascorse delle ore, mentre invece sono trascorsi pochi minuti. Mi sembra un tempo enorme che sta passando. Il tempo non passa mai, guardo l'orologio, ma le lancette sono sempre ferme nella stessa posizione, non si muovono mai, non vanno mai avanti; allora sento se l'orologio si è fermato, sento che batte, ma le lancette sono sempre ferme. Al passato non penso, me ne ricordo ma non ci penso molto. Quando sto così male non penso mai al mio passato. Non mi veniva in mente niente, niente... non riuscivo a pensare a niente. Non riuscivo a prospettarmi niente, niente in futuro. Per me il tempo presente non esiste quando sto così male... non esiste il passato, non esiste il futuro."

An extraordinary vivid and detailed account of this type of the psychopathology of time was found in ref. 77 (p. 311):

"...Das seltsame war, daβ mir hier und da das normale Zeitbewuβtsein angesichts dieser Figuren vollkommen verloren ging; die Zeit war nich mehr ein Strom, der vorbeiflo β und dessen Fließen man messen konnte, sondern sie war gleichsam ein Meer, das als Ganzes stillstand und nur in sich ein chaotisches Durcheinander war. Ich vermochte das ständige Werden der Figuren dann nicht mehr als eine Sukzession in einer bestimmten zeitlichen Richtung aufzufassen, sondern manchmal flossen die Farben und Formen zu einem unbeschreiblichen Durcheinander zusammen, wie wenn die vorher abwechselnden Figuren jetzt alle gleichzeitig erlebt würden. Hatte ich vorher die Figuren in ständiger Bewegung gesehen, so war jetzt nur noch eine bunte, unsäglich verwirrende Mannigfaltigkeit da, in der ich keine Bewegung mehr wahrnehmen konnte. Wenn ich in die Schau der Figuren ganz versank, so geschah es hier und da, daβ ich zugleich auch in diesen Zeitenstillstand versank, wo das Aufeinanderfolgende in einer stillstehenden Gegenwart zusammenflo β . Diese Unterbrechungen im normalen Zeiterleben kann ich jetz nicht nur nicht formulieren, sondern mir auch erlebnismäβig kaum mehr vergegenwärtigen. Wenn ich mich von den Figuren losriβ und mich gewaltsam der Außenwelt zuwandte, so war dieses abnorme Zeiterlebnis nich mehr vorhanden, aber die Zeitsinnstörung äußerte sich dann in der Täuschung, daβ seit dem letzten Aufraffen ungeheuer lange Zeit verflossen sein müßte..."

The third illuminating description of time standing still is picked up from ref. 15 (p. 13); this account is interesting by the fact that the author also pays attention to the corresponding distortion in spatial perception (the last line of the vignette):

"Ich merke wohl den Zeitflu β , aber ich erlebte ihn nicht. Ich wei β , da β morgen auch ein Tag ist, ich fühle ihn aber nicht kommen. Die Vergangenheit kann ich jahremä β ig abschätzen, ich habe aber keine Beziehung mehr dazu. Der Zeitstillstand ist unendlich, ich lebe in einer stehenden Ewigkeit. Ich sehe, da β die Uhren sich drehen, aber für mich vergeht die Zeit nicht...

^{*} In this and all subsequent excerpts, italics are used to emphasize those parts of the narrative that most directly relate to the subject discussed.

Alles liegt in einer Linie, es sind keine Tiefenunterschiede mehr... Alles ist wie eine feststehende Fläche."

In the last example, the subject makes it clear that it is only their internal time that comes to a complete halt, while the objective/external time still keeps its course. However, there are also experiences where an opposite phenomenon seems to take place, as in the following intriguing case (ref. 132, p. 158):

"Now I was walking and I walked not. And I looked up to the air and saw the air in amazement. And I looked up unto the pole of the heaven and saw it standing still, and the fowls of the heaven without motion. And behold there were sheep being driven and they went not forward but stood still; and the shepherd lifted his hand to smite them with his staff and his hand remained up. And I looked upon the stream of the river and saw the mouths of the kids upon the water and they drank not. And of a sudden all things moved onwards in their course."

4.2 Pure Present of the Second Kind

Time standing still may be viewed as an extreme case of the *de*celeration of the lapse of subjective time, viz. by its infinite slowing-down. This is one limit of the 'present-preserving' disappearance of the flux of time. The other limiting case is represented by an infinite *ac*celeration/speeding-up of time's pace. It is intuitively obvious that this extinction of temporal flow is of a different nature than that characterizing the previous case. And the *second kind* of pure present, Fig. 3h, is indeed found to differ from its 1st kind counterpart. Although it is also a transitory mode, it separates two *identical* modes, Figs. 3a, g; they both are perfect arrows, and of the same polarity at that.

This other kind of pure present is also a well-documented and invariable feature of a certain group of the altered states of consciousness, being most often called an 'eternity', or 'timelessness' ^{14,18,20,28,36,45–47,52,56–57,64–67,69–70,75–79,83–85,87–94,126,130–131}. It is a sort of telescoping of past, present, and future into the present that is experienced as a 'timeless now', where everything appears to be happening at once. Perhaps the best account of what this is like we found in ref. 76 (p. 46); although the author almost exclusively focuses on the spatial fabric of existence throughout the book, it is the temporal aspect of this particular experience that fascinates most:

"Ich erwachte in einer andern Welt, in der das Welträtsel auf eine unendlich einfache Weise gelöst war in der Art eines andern Raumes. Ich staunte über das Wunder dieses andern Raumes, und in dem Staunen war das Urteil verborgen, er ist grundverschieden von unserem Raum. Er hatte andere Dimensionen, alles war in allem enthalten. Ich war in diesem Raum, und der Raum war ich. Das Weltall war in dem Raum enthalten, ich im All and es in mir...

Jedenfalls erlebte ich in der zweiten Phase dieses Traumes erst die Zeit, Weltallzeit, Äonen. In kosmischen Zeitläufen sah man Welten entstehen, aufblühen wie Blumen, bestehen und wieder vergehen. Es war ein ewiges Spiel, in dem es kein Ende gab. Blickte man zurück in die Vergangenheit, sah man Äonen, blickte man vorwärts in die Zukunft dehnten sich Äonen in Ewigkeit aus, und die Ewigkeit war im Punk der Gegenwart enthalten. Man befand sich in einem Sein, in dem das Werden und Vergehen bereits enthalten war, und dieses Sein war mein Bewuβtsein. Es enthielt alles…"

Another gripping picture is taken from ref. 130 (Chapt. 2), based on the author's personal near-death experience:

"This time, I moved, not my environment, and I moved rapidly... My speed accelerated until I noticed a wide but thin-edged expanse of bright light ahead, like a "parting" in space or a "lip", with a brightness so brilliant it was beyond light yet I could look upon it without pain or discomfort... The closer I came the larger the parting in space appeared until... I was absorbed by it as if engulfed by a force field...

Further movement on my part ceased because of the shock of what happened next. Before me there loomed two gigantic, impossibly huge masses spinning at great speed, looking for all the world like cyclones. One was inverted over the other, forming an hourglass shape, but where the spouts should have touched there was instead incredible rays of power shooting out in all directions... I stared at the spectacle before me in disbelief...

As I stared, I came to recognize my former Phyllis self in the midupperleft of the top cyclone. Even though only a speck, I could see my Phyllis clearly, and superimposed over her were all her past lives and all her future lives happening at the same time in the same place as her present life. Everything was happening at once! Around Phyllis was everyone else she had known and around them many others... The same phenomenon was happening to each and all. Past, present, and future were not separated but, instead, interpenetrated like a multiple hologram combined with its own reflection.

The only physical movement anyone or anything made was to contract and expand. There was no up or down, right or left, forward or backward. There was only in and out, like breathing, like the universe and all creation were breathing - inhale/exhale, contraction/expansion, in/out, off/on."

The third impressive description of a timeless state is taken from ref. 131. Although slightly out of date, the book can still be regarded as a landmark treatise on various aspects of altered states of consciousness. The account in question is from the paper by Pahnke and Richards (ref. 131, p. 403):

"From the perspective of the Timeless, I could see my life in retrospect and prospect. It was as if it had all been lived through before, as if we had all been here before, and would be here again. There was a strong pre-ordained feeling about this. I began to see a bit into the future. I understood that I should go back and try to work through some unresolved problems in my relationships with others, and that there would be considerable suffering ahead. My own death was also dimly sensed and strangely accepted. I saw the unbroken continuity of my past with my future, which was not contradicted by the feeling that this present experience

would remain with me and bring about deep changes. The fact that all was pre-ordained did not contradict living in freedom, fighting for truth and against evil..."

4.3 Past-Present-Past Configuration

Our next discussion is devoted to a past-present-past mode, Fig. 3d. A comparison of the latter figure with Fig. 3a shows that this mode can simply be looked upon as an anomalous arrow lacking the domain of the future, the latter being replaced by the second domain of the past. Moreover, it is also a particular case of the 'fluxless' time; for (see Figs. 3c-e) it represents a borderline between two kinds of the a-w-a configurations, and these have principal arrows of the opposite polarities (compare Figs. 3c and e). A few first-hand accounts given below illustrate sufficiently well this past-prevailing mode. A couple of them are picked up from a very intriguing paper by Franz Fischer²⁹, which, in our opinion, collects the best descriptions of the schizophrenic's disturbed sense of time. Both the experiences belong to one and the same patient. On one occasion, the patient reports (ref. 29, pp. 560-1):

"Das Denken ist anders, es hat keinen Stil mehr. Was ist mit der Zukunft? Man kann sie nicht erreichen. Man kann erzählen von der Gegenwart und von der Vergangenheit, vorstellen kann man sich das nicht mehr. Man kann es sich nicht ausmalen, und man muβ zurücksinken. Alles ist wie ein Fragezeichen...

Es zieht mich zurück, ja, wohin? Da, wo es herkommt, dorthin, wo es früher war. In die Vergangenheit geht es hinein. Das ist so ein Gefühl, als ob man zurückfallen müsse. Das ist das Verschwinderische, das Vergehen. Die Zeit schlupft in die Vergangenheit hinein, die Mauern sind gefallen. Früher stand Alles fest. Es ist wie zum Greifen, als ob man es wieder herziehen müsse. Ist das die Zeit? Weit hergerückt!

Morgens beim Aufwachen, ja wie soll ich sagen, das Verschwinderische ist dann wieder da; da setzt es mir ganz besonders zu! Ob ich wei β , wo ich bin? Ja schon. Aber das Verschwinderische, da β keine Zeit da ist, und wo man mit der Zeit anpacken soll, und wann gestern war! Da langt es innen in mir aus, immer weiter zurück, - aber wohin? Die Zeit ist im Abbruch."

In another situation (ref. 29, p. 563), she puts it as:

"Ich bleibe stehen; ich bin zurückgeworfen in die Vergangenheit durch Worte, die man im Saal spricht. Aber das ist doch Alles selbstverständlich, das muβ so sein! Es gibt keine Gegenwart mehr, sondern nur ein Zurückbezogensein, das ist mehr als ein Gefühl, es geht durch und durch...

Gibt es denn eine Zukunft? Früher hat es für mich eine Zukunft gegeben, jetzt schrumpft sie immer mehr zusammen. Die Vergangenheit ist so aufdringlich; sie wirft sich über mich; sie zieht mich zurück...

Ich bin wie ein brennender Pfeil, den man nach vorn wirft, dann bleibt er stecken, fällt zurück und löscht aus wie in einem luftleeren Raum. Er wird zurückgeworfen. Damit will ich

sagen, $da\beta$ es keine Zukunft gibt, und $da\beta$ ich zurückgeworfen werde. Fremde Gedanken kommen herein und treiben mich in die Vergangenheit ab. Es ist furchtbar. Es geht durch und durch!..."

An almost 'textbook' example has been borrowed from⁶²; in this case, a patient suffers an endogenous depression (ref. 62, p. 229):

"Ich kann nichts mehr voraussehen, als wenn es keine Zukunft mehr gäbe. Früher habe ich mir immer überlegt, was ich nächste Woche tue und was ich mir im Sommer kaufe; jetzt kann ich das nicht mehr, ich meine immer, es hört jetzt alles auf und morgen ist überhaupt nichts mehr. Ich komme mir vor wie eine Puppe, die leblos ist und der erst wieder neues Leben eingehaucht werden muβ. An die Zukunft kann ich nicht denken - ich kann nicht vorausdenken." (Und wie steht es mir der Vergangenheit?) "Ja, an die denke ich sehr stark sogar. Es ist mir sehr arg, daβ die schönen Stunden so vorbei sind." (Und die Gegenwart?) "An die denke ich weniger, die nehme ich halt so hin, der Tag vergeht. Ich studiere halt immer, wo das alles herkommt." (Wie geht es mit dem Denken?) "Jetzt kann ich schon denken, bloβ voraus, das geht einfach nicht, es ist einfach, als wenn es abgeschnitten wäre."

To conclude this subsection, we introduce one more experience where the existence of the two domains of the past is explicitly mentioned (ref. 90, pp. 112-3):

"Every day is a thousands of years, yet the days behind me all collapse into nothing, like a pack of cards. Each day is so long that a normal human being can't imagine it. Each moment is the same way - long. Nothing within this time world has any meaning for me, which is why the time is so long... There are no natural divisions...

I have two pasts, the past before I lost my feelings, which my mind struggles to recapture, and the undifferentiated past of my "time world", which does not interest me. The two pasts are separated by the barrier known as "loss of feelings." The past, or before loss of feelings, is steadily fading in my memory, and since my "new past" is meaningless I don't remember it; so, soon, I will have no past whatsoever, and no future; I will exist only in the endless present..."

4.4 Past-Related Arrow Within Arrow

All the anomalous temporal patterns so far dealt with are characterized by the absence of the flux of time. The remaining two configurations to be discussed, viz. the a-w-a structures, show the contrary. Not only are they characterized by the flux of time, but each of them exhibits a couple of flows - principal and subsidiary, the two pointing against each other (see Figs. 3c, e). Let us examine first the *past*-related mode. As the principal arrow of the latter has the opposite polarity than the reference arrow of our ordinary state (Sec. 3), the psychotic subject experiencing such a configuration will feel that time moves *backward*. Moreover, as the present of the subsidiary arrow is located in what would otherwise be the domain of the past of the reference arrow, the subject will also report to have a direct

access to his *past* events, re-living them in the ordinary sequence (as the subsidiary arrow has the same polarity as the reference arrow, compare Fig. 3c with Fig. 3a). From all the 'psycho-time-related' references^{9–133} we have had a possibility to look into, no account draws a portrait of the essential properties of this mode better than that found in ref. 29 (p. 556):

"Gestern am Mittag, als das Essen aufgetragen wurde, schaute ich zur Uhr: Warum sollte man das auch nicht tun, aber da war etwas Besonderes dabei. Denn die Uhr konnte auch nicht mehr helfen und hatte mir nichts mehr zu sagen. Wie sollte ich mich mit der Uhr zusammenbringen? Ich fühlte mich wie zurückversetzt, als ob Etwas das vergangen sei, gewissermaßen auf mich zukäme, als ob ich selbst einen Weg zurücklegte. Es war mir so, als sei um 111/2 Uhr wieder 11 Uhr, aber nicht nur die Zeit war wieder dieselbe und kam zurück, sondern auch, was in dieser Zeit sich für mich zugetragen hatte. Aber eigentlich ist das Alles viel tiefer, als daβ ich es sagen kann. Mitten drin, als ob es nicht dazugehöre, kam Etwas Anderes herauf. Es war mit einem Male nicht nur wieder 11 Uhr, nein, eine längst vergangene Zeit war da, und darin - habe ich Ihnen das schon erzählt, vom Samenkorn in der großen, harten Schale? So war das auch wieder: Mitten drin in der Zeit aus die Vergangenheit komme ich auf mich selbst zu. Wie entsetzlich war das. Ich dachte mir, vielleicht ist die Uhr zurückgestellt, da haben die Pfleger einen dummen Streich mit der Uhr gespielt. Ich beschäftigte mich damit, die Zeit als normal zu betrachten, aber es gelang mir nicht; und dann kam ein Gefühl einer furchtbaren Erwartung für mich, ich könne in das Vergangene hineingezogen werden, oder das Vergangene könne über mich kommen. Es war unheimlich, daβ man so mit der Zeit spielt, irgendwie dämonisch..."

Here, we use a sans serif type style to mark those parts of the narrative that indirectly point out the existence of the subsidiary arrow. A psychotic patient of Laing (ref. 133, p. 148) gives a very succinct description:

"... ebbi la sensazione che il tempo stesse andando all'indietro; sentii che il tempo procedeva al contrario, ebbi proprio questa straordinaria sensazione, già... la sensazione più importante in quel momento era quella, del tempo alla rovescia... La sensazione era tanto viva che guardai l'orologio e non so come ebbi l'impressione che l'orologio mi confermasse quest'idea, benchè non riuscissi a scorgere il movimento delle lancette..."

A strikingly similar portrayal of time-reversal is also provided by a(nother) depressive patient of Kloos (ref. 62, p. 237):

"Bei meinem plötzlichen Zusammenbrechen war es so in mir, daβ die Zeit abgelaufen ist. Ich hatte nach dem Krankenlager nach 3 Wochen das Gefühl, die Uhrzeiger laufen leer, sie haben keinen Halt. Das war mir auf einmal so. Ich fand sozusagen keinen Halt mehr an der Uhr und auch am Leben, ich hatte einen furchtbaren seelischen Zusammenbruch. Wie ich gerade auf die Uhr aufmerksam wurde, weiß ich selber nicht. Zugleich hatte ich das Gefühl, daß die Uhrzeiger nach rückwärts laufen... Das bleibt ein einziges Stück sozusagen, und das bleibt stehen. Ich konnte nicht glauben, daß die Zeit wirklich fortschreitet, deswegen meinte ich wohl, die Uhrzeiger hätten keinen Halt und liefen leer... Wie ich immer arbeitete und arbeitete und mich plagte und nichts zustande brachte, hatte ich eben das Gefühl, es geht mit uns alles zurück... Ich kam einfach in meiner Krankheit nicht mehr mit, und da hatte ich

einfach diesen Wahn in mir, da β die Zeit nach rückwärts läuft... Ich habe mich nicht mehr ausgekannt und hab immer gedacht, ich verlier den Verstand. Ich glaubte immer, die Uhrzeiger laufen verkehrt, sie sind soviel wie zwecklos. Ich stand gerade auf vom Krankenlager und sah auf die Uhr - da fiels mir auf einmal ein: ja was ist denn das, die Zeit laufen ja verkehrt?!... Ich sah natürlich, da β die Zeiger vorwärts gingen, aber wie ich es nicht glauben konnte, meinte ich immer, die Uhr geht in Wirklichkeit doch zurück..."

4.5 Future-Related Arrow Within Arrow

To move backward in time is quite a fascinating thing, but going through time into the future is even more astonishing; for the barrier between the present and the future seems to be far more opaque to break than that separating the present from the past. And yet in our last mode, viz. in the future-related arrow within arrow (Fig. 3e), this boundary is fully transparent. Really, comparing Fig. 3e with Fig. 3a we see that the present of the subsidiary arrow of Fig. 3e corresponds to an event of the *future* of the reference arrow of Fig. 3a. So, the subject can now glimpse some part of his/her own future; this will be done in a reverse order as, in this case, the polarity of the subsidiary arrow is opposite to that of the reference arrow. In this particular state, there is thus nothing mysterious about such phenomena as clairvoyance (the power of seeing future events in the mind), precognition (the knowledge that something will happen before it does), and/or déjà-vu experiences (the feelings that one has previously experienced exactly the same thing as one is experiencing now). To illustrate the case, it is worth introducing the following premonitory dream of J. W. Dunne (ref. 132, p. 165):

"I seemed to be standing on high ground. Here and there in this were little fissures, and from these jets of vapor were spouting upward. In my dream I recognized the place as an island of which I had dreamed before - an island which was in imminent peril from a volcano. And, when I saw the vapor spouting from the ground, I gasped: 'It's the island! Good Lord, the whole thing is about to blow up!'... All through the dream the *number* of the people in danger obsessed my mind. I repeated it to everyone I met, and, at the moment of waking, I was shouting 'Listen! Four thousand people will be killed unless...'

I am not certain when we received our next batch of papers but when they did come the Daily Telegraph was amongst them, and, on opening the centre sheet, this is what met my eyes: *Volcano Disaster in Martinique. Probable Loss of Over 40,000 Lives.*"

As for the phenomenon of clairvoyance, it can be well pinpointed by the case of a famous Dutch clairvoyant Gerard Croiset (ref. 134, pp. 11-12):

The great gift which the Utrecht paragnost possesses is being able to break through the time and space barriers. Telepathy and clairvoyance occur at the present. But Croiset's precognition powers sweep across time, enabling him to glimpse the future. Although most of us periodically reveal these abilities, Gerard Croiset regularly sees things not visible to the

normal human eye or mind. His images aren't confined to the limits of conventional time and space. Croiset not only describes events occurring in the present hundreds of miles away,... but constantly dips into the past and future as if into an endless sea. "The past, present, and the future are difficult to separate for me", he says.

Though Professor Tenhaeff has spent nearly half a century studying these phenomena, even he is unsure what causes these impressions. But he is confined that someday soon we shall have the answer. "It very likely has something to do with a new concept of time", he explains. "Today not only our old concept of space requires revising, but even our ideas about time."

The future may also be accessible for the subject experiencing the so-called near-death experience (NDE). The first to draw attention to this fact seems to have been Kenneth Ring (ref. 135, p. 183):

"... the material I have collected that bears upon a remarkable and previously scarcely noted precognitive feature of the NDE I have called *the personal flashforward* (PF). If these experiences are what they purport to be, they not only have extremely profound implications for our understanding of the nature of time but also possibly for the future of our planet...

Personal flashforwards usually occur within the context of an assessment of one's life during an NDE (i.e. during a life review and preview), although occasionally the PF is experienced as a *subsequent* vision. When it takes place while the individual is undergoing an NDE, it is typically described as an image or vision of the future. It is as though the individual sees something of the whole trajectory of his life, not just past events, as some previous accounts have implied. The understanding I have of these PFs is that to the NDEr they represent events of a conditional future - i.e., if he chooses to return to life, then these events will ensue. In this sense, from the standpoint of an NDEr, a PF may be likened to a "memory" of future events..."

5. FINITE FIELDS AND 'DISCRETE', 'DISORDERED' SUBJECTIVE TIME

The classification of the 'pathological' time configurations given in the preceding section relies on the fact that we change the position of d-line in the projective plane whereas the ground field of the latter remains intact, being identical to that of the real numbers. In this section we shall briefly discuss a complementary case. That is, we shall keep d-line in a fixed position, and replace the real numbers by some other field(s). In a slightly different context, this case has been dealt with in detail in refs. 4-6, where the interested reader is referred to for the corresponding mathematical background. Here, we shall introduce only those elements of the formalism that are necessary for grasping the idea behind, focussing on the case when d-line is in a general position, i.e. when it incorporates neither the base point (B, B'), nor the singular point S.

To this end, it is necessary to generalize our concepts of the past, present and future, given in Sect. 3 for the case of the field of real numbers R, to an arbitrary ground field F. This is quite easy if we take into account factorization properties of the polynomial P(x) represented by eqn. (18). Obviously, if d-line is in general position, then both a and b in eqn. (17) are non-zero. As these two parameters are now kept fixed, the factorization properties of the polynomial P(x) depend solely on the value of the parameter v. From what has been said in the first paragraph of Sect. 3 it follows that if this polynomial splits into two distinct factors, two coinciding (repeated) factors, or does not factor at all, then the conic $\mathcal{L}(v)$, eqn. (14), is (stands for) a hyperbola (an event of the past), parabola (the present moment), or an ellipse (an event of the future), respectively. And this is the definition of the three temporal domains that can clearly be extended to any field.

Let us now take an event of the R-future, for example the one defined by $\vartheta = \vartheta_0 = 2$ for $\kappa = 1$, a = 19 and b = 3; then

$$P(x) = x^2 + 16x + 76. (21)$$

Next, switch to a finite (Galois) field, say to F_7 - the field of residues modulo 7. Since in this field +16=-5 and 76=6, the above polynomial factors as

$$P(x)=x^2+16x+76=x^2-5x+6=(x-2)(x-3)$$
(22)

which implies that the event in question belongs to the F_7 -past! Moreover, as

$$\widetilde{\vartheta}_0 = \vartheta + 7l, \ l \in N,$$
 (23)

represents one and the same conic in F_7 , we see that this past moment of the F_7 -temporal corresponds to an *infinite* sequence of events of the R-time: the events with non-negative values of l being in the R-future, while those for which l is negative falling into the R-past. Hence, what we perceive as an ordered sequence of events in our ordinary state, i.e. the state backed by the reals³, will appear as a disordered, chaotic set in an 'extraordinary' state that corresponds to a modular field⁴⁻⁶.

This is another fascinating aspect of our theory that finds its counterpart in the realm of psychopathology. Christopher Mayhew, a (former) member of the British parliament, volunteered to participate in a series of laboratory experiments with the drug called mescaline. He reported his experience in the article "An Excursion Out of Time" in the London *Observer* on

October 28, 1956, and the way how he perceived time under influence of this drug conforms nicely to the above-outlined pattern (ref. 89, p. 295):

"For half an hour nothing happened. Then I began feeling sick; and various nerves and muscles started twitching unpleasantly. Then, as this wore off, my body became more or less anaesthetized, and I became 'de-personalized', i.e., I felt completely detached from my body and the world...

This experience alone would have fully justified the entire experiment for me..., but at about 1.30 all interest in these visual phenomena was abruptly swept aside when I found that time was behaving even more strangely than color. Though perfectly rational and wideawake... I was not experiencing events in the normal sequence of time. I was experiencing the events of 3.30 before the events of 3.0; the events of 2.0 after the events of 2.45, and so on. Several events I experienced with an equal degree of reality more than once.

I am not suggesting, of course, that the events of 3.30 happened before the events of 3.0, or that any event happened more than once. All I am saying is that I experienced them, not in the familiar sequence of clock time, but in a different, apparently capricious sequence which was outside my control.

By 'I' in this context I mean, of course, my disembodied self, and by 'experienced' I mean learned by a special kind of awareness which seemed to comprehend yet be different from seeing, hearing, etc... I count this experience, which occurred when, as I say, I was wide awake and intelligent, sitting in my own armchair at home, as the most astounding and thought-provoking of my life."

That mescaline intoxication seems to favor this kind of time's manifold is also evidenced by the following interlude (ref. 77, p. 148):

"... Im Treppensteigen plötzlich wie festgenagelte Momentaufnahme, die momentane Stellung von Dr. M., Dr. St. und mir im Raum aufgefallen. Dies wiederholte sich auf verschiedenen Treppenstufen. Oben angekommen schien keine Kontinuität der Zeit vorhanden gewesen zu sein, ganzer Vorgang aufgelöst in unzusammenhängende Einzelsituationen, die nachträglich wie beim Betrachten eines Filmstreifens aktiv daran arbeitend verbunden werden konnten, die aber sowohl im Erleben wie in der unmittelbaren Reproduktion des Geschehens danach durchaus den Charakter des Unabhängigen, Unverbundenen trugen. Ein seltsames Nebeneinander, nicht Hintereinander, sie haben keine Stelle in der Zeit, Zeit hat hier keinen Sinn"

'Modular' time patterns might not also be foreign to other mental psychoses, like, e.g., those suffered by shizophrenics (ref. 78, pp. 104, 106-7):

...Die Unterscheidung von Gegenwart und Zukunft ist ja nicht etwa aufgehoben, da sie doch von beiden Dimensionen immer noch spricht, aber die Grenzen zwischen real Gegenwärtigem und blo β möglichem, irreal Zukünftigem werden schwankend und überspringbar. Die beiden Dimensionen verschachteln und überlagern sich ohne stetigen Übergang. Das Zukünftige schlägt ins Gegenwärtige hinein und umgekehrt, und das Erleben bekommt zwischen ihnen etwas flackernd Zwielichtiges, welches sich in seiner Diskontinuität radikal unterscheidet von der Art, wie etwa der Gesunde in Wachträumen und ähnlichem die Zukunft antizipiert...

Auch die Grenze von der Gegenwart zur Vergangenheit hin ist schwankend, und dabei wird Vergangenes auf ganz andere Weise in das Gegenwartsgeschehen einbezogen und eingeschmolzen, als auch normalerweise in der Gegenwart das Vergangene mitenthalten ist. Es ist eine Art von Raffung da; die Gegenwart setzt sich nicht mehr in kontinuierlich-stetigem $Flu\beta$ von der Vergangenheit ab, sondern das Gegenwärtige ist zugleich von etwas Vergangenem nicht wie beim Normalen nur durchdrungen, sondern überlagert und überlappt.

In den völlig auf die Zukunft bezogenen... Halluzinationen sahen wir den Schwerpunkt nun eindeutig in das Zukünftige verlagert, welches sinnlich und leibhaftig in die Gegenwart einbrach, wobei sich die beiden zeitlichen Erlebnisebenen ohne kontinuierlichen Übergang verquickten und verschachtelten. Durch das Phänomen des zur Realität gewordenen "déjà-vu" erwiesen sich die Grenzen zur Vergangenheit hin ebenfalls als schwankend: eine der übrigen Vergangenheit nicht integrierte Vergangenheitsdimension drängte sich in diskontinuierlicher Weise ins Gegenwartserleben ein...

6. DISCUSSION AND CONCLUSION

The findings and results of the previous sections provide us with further evidence, much stronger and more provoking than that already given in refs. 4-8, that not only are the manifestations of mental or psychological time so diverse and unusual that they totally fail to conform to the generally adopted picture of the macroscopic physical world, but that there also does exist a unique mathematical framework which, at least qualitatively, underlies and unifies their seemingly bizarre properties. Hence, any attempt to disregard these psychopathological temporal constructs as pure hallucinatory phenomena would simultaneously cast a doubtful eye on the very role of mathematics in our understanding of Nature. To the contrary, it is just mathematics (algebraic projective geometry here) that plainly tells us that it is far more natural to expect all these 'unusual' perceptions of time to be simply as real as our ordinary ('normal') one - this being one of the principal messages the paper should convey to the reader. We are, however, fully aware that this point of view is very likely to meet with scepticism, and even with fierce opposition, from the side of 'hard-line' instructional scientists, as envisaged - on a slightly different ground - by St. John (ref. 98, pp. 161-2):

"...the traditional scientific mind refuses to recognize even the possibility of what I am concerned with - the actual nature and quality of the subjective, mystical experience. Materialist scientists of course have no difficulty in recognizing intangible phenomena such as magnetism or cosmic rays passing through a cloud chamber, because they can measure them. Anything that cannot be measured and enumerated doesn't, for the practical purposes of science, exist... By not being able to observe beyond the physical correlates, to reach the internal mental states themselves, science fails to get to grips with what really matters, to recognize that for the individual experiencing them these mental states are much more than a

by-product and quite, quite different from a series of (maybe related) biochemical and neurophysiological happenings..."

The main objection will probably be raised against an *anecdotal* character inherent in describing the variety of time's multifaceted phenomena given in the last section. However, we believe this to be the inevitable and the only possible way to follow in pursuit of the research on time in its qualitative aspect, bringing profit to both psychology and physics. As very well argued by Shallis (ref. 132, p. 153):

"Quality and quantity are somewhat like the ingredients of descriptive and instructional science, respectively. Because the two approaches are so different the sorts of evidence employed in each will also differ. Whereas the instructional approach requires, indeed demands, rigorous, quantitative and reproducible evidence, the descriptive attitude, which often deals with the unique and individual, is mainly anecdotal. This does not mean it is uncritical or sloppy, but in trying to find the whole truth everything must be taken into account. If some evidence turns out to be false, that too is part of the picture. In instructional science anecdotal evidence, *even if true*, can be dismissed as unquantifiable and impossible to assess.

The techniques of instructional science cannot handle individual experience or admit to the quality of time. Descriptive science can;..."

We are firmly convinced that anything that shows a definite mathematical structure, whatever bizarre and contraintuitive it may appear, deserves effort and ingenuity to be thoroughly explored and examined, all the more that (ref. 132, pp. 153-4):

"... the fact that the experience of time is not quantifiable puts it into arena of human perceptions that are at once richer and more meaningful than are those things that are merely quantifiable... The lack of quantification of temporal experiences is not something that should stand them in low stead, to be dismissed as nothing more than fleeting perceptions or as merely anecdotal; rather that lack should be seen as their strength. It is because the experience of time is not quantifiable and not subject to numerical comparison that makes it something of quality, something containing the essence of being..."

Last but not least, since the complexity of time is, in our opinion, very intimately tied up with that of consciousness, and the latter seems to be a basic characteristic of the world, the above-discussed geometrical patterns of psycho-(space)time may well turn out to be a decisive clue to breaking the enigma of time; for ref. 136 (p. 1):

"...the creation most suited to probing time's subtle secrets is the human mind, which could, were its cranial computer to be suitably programmed, produce more astounding revelations."

And if nothing else, they should at least stir up some controversy, and perhaps encourage some research along what seems to be an extremely promising line of scientific inquiry.

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